# 4

# INTERFERENCE

*"Light + light does not always give more light, but may in certain circumstances give darkness."*

– **Max Born**

The wave theory of light was first put forward by Huygen in 1678. On the basis of his theory, Huygen explained satisfactorily the phenomena of reflection, refraction and total internal reflections (TIR). According to his theory, a luminous body is a source of disturbance in a hypothetical medium, called *ether*. The medium pervades all space. The disturbance from the source is propagated in the form of waves through space and energy is distributed equally in all directions. When these waves carrying energy are incident on the eye, the optic nerves are excited and the sensation of vision is produced. Huygen's theory predicted that the velocity of light in medium shall be less than the velocity of light in free space, which is just converse of the prediction made from Newton's corpuscular theory. The experimental evidence for the wave theory in Huygen time was very small. In 1801, however Thomas Young obtained evidence that light could produce wave effects. Very shortly, diffraction was explained by Fresnel and Fraunhofer, while the transverse nature of light was explained by polarisation experiments. The subject of interference, diffraction and polarisation is called *physical optics* or *wave optics* and should be explained by using wave theory of light.

The phenomenon of interference of light has proved the validity of the wave theory of light. When two or more light waves of the same frequency travel in approximately the same direction and have a phase difference that remains constant with time, the resultant intensity of light is not distributed uniformly in space. The non-uniform distribution of the light intensity due to the superposition of these waves is called *"interference"*. At some points the intensity is a maximum and the interference at these points is called *"constructive interference"*. At some points the intensity is a minimum (possibly even zero), and the interference at these points is called *"destructive interference"*. Usually when two or more light waves are made to interfere, we get alternate dark and bright bands of a regular or irregular shape. These bands are called *interference fringes*.

The interference of light waves is of two types :

| ***Interference produced by the division of wavefront*** | ***Interference produced by the division of amplitude*** |
|---|---|
| In this case the incident wavefront is divided into two parts by making use of phenomenon of reflection, refraction or diffraction. Two parts of the wavefront travels unequal distances and reunite to produce interference fringes. | In this class, the amplitude of incident light is divided into two parts either by parallel reflection or refraction. These light waves with divided amplitude reinforce after travelling different distances and produce interference fringes. |
| *Examples* : Young's double slit experiment, Fresnel's biprism. | *Examples* : Newton's ring, Michelson's interferometer. |

## 4.1 WAVEFRONT AND RAYS

### 4.1.1 Wavefront

Suppose a stone is thrown on the surface of still water. Circular patterns of alternate crests and troughs begin to spread out from the point of impact. Clearly, all the particles lying on a crest are in the position of their maximum upward displacement and hence in the same phase. Similarly, all particles lying on a trough are in the position of their maximum downward displacement and therefore, in the same phase. The locus of all such points oscillating in the same phase is called a *wavefront*. Thus every crest or a trough is a wavefront.

> *"A **wavefront** is defined as the continuous locus of all such particles of the medium which are vibrating in the same phase at any instant."*

Thus a wavefront is a surface of constant phase. The speed with which the wavefront moves towards from the source is called the *phase speed*.

**Types of Wavefronts**

The geometrical shape of a wavefront depends on the source of disturbance. Some of the common shapes are :

(*i*) ***Spherical wavefront.*** In case of waves travelling in all directions from a point source, the wavefronts are spherical in shape. This is because all such points which are equidistant from the point source will lie on a sphere as shown in Fig. 4.1 and the disturbance starting from the source *S* will reach all these points simultaneously.

(*ii*) ***Cylindrical wavefront.*** When the source of light is linear in shape, such as a fine rectangular slit, the wavefront is cylindrical in shape. This is because the locus of all such points which are equidistant from the linear source will be a cylinder as shown in Fig. 4.2.

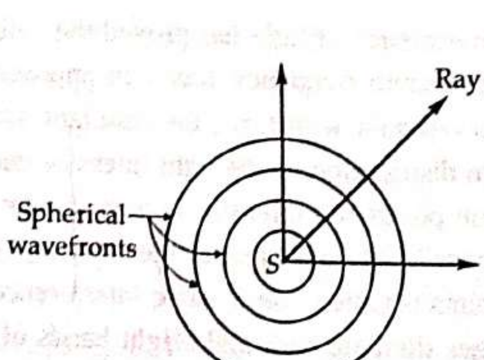


**Fig. 4.1** Spherical wavefront.

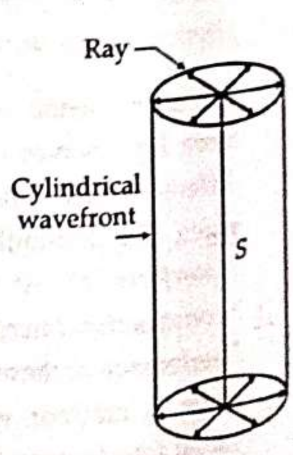


**Fig. 4.2** Cylindrical wavefront.

(iii) ***Plane wavefront.*** As a spherical or cylindrical wavefront advances, its curvature decreases progressively. So a small portion of such a wavefront at a large distance from the source will be a plane wavefront as shown in Fig. 4.3.

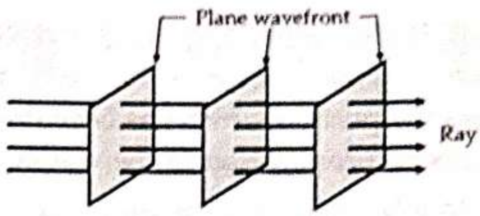


**Fig. 4.3** Plane wavefront.

### 4.1.2 Ray of Light

It is seen that whatever is the shape of a wavefront, the disturbance travels outwards along straight lines emerging from the source *i.e.*, the energy of a wave travels in a direction perpendicular to the wavefront.

*"An arrow drawn perpendicular to a wavefront in the direction of propagation of a wave is called a* ***ray.****"*

A ray of light represents the path along which light travels. If we measure the separation between a pair of wavefronts along any ray, it is found to be a constant.

This illustrates *two* general principles :

1. Rays are perpendicular to wavefronts.
2. The time taken for light to travel from one wavefront to another is the same along any ray.

In case of a plane wavefront, the rays are parallel as shown in Fig. 4.4.

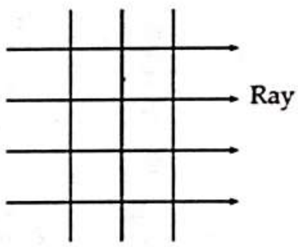


**Fig. 4.4** Rays in case of plane wavefront.

A group of parallel rays is called a *beam of light*. In case of a spherical wavefront, the rays either converge to a point [Fig. 4.5] or diverge from a point (Fig. 4.6).

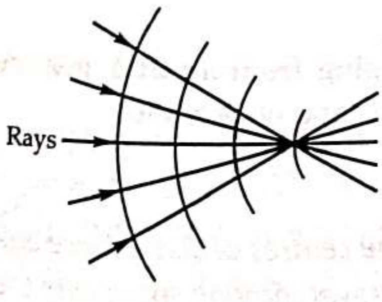


**Fig. 4.5** Rays in converging spherical wavefront.

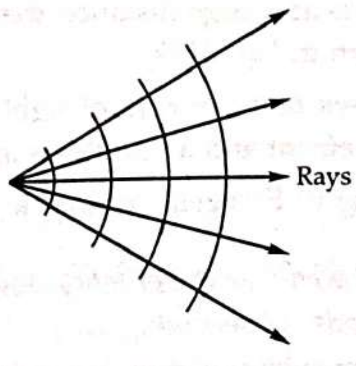


**Fig. 4.6** Rays in diverging spherical wavefront.

## 4.2 HUYGEN'S PRINCIPLE OF SECONDARY WAVELETS

According to Huygen : A source of light in a homogeneous hypothetical medium called the ether sends out waves in all directions. These waves carry energy which is transmitted in all directions.

If $S$ is the source of light, it sends energy in the form of waves in all directions. After an interval of time $t$, all the particles of the medium lying on the surface $AB$ are vibrating in the same phase. $AB$ is thus the portion which has been drawn with $S$ as centre and radius $SA$ equal to $ct$ where $c$ is the velocity of propagation of the waves. The surface $AB$ is called the *primary wavefront*.

In a homogenous medium, for a point source of light, if distance is small, the wavefront is a sphere, as shown in Fig. 4.7(*a*).

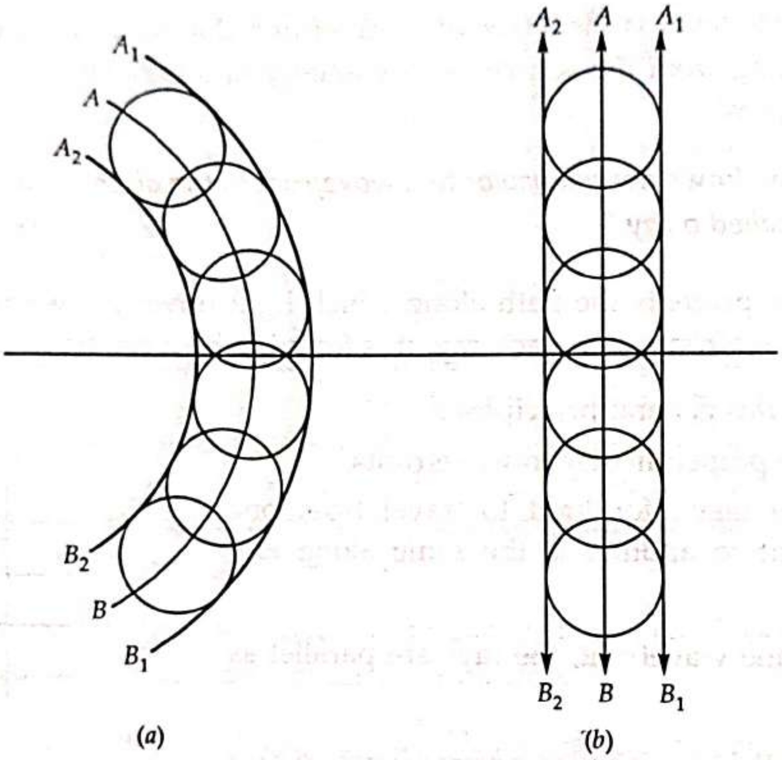


**Fig. 4.7** Huygen's principle.

If source is at a large distance, then a small portion of the wavefront can be considered to be plane as shown in Fig. 4.7(*b*).

This shows that the rays of light, converging to or diverging from a point, give rise to a spherical wavefront and a parallel beam of light gives rise to a plane wavefront.

According to **Huygen's principle,**

> ***All points on the primary wavefront are considered to be centres of disturbance and sends out secondary waves in the all directions which travel through space with the same velocity in an isotropic medium.***

After a given interval of time the envelope of all these secondary waves gives rise to the secondary wavefront.

To find the position of new wavefront after $t$ seconds, take a number of points on $AB$ With each point in turn as centre and radius $ct$, draw spheres. These spheres represent the secondary waves starting from these points respectively. A surface $A_1B_1$ touching all these spheres in the forward direction is the new wavefront.

## Division of Wavefront

### 4.3 YOUNG'S DOUBLE SLIT EXPERIMENT

Thomas Young[1], in 1801, demonstrated the phenomenon of interference of light. The arrangement is shown in Fig. 4.8.

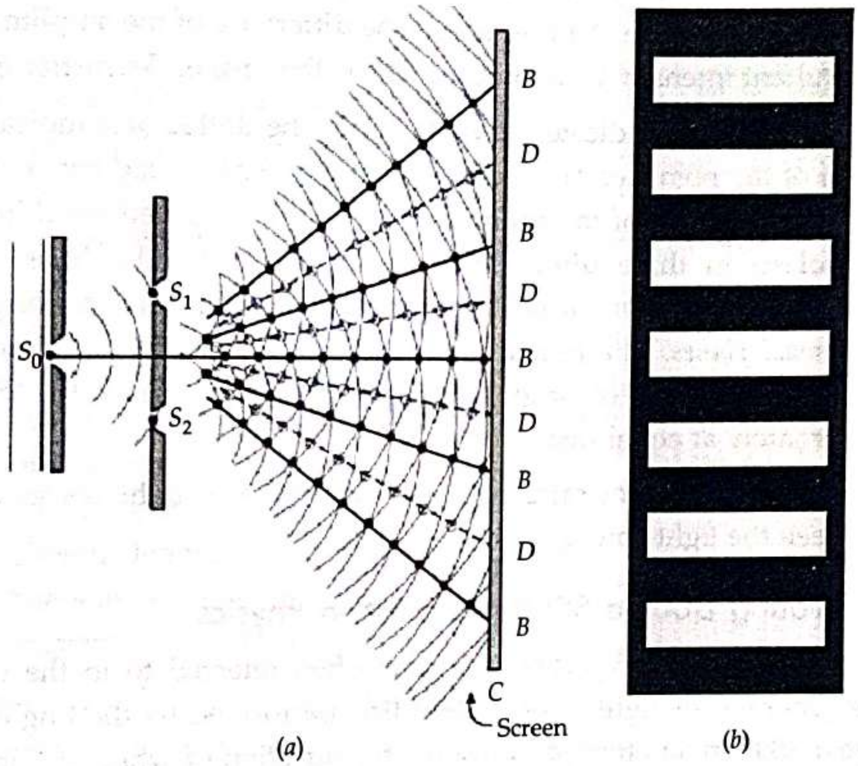


**Fig. 4.8** Young's double slit experiment.

Sunlight was first allowed to pass through a pinhole *S*, and then through two pinholes $S_1$ and $S_2$ placed at a considerable distance away from *S*. Finally the light was received on a screen. The two sets of spherical waves emerging from $S_1$ and $S_2$ interfered with each other and a few coloured fringes of varying intensity were seen on the screen.

As an improvement of the original arrangement, the pinholes $S_1$ and $S_2$ are replaced by narrow slits and sunlight by monochromatic light. The interfering waves are then cylindrical, and a number of alternate bright and dark fringes running parallel to the length of the slits are observed on the screen.

1. Thomas Young read fluently at the age of 2 ; by 4, he had read Bible twice ; by 14, he knew eight languages. In his adult life, he was physician and scientist, contributing to an understanding of fluids, work, and energy and the elastic properties of material. He was first person to make progress in deciphering Egyptian hieroglyphics. No doubt about it – Thomas Young was bright guy !

*Explanation.* The formation of bright and dark fringes on the screen can be explained on the basis of wave theory of light. The cylindrical wavefront starting from $S$ falls on $S_1$ and $S_2$. According to Huygen's principle, $S_1$ and $S_2$ become the centres of secondary wavelets. The two cylindrical wavefronts issued out, one from $S_1$ and other from $S_2$.

Their radii increase as they move away from $S_1$ and $S_2$, so that they superimpose more and more on each other. At points where a crest (or trough) due to one falls on a crest (or trough) due to other, the resultant amplitude is the sum of the amplitudes due to each wave separately. The intensity, which is proportional to the square of the amplitude, at these points is therefore a maximum. This is a case of *constructive interference*. At points where a crest due to one wave falls on a trough due to other, the resultant amplitude is the difference of the amplitudes due to separate waves and the resultant intensity is minimum. This is the case of *destructive interference.*

In Fig. 4.8, the solid arcs indicate the crests while the dotted arcs indicate the troughs. The solid lines are loci of the points of maximum intensity and are called *antinodal lines*. The broken lines are the loci of the points of minimum intensity and are called *nodal lines*. (Actually these lines are hyperbolas.) In three dimensional space, the antinodal lines describe planes of maximum intensity, called the *antinodal planes* while the nodal lines describe planes of minimum intensity, called *nodal planes*. The intersections of these lines on the screen at points $B$ and $D$ respectively give the positions of bright and dark fringes respectively. The bright and dark fringes occur alternately at equal distances.

Young's experiment demonstrates both the diffraction of light waves at the slits and the interference between the light emerging from the slits.

## Importance of Young Double Slit Experiment in Physics

Although the double-slit experiment is now often referred to in the context of quantum mechanics, it is generally thought to have been first performed by the English scientist Thomas Young in the year 1801 in an attempt to resolve the question of whether light was composed of particles (Newton's corpuscular theory), or rather consisted of waves traveling through some ether, just as sound waves travel in air. The interference patterns observed in the experiment seemed to discredit the corpuscular theory, and the wave theory of light remained well accepted until the early 20th century, when evidence began to accumulate that seemed instead to confirm the particle theory of light.

The double-slit experiment, and its variations, then became a classic thought experiment for its clarity in expressing the central puzzles of quantum mechanics.

It was shown experimentally in 1972 that in a Young slit system where only one slit was open at any time, interference was nonetheless observed provided the path difference was such that the detected photon could have come from either slit. The experimental conditions were such that the photon density in the system was much less than unity.

A Young double slit experiment was not performed with anything other than light until 1961, when Clauss Jönsson of the University of Tübingen performed it with electrons and not until 1974 in the form of "one electron at a time", in a laboratory at the University of Milan, by researchers led by Pier Giorgio Merli, of LAMEL-CNR Bologna.

The results of the 1974 experiment were published and even made into a short film, but did not receive wide attention. The experiment was repeated in 1989 by Tonomura *et al.* at Hitachi in Japan. Their equipment was better, reflecting 15 years of advances in electronics and a dedicated development effort by the Hitachi team. Their methodology was more precise and elegant, and their results agreed with the results of Merli's team. Although Tonomura asserted that the Italian experiment had not detected electrons one at a time—a key to demonstrating the wave-particle paradox—single electron detection is clearly visible in the photographs and film taken by Merli and his group.

## 4.4 COHERENCE

If a fixed and predictable phase difference between several light waves traveling in a particular direction be maintained, then we may say the motion is coordinated or *coherence.* The corresponding waves are called *coherent waves* and sources emitting them, the *coherent sources.*

Coherence effects are mainly *two* types : (*i*) Temporal coherence and (*ii*) Spatial coherence.

### 4.4.1 Temporal Coherence

If the phase difference at a single point in the bundle of light waves propagating in space, at the beginning and end of a fixed time interval does not change with time then the waves are said to have *temporal coherence.* The phase difference between any two fixed points $P_1$ and $P_2$ as shown in Fig. 4.9 along any ray will be independent of time but depends on $P_1P_2$ and the coherent length[2] ($l_c$) of light beam, *i.e.,* the distance $P_1P_2 < l_c$. The waves will correlated in their rise and full maintaining a *constant phase difference.*

If $P_1P_2 > l_c$, then the points $P_1$ and $P_2$ would not maintain any phase relationship. In that case many wave trains will span the distance $P_1P_2$. At any instant of time the plane at $P_1$ and $P_2$ will be *time independent*. The degree of correlation of phases is the amount of *longitudinal coherence.*

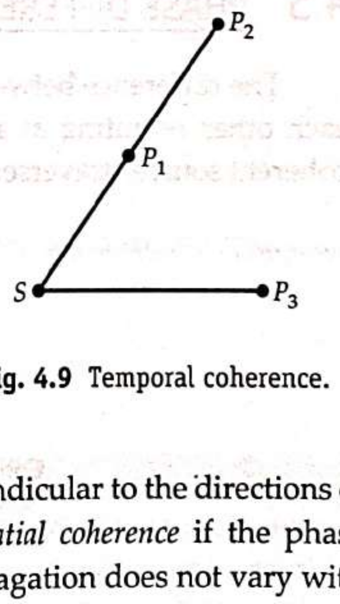

**Fig. 4.9** Temporal coherence.

### 4.4.2 Spatial Coherence

The continuity and uniformity of a light wave in a direction perpendicular to the directions of propagation refers to *spatial coherence.* The move is said to have *spatial coherence* if the phase difference for any two fixed points in a plane normal to the wave propagation does not vary with time. In Fig. 4.9, if $SP_1 = SP_3$, then field points $P_1$ and $P_3$ would have phase as shown in Fig. 4.10. Since the wave produced by an ideal source exhibit spatial coherence as the phases of the waves at any two points, which are equidistant from the source are equal. An extended source, however, exhibits less lateral coherence.

The *degree of contrast* of interference fringes is a measure of the degree of spatial coherence of the source resulting the waves. Spatial coherence is better if the contrast is higher.

2. The coherent length is the product of the number of waves ($N$) and the wavelength $\lambda$, *i.e.,* $l_c = N\lambda$.

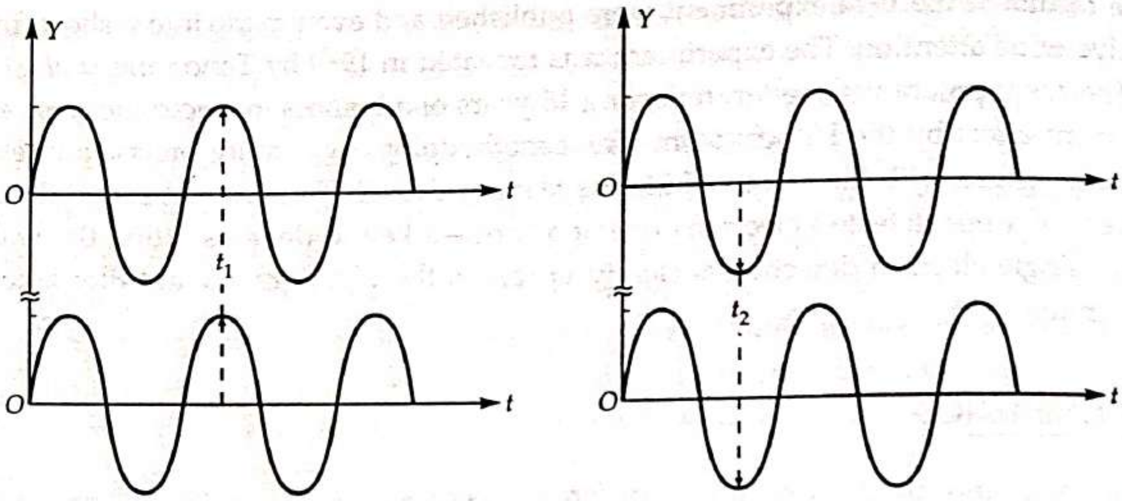


**Fig. 4.10** The spatial coherence between two waves.

### 4.4.3 Coherent and Incoherent Sources

Two or more sources of light, which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them are called *coherent sources.*

Two or more sources of light which do not emit light waves with a constant phase difference are called *incoherent sources.*

## 4.5 PHASE DIFFERENCE AND PATH DIFFERENCE

The difference between optical path of two rays, which are in constant phase difference with each other reuniting at a particular point is known as *path difference.* For example, let the two coherent sources traversed different paths and meet at a particular point $P$ as shown in Fig. 4.11.

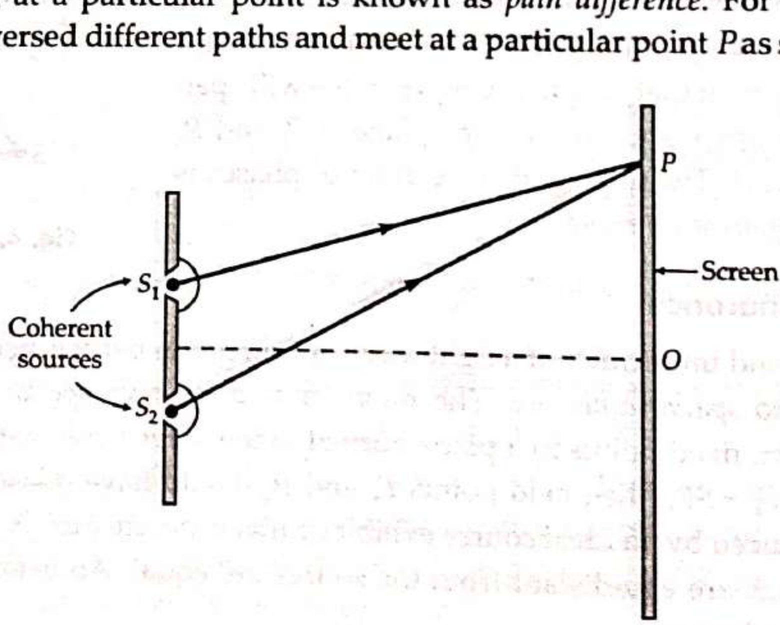


**Fig. 4.11** Path difference between two light waves $S_1P$ and $S_2P$.

Then the path difference is given as

$$\text{Path difference } \Delta = S_2P - S_1P$$

Suppose for a path difference $\lambda$, the phase difference is $\varphi$.

$\because$ For a path difference $\lambda$, the phase difference $= 2\pi$

$\therefore$ For a path difference $x$, the phase difference $= \frac{2\pi}{\lambda} x$

$\therefore$ Phase difference $(\varphi) = \frac{2\pi}{\lambda} x = \frac{2\pi}{\lambda} \times$ Path difference

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$$

## 4.6 CONDITIONS FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE

### 4.6.1 Expression for Intensity at any Point in Interference Pattern

Let us consider a monochromatic source of light $S$ emitting waves of wavelength $\lambda$. $S_1$ and $S_2$ are two similar parallel slits of very close together and equidistant from $S$. [Fig. 4.12]

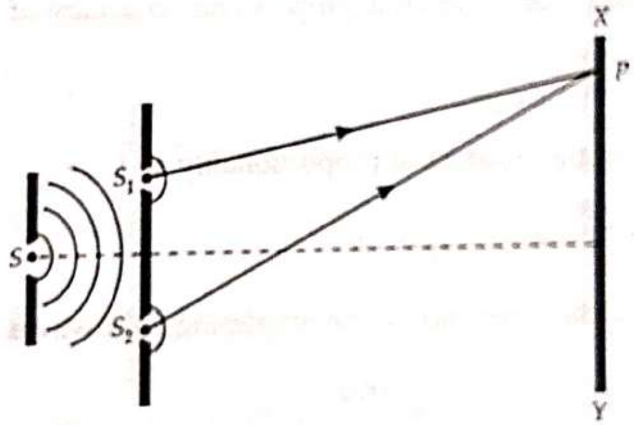


Fig. 4.12 Interference of two waves.

Let $a_1$ and $a_2$ be the amplitudes at $P$ due to the waves from $S_1$ and $S_2$ respectively. The waves reunite after transversing different paths $S_1P$ and $S_2P$.

Let the phase difference between the waves be $\varphi$,

i.e.,
$$\varphi = \frac{2\pi}{\lambda} \times (S_2P - S_1P)$$

If $y_1$ and $y_2$ are displacements of two waves as

$$y_1 = a_1 \sin \omega t \quad \text{...(4.1)}$$

and
$$y_2 = a_2 \sin(\omega t + \varphi) \quad \text{...(4.2)}$$

where $\frac{\omega}{2\pi} = \nu =$ common frequency of two waves.

**NOTE**

By using the principle of superposition, the resultant displacement is equal to the sum of the individual displacements of the two or more waves.

Hence the resultant displacement is

$$Y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin(\omega t + \varphi)$$

$$Y = a_1 \sin \omega t + a_2 \sin \omega t \cos \varphi + a_2 \cos \omega t \sin \varphi$$

$$\Rightarrow \quad Y = \sin \omega t (a_1 + a_2 \cos \varphi) + a_2 \cos \omega t \sin \varphi \quad \text{...(4.3)}$$

Now, let

$$a_1 + a_2 \cos\varphi = R\cos\theta \quad ...(4.4)$$

and

$$a_2 \sin\varphi = R\sin\theta \quad ...(4.5)$$

where $R$ and $\theta$ are new constants ; this gives

$$Y = \sin\omega t\, R\cos\theta + \cos\omega t\, R\sin\theta = R\sin(\omega t + \theta)$$

Hence the resultant displacement at $P$ is simple harmonic and of amplitude $R$.

Squaring and adding the Eqs. (4.4) and (4.5), we get

$$R^2(\cos^2\theta + \sin^2\theta) = (a_1 + a_2\cos\varphi)^2 + (a_2\sin\varphi)^2$$

$$= a_1^2 + a_2^2\cos^2\varphi + 2a_1a_2\cos\varphi + a_2^2\sin^2\varphi$$

$$R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\varphi \quad ...(4.6)$$

The resultant intensity $I$ at $P$, which is proportional to square of the resultant amplitude is given by

$$I = R^2 \quad ...(4.7)$$

For simplicity, taking the constant of proportionality as 1.

Thus $$I = a_1^2 + a_2^2 + 2a_1a_2\cos\varphi$$

Now if $I_1$ and $I_2$ are the intensities of the interfering light waves, then

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\varphi \quad ...(4.8)$$

Hence, the resultant intensity is not just the sum of the intensities due to the separate waves *i.e.*, $(a_1^2 + a_2^2)$.

Suppose $a_1 = a_2 = a$ (*i.e.*, amplitudes of two waves are same)

Then $$I = a^2 + a^2 + 2a^2\cos\varphi$$

$$= 2a^2(1+\cos\varphi) = 2a^2 \times 2\cos^2\varphi/2$$

$$\Rightarrow \quad I = 4a^2\cos^2\frac{\varphi}{2} \quad ...(4.9)$$

### 4.6.2 Constructive Interference

The intensity $I$ is maximum when $\cos\varphi = +1$, *i.e.*, when phase difference $\varphi = 2n\pi$, where $n = 0, 1, 2, 3,...$

$$\text{Path difference} = \frac{\lambda}{2\pi} \times 2n\pi = n\lambda$$

Hence the resultant intensity at a point is maximum when the phase difference between the two superposing waves is an even multiple of $\pi$ or path difference is an integral multiple of wavelength $\lambda$. This is condition of *constructive interference*.

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{max} = (a_1 + a_2)^2$$

$$I_{max} > (I_1 + I_2)$$

If $a_1 = a_2 = a$ (let)

then

$$I_{max} = 4a^2 \qquad ...(4.10)$$

### 4.6.3 Destructive Interference

The intensity $I$ is minimum when $\cos\varphi = -1$, *i.e.*, when phase difference $\varphi = (2n \pm 1)\pi$, $n = 0, 1, 2, 3, \ldots$

$$\text{Path difference} \quad (S_2P - S_1P) = (2n \pm 1)\frac{\lambda}{2}$$

Hence the resultant intensity at a point is minimum when the phase difference between the two superposing waves is an odd multiple of $\pi$ or the path difference is an odd multiple of $\frac{\lambda}{2}$. This is condition of *destructive interference.*

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$$

$$= (a_1 - a_2)^2 = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$I_{min} < I_1 + I_2$$

If $a_1 = a_2$, then

$$I_{min} = 0 \qquad ...(4.11)$$

**NOTE**

Thus on screen there is a variation in the intensity of light being alternatively maximum or minimum. This is called the 'interference pattern'.

### 4.6.4 Conservation of Energy in Interference : Average Intensity

The average intensity is given as

$$I_{av} = \frac{\int_0^{2\pi} I\, d\varphi}{\int_0^{2\pi} d\varphi} = \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1a_2\cos\varphi)\, d\varphi}{\int_0^{2\pi} d\varphi}$$

$$= \frac{[a_1^2\varphi + a_2^2\varphi + 2a_1a_2\sin\varphi]_0^{2\pi}}{2\pi} = \frac{2\pi(a_1^2 + a_2^2)}{2\pi} = a_1^2 + a_2^2$$

*i.e.,*

$$I_{av} = I_1 + I_2 \qquad ...(4.12)$$

If $a_1 = a_2 = a$

$$I_{av} = 2a^2 \qquad ...(4.13)$$

Thus the average intensity is equal to the sum of the separate intensities. That is, whether energy apparently disappears at the minima is actually present at maxima. Thus *there is no violation of the law of conservation of energy in the phenomenon of interference.*

### 4.6.5 Visibility of Fringes

The quality of fringes produced by interferometric system can be described quantitatively using visibility ($V$), which is given by

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \quad ...(4.14)$$

It was first formulated by Michelson. Here $I_{max}$ and $I_{min}$ are the intensities corresponding to the maximum and adjacent minimum in the fringe system. There will be best contrast in fringes or visibility when the difference $I_{max}$ and $I_{min}$ is maximum. As we know that

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \quad \text{and} \quad I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Then visibility will be $V = \dfrac{2\sqrt{I_1 I_2}}{I_1 + I_2}$ ...(4.15)

### 4.6.6 Intensity Variation in Interference

Due to interference of two coherent waves the resultant intensity at a point depends on the phase difference between the waves at that point. As the phase difference is a function of the position of that point there shall be variation in the resultant intensity from point to point. Further the intensity is a measure of net energy of disturbance at the given point so there shall be a *non-uniform energy distribution* in space. Corresponding to a path difference Δ, the resultant intensity at that point is given by

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\varphi = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$$

If we plot a graph between resultant intensity $I$ and phase difference φ. The variation of $I$ and φ will be shown in Fig. 4.13. When amplitudes $a_1$ and $a_2$ are different the intensity maxima will have a value $(a_1 + a_2)^2$ or $(\sqrt{I_1} + \sqrt{I_2})^2$, while the intensity minima will have value $(a_1 - a_2)^2$ or $(\sqrt{I_1} - \sqrt{I_2})^2$ as shown in Fig. 4.13($a$).

When the amplitudes are equal *i.e.*, $a_1 = a_2 = a$ or $I_1 = I_2 = I_0$, then $I_{max} = 4a^2$ or $I_{max} = 4I_0$ and $I_{min} = 0$ as shown in Fig. 4.13($b$).

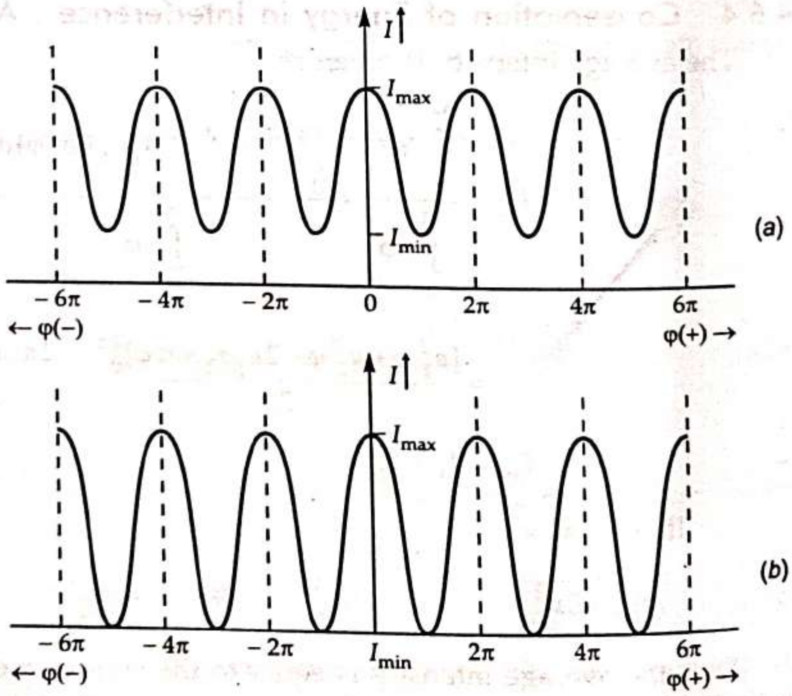


**Fig. 4.13** ($a$) Energy distribution curve for different valued amplitudes. ($b$) Energy distribution curve for equal amplitudes.

**Example 4.1** *Find the resultant of superposition of two waves* $y_1 = 2.0 \sin \omega t$ *and* $y_2 = 5.0 \sin(\omega t + 30°)$. *Symbols have their usual meanings.* **[GGSIPU, Dec. 2004, (4 marks)]**

***Solution.*** According to superposition principle,

*Method I*

According to superposition principle, we have $Y = y_1 + y_2$

$$Y = y_1 + y_2 = 2.0 \sin(\omega t) + 5.0 \sin(\omega t + 30°)$$

$$= 2.0 \sin \omega t + 5.0 (\sin \omega t \cos 30° + \cos \omega t \sin 30°)$$

$$= 2.0 \sin \omega t + \frac{5.0 \times \sqrt{3}}{2} \sin \omega t + \frac{5.0}{2} \cos \omega t$$

$$= (2.0 + 2.5 \times 1.732) \sin \omega t + 2.5 \cos \omega t$$

$$= 6.33 \sin \omega t + 2.5 \cos \omega t$$

$$= R \cos \theta \sin \omega t + R \sin \theta \cos \omega t$$

Here $R \cos \theta = 6.33$ ; $R \sin \theta = 2.5$

$$R^2 (\sin^2 \theta + \cos^2 \theta) = 46.3189$$

$$R = 6.8$$

$$\tan \theta = \frac{R \sin \theta}{R \cos \theta} = 0.394$$

$$\theta = 21.55°$$

Then $$Y = R \sin(\omega t + \theta)$$

$$= 6.8 \sin(\omega t + 21.55°)$$

*Method II*

Given $a_1 = 2.0$, $a_2 = 5.0$, $\varphi = 30°$, the resultant amplitude

$$R = \sqrt{(a_1^2 + a_2^2 + 2a_1 a_2 \cos 30°)}$$

$$= \sqrt{4 + 25 + 2 \times 2 \times 5 \times \sqrt{3}/2} = 6.8$$

and $$\tan \theta = \frac{R \sin \theta}{R \cos \theta} = 0.394$$

then $$\theta = 21.55°$$

Hence $$Y = R \sin(\omega t + \theta)$$

$$= 6.8 \sin(\omega t + 21.55°)$$

**Example 4.2** *The coherent sources of intensity ratio* $\alpha$ *interfere. Prove that in the interference pattern,*

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\alpha}}{1 + \alpha}.$$

*where symbols have their usual meanings.*

***Solution.*** We know that the resultant intensity at a point due to two waves of amplitudes $a_1$ and $a_2$ is given by

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\varphi$$

and

$$I_{max} = (a_1 + a_2)^2$$

$$I_{min} = (a_1 - a_2)^2$$

Given $\alpha = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$

where $I_1$ and $I_2$ are the intensities of two sources of respective amplitudes $a_1$ and $a_2$. Then

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a_1 + a_2)^2 - (a_1 - a_2)^2}{(a_1 + a_2)^2 + (a_1 + a_2)^2}$$

$$= \frac{4a_1 a_2}{2(a_1^2 + a_2^2)} = \frac{2a_1 a_2}{a_1^2 + a_2^2}$$

$$= \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{2\sqrt{\frac{I_1}{I_2}}}{1 + \frac{I_1}{I_2}} = \frac{2\sqrt{\alpha}}{1+\alpha}.$$ Hence proved.

## 4.7 THEORY OF INTERFERENCE FRINGES

### 4.7.1 Expression for Fringe Width

Consider a narrow monochromatic source $S$ and two parallel narrow slits $S_1$ and $S_2$ very close together and equidistant from $S$. Let $2d$ is the separation between slits $S_1$ and $S_2$. $D$ is the distance of screen $XY$ from $S_1$ and $S_2$.

In Fig. 4.14, the point $O$ is equidistant from $S_1$ and $S_2$. Initially path difference between two rays from $S_1$ and $S_2$ is zero. Intensity at $O$ is maximum. Point $P$ is at a distance of $x$ from $O$. We shall now consider the conditions for a bright or dark fringes at this point $P$.

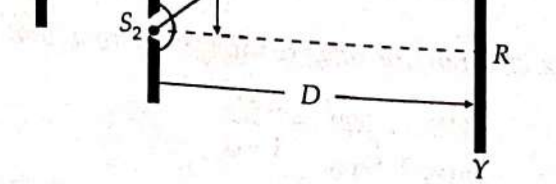


**Fig. 4.14** Geometric construction for describing theory of interference fringes using Young double-slit experiment.

From $\Delta S_1QP$, $(S_1P)^2 = (S_1Q)^2 + (QP)^2 = D^2 + (x-d)^2$

and from $\Delta S_2PR$, $(S_2P)^2 = (S_2R)^2 + (RP)^2 = D^2 + (x+d)^2$

Then, $(S_2P)^2 - (S_1P)^2 = (x+d)^2 + (x-d)^2 = 4xd$

$\Rightarrow$ $(S_2P - S_1P)(S_2P + S_1P) = 4xd$ $[\because a^2 - b^2 = (a-b)(a+b)]$

In Young's experiment, $D > 1000(2d)$ and $D > 1000x$

So that $(S_2P + S_1P)$ is replaced by $2D$, the error is not more than a fraction of 1%.

$$(S_2P - S_1P)2D = 4xd$$

$$(S_2P - S_1P) = \frac{4xd}{2D} = \frac{2xd}{D} = \frac{x(2d)}{D} \quad ...(4.16)$$

## Position and Spacing of Fringes

Now we shall consider the following *two* cases :

*(i) Bright Fringes*

$P$ is bright, when $\varphi = \frac{2\pi}{\lambda}x$.

where $\varphi$ be the phase difference, $x$, the path difference and $n$ is the whole number multiple of wavelength $\lambda$.

*i.e.,* $S_2P - S_1P = n\lambda$, where $n = 0, 1, 2, 3, ...$

Substituting $S_2P - S_1P$ from Eq. (4.16)

$$\frac{2xd}{D} = n\lambda$$

$$x = \frac{n\lambda D}{2d} \quad ...(4.17)$$

Equation (4.17) gives the distance of the bright fringes from $O$, the path difference is zero. Hence, there is a bright fringe.

The next fringes are when $n = 1, 2, 3, ...$ and so on.

When $n = 1$ $x_1 = \frac{\lambda D}{2d}$

$n = 2$ $x_2 = \frac{2\lambda D}{2d}$

$n = 3$ $x_3 = \frac{3\lambda D}{2d}$

..................................

..................................

$n = n-1$ $x_n = \frac{(n-1)\lambda D}{2d}$

$n = n$ $x_n = \frac{n\lambda D}{2d}$

The distance between any two consecutive bright fringes is

$$(x_n - x_{n-1}) = \frac{n\lambda D}{2d} - \frac{(n-1)\lambda D}{2d} = \frac{\lambda D}{2d} \quad ...(4.18)$$

*(ii) Dark Fringes.*

If point $P$ is dark, when path difference is an odd number of multiple of half wavelength.

i.e.,
$$S_2P - S_1P = \frac{(2n \pm 1)\lambda}{2}, \quad \text{where } n = 0, 1, 2, 3, 4, ...$$

Substituting $S_2P - S_1P$, from Eq. (4.16)

$$\frac{2xd}{D} = \frac{(2n \pm 1)\lambda}{2}$$

or
$$x = \frac{(2n \pm 1)\lambda D}{4d} \quad ...(4.19)$$

Equation (4.19) gives distances of the dark fringes from point $O$. The dark fringes are formed as follows :

When $n = 0$ $\quad x_0 = \frac{\lambda D}{4d}$

$n = 1$ $\quad x_1 = \frac{3\lambda D}{4d}$

$n = 2$ $\quad x_2 = \frac{5\lambda D}{4d}$

..................................................

..................................................

$n = n-1$ $\quad x_{n-1} = \frac{(2n-1)\lambda D}{4d}$

$n = n$ $\quad x_n = \frac{(2n+1)\lambda D}{4d}$

The distance between any two consecutive dark fringes is

$$(x_n - x_{n-1}) = \frac{(2n+1)\lambda D}{4d} - \frac{(2n-1)\lambda D}{4d} = \frac{2\lambda D}{4d} = \frac{\lambda D}{2d} \quad ...(4.20)$$

Hence, the spacing between any two consecutive maxima and minima is the same. This is expressed by $\beta\left(= \frac{\lambda D}{2d}\right)$ and is known as *fringe width.*

It is obvious from Eqs. (4.18) and (4.20), the spacing :

*(i)* is directly proportional to the wavelength of light *i.e.,* $\beta \propto \lambda$

*(ii)* is directly proportional to the distance of screen from two sources *i.e.,* $\beta \propto D$

and *(iii)* is inversely proportional to the separation between two coherent sources *i.e.,* $\beta \propto 1/2d$.

Hence the fringe width (spacing) increases with increase in wavelength and distance $D$ and bringing the two coherent sources close to each other.

### 4.7.2 Shape of the Interference Fringes

Let $S_1$ and $S_2$ be the two coherent sources. At the point $P$, there is maximum and minimum intensity according to following conditions :

$$S_2P - S_1P = n\lambda \qquad \text{(maximum)}$$

$$S_2P - S_1P = (2n \pm 1)\frac{\lambda}{2} \qquad \text{(minimum)}$$

Thus for a given value of $n$, locus of points of maximum or minimum intensity is given by

$$S_2P - S_1P = \text{constant} \qquad ...(4.21)$$

Which is the equation of a hyperbola with $S_1$ and $S_2$ as foci of hyperbolas. This establishes that interference fringes are hyperbolas in shape as shown in Fig. 4.15. Since the wavelength of light waves is extremely small ($\cong 10^{-7}$ m), the value of $S_2P - S_1P$ is also of that order. Therefore, *the eccentricity of fringes is quite large and hence these hyperbolas appear, more or less as straight line.*

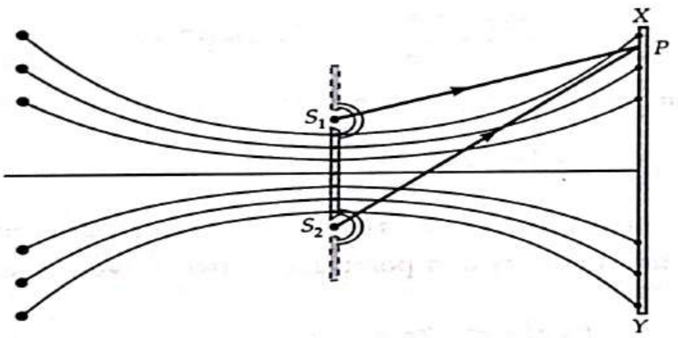


**Fig. 4.15** Shape of interference fringes.

### 4.7.3 Angular Fringe Width

The angular fringe width is defined as the angular separation between consecutive bright and dark fringes and is denoted by θ

As $\qquad \text{Angle} = \dfrac{\text{Arc}}{\text{radius}}$

$$\theta = \theta_{n+1} - \theta_n = \frac{x_{n+1}}{D} - \frac{x_n}{D} = \frac{x_{n+1} - x_n}{D} = \frac{\beta}{D}$$

$$= \frac{\frac{D\lambda}{2d}}{D}$$

or $$\theta = \frac{\lambda}{2d} \text{ radian} \qquad ...(4.22)$$

**Example 4.3** *In Young's double slit experiment, a source of light of wavelength 4200 Å is used to obtain interference fringes of width $0.64 \times 10^{-2}$ m. What should be the wavelength of the light source to obtain fringes $0.46 \times 10^{-2}$ m wide, if the distance between screen and the slits is reduced to half the initial value ?*

***Solution.*** In *first* case, $\lambda = 4200\ \text{Å} = 4200 \times 10^{-10}$ m, $\beta = 0.64 \times 10^{-2}$ m

$$\therefore \quad 0.64 \times 10^{-2} = \frac{4200 \times 10^{-10} \times D}{(2d)} \quad ...(i) \quad \left[\because \beta = \frac{\lambda D}{(2d)}\right]$$

In *second* case, $\beta = 0.46 \times 10^{-2}$ m, $\lambda = ?$, $D = \left(\frac{D}{2}\right)$

$$0.46 \times 10^{-2} = \frac{\lambda \times \left(\frac{D}{2}\right)}{(2d)} = \frac{\lambda D}{2\,(2d)} \quad ...(ii)$$

Dividing Eq. (*i*) by Eq. (*ii*),

$$\frac{0.64 \times 10^{-2}}{0.46 \times 10^{-2}} = \frac{4200 \times 10^{-10} \times D \times 2d \times 2}{(2d)\lambda D}$$

$$\therefore \quad \lambda = \frac{4200 \times 10^{-10} \times 2 \times 0.46}{0.64} = 6037.5\ \text{Å}$$

***Example 4.4*** *Show that in a two-slit interference pattern the intensity at a point is given by*

$$I = A + B\cos^2\left(\frac{kx}{2}\right)$$

*where A, B and k are constants of the set up and x is the linear distance of this point from the central fringe.*

***Solution.*** The resultant intensity at a point due to two waves of amplitudes $a_1$ and $a_2$ are given by

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\varphi$$

where $\varphi$ is the phase difference at the point

$$\varphi = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda}(S_2P - S_1P) = \frac{2\pi}{\lambda}\frac{xd}{\Delta}$$

where $x$ is the linear distance of the point $P$ from the central ring, $d$ is the separation between the sources and $D$ is the distance between slit and screen.

$$\varphi = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right) = kx$$

where $k$ is a constant and its value is $\frac{2xd}{\lambda D}$

$$\therefore \quad I = a_1^2 + a_2^2 + 2a_1 a_2 \cos kx$$

$$= a_1^2 + a_2^2 + 2a_1 a_2\left(2\cos^2\frac{kx}{2} - 1\right) = (a_1 - a_2)^2 + 4a_1 a_2 \cos^2\frac{kx}{2}$$

$$= A + B\cos^2\frac{kx}{2}$$

where $A = (a_1 - a_2)^2$ and $B = 4a_1 a_2$ are the other constants of the set up.

*Example 4.5* *In Young's double slit experiment the angular width of a fringe formed on a distant screen in 0.1°. The wavelength of light used in 600 nm. What is the spacing between the slits ?*

***Solution.*** Given $\theta = 0.1° = \frac{0.1 \times 3.14}{180} = 1.74 \times 10^{-3}$ rad ;

$$\lambda = 600 \text{ nm} = 6.0 \times 10^{-7} \text{ nm}$$

We know the angular fringe width

$$\theta = \frac{\lambda}{2d}$$

Then $$2d = \frac{\lambda}{\theta} = \frac{6.0 \times 10^{-7}}{1.74 \times 10^{-3}} = 3.4 \times 10^{-4} \text{ m} = 0.34 \text{ mm}$$

## 4.8 CONDITIONS FOR INTERFERENCE OF LIGHT WAVES

To obtain a well defined observable interference pattern, the following conditions must be fulfilled :

### Conditions for Sustained Interference

By sustained interference, we mean that the nature and order of interference at a point of the medium should remain unchanged with time.

For this to happen there are *two* conditions :

(*i*) The two sources must be *monochromatic, i.e.,* they must emit light of same wavelength or frequency.

(*ii*) The two sources must have either no phase difference or if there is a phase difference, it must remain unchanged with time.

If the above conditions are not satisfied the phase difference between interfering waves at a point will go on changing, and hence the resultant amplitude (or resultant intensity) at the point will go on changing with time. This will result in either uniform intensity or fluctuating intensity at the point.

### Conditions for Good Visibility

(*i*) The separation between two coherent sources *i.e.,* $2d$ should be small so that the width of bright and dark fringes formed will increase giving rise to increase resolving power and hence good visibility of the fringes.

(*ii*) The separation of screen from the two coherent sources *i.e., D* should be large so that the width of the fringes increases, and hence they are clearly seen.

(*iii*) The background in which the fringes are seen should be dark.

### Conditions for Good Contrast

By good contrast, we mean the difference between the maximum and minimum intensity or the difference between the intensities of bright and dark fringes should be as large as possible.

For this the following conditions are necessary :

(*i*) The amplitude of the two interfering waves must be nearly the same or equal. In this case $a_1 = a_2 = a$ ;

$$I_{max} = (a+a)^2 = 4a^2 \quad \text{and} \quad I_{min} = (a-a)^2 = 0$$

so that the difference between $I_{max}$ and $I_{min}$ is maximum and equal to $4a^2$.

(*ii*) The two light sources should be very narrow. If the sources are wide, they contain a large number of narrow sources giving rise to many interference patterns which overlap on the screen resulting in the decreased contrast.

(*iii*) Light sources should be monochromatic or should have wavelengths with smaller difference, otherwise due to overlapping of interference fringes of different colours the interference pattern is seen white.

## 4.9 INTERFERENCE BANDS WITH FRESNEL'S BIPRISM

A biprism, as its name suggests, is a combination of two thin prisms with their bases joined and their two faces making an obtuse angle of about 179° so that the other angles are each of about 30′. In actual practice the biprism is grounded from a single optically plane glass plate.

When a monochromatic source of light illuminates a narrow vertical slit *S* held symmetrically at a short distance from a biprism *ABC* with its refracting edge vertical and parallel to the slit, each half of the biprism produces a virtual image of *S* by refraction. The distance between *S* and the biprism is so adjusted that the two virtual images $S_1$ and $S_2$ are quite close together. A horizontal cross-section of the arrangement is shown in Fig. 4.16. The two sources $S_1$ and $S_2$ give out light waves parallel to each other in the constant phase having the same amplitude. Closely spaced interference fringes are produced in the superposition region *QR*, while the wide set of fringes at the edges of the pattern is on account of diffraction*.

**NOTE**

*These wider fringes are produced by the vertex of the prism which acts as a straight edge.

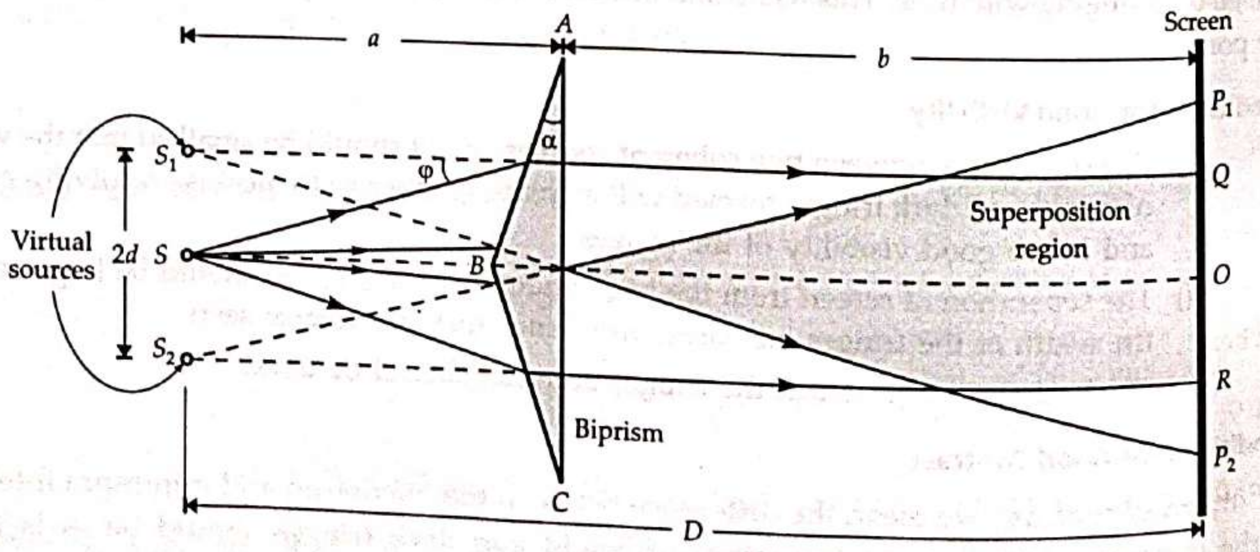


**Fig. 4.16** Interference due to biprism.

### Theory

As the point $O$ (Fig. 4.17) is equidistant from $S_1$ and $S_2$, the displacement will be in the same phase and so the intensity here is maximum.

*To find the intensity at a point P, which is at a distance x from O; we proceed as follows :*

Let $2d$ be the separation between virtual sources $S_1$, $S_2$ and $D$ be the distance between the slits and the screen.

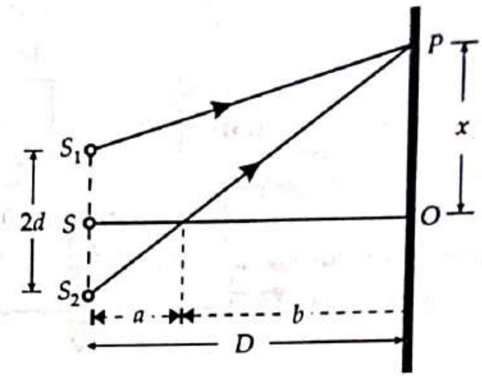


**Fig. 4.17** Illustration for theory of Fresnel's biprism.

Then from Fig. 4.17,

$$(S_2P)^2 = D^2 + (x+d)^2$$

and

$$(S_1P)^2 = D^2 + (x-d)^2$$

$$\therefore \quad (S_2P)^2 - (S_1P)^2 = (x+d)^2 - (x-d)^2 = 4xd$$

$\therefore$ Path difference

$$S_2P - S_1P = \frac{4xd}{S_2P + S_1P} \qquad ...(4.23)$$

We assumed here $(S_2P = S_2P) = D$, then Eq. (4.23) becomes

$$S_2P - S_1P = \frac{4xd}{2D} = \frac{2xd}{D} \qquad ...(4.24)$$

The intensity at $P$ is maximum if the path difference $(S_2P - S_1P)$ is an even multiple of half a wavelength and minimum if it is an odd multiple of half a wavelength.

*For maximum intensity,*

$$\frac{x(2d)}{D} = n\lambda \quad \text{or} \quad x = \frac{D}{2d}n\lambda$$

Fringe width $\beta = x_n - x_{n-1}$

$$= \frac{D}{2d}n\lambda - \frac{D}{2d}(n-1)\lambda = \frac{D}{2d}\lambda$$

or

$$\lambda = \frac{2d}{D}\beta$$

$\Rightarrow$

$$\lambda = \frac{2d}{(a+b)}\beta \qquad ...(4.25)$$

where $D = (a+b)$

### 4.9.1 Applications of Fresnel's Biprism

**(a) *Determination of the Wavelength of Sodium Light using a Fresnel's Biprism :***

The experimental arrangement is shown in Fig. 4.18. The various devices are arranged at the same height above the optical bench. As we know that a closely spaced interference fringes are produced by a biprism. The fringes are actually anywhere in the space between the biprism and the eyepiece ; and are called the *non-localised fringes*. The fringes are observed with the microscope, with the fringes lying in the focal plane of its eyepiece.

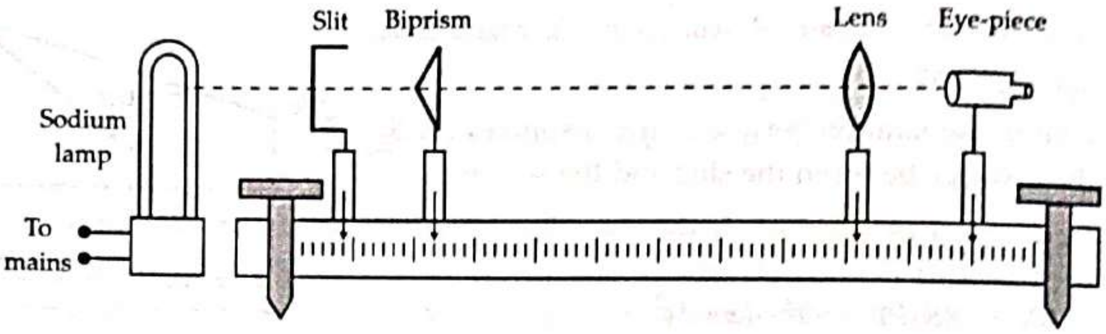


**Fig. 4.18** Experimental arrangement of Fresnel's biprism.

If $2d$ is the distance between $S_1$ and $S_2$, $D$ is the distance between the slit $S$ and cross-wire of the eyepiece, and $\beta$ is the fringe width, then from Eq. (4.25),

$$\lambda = \frac{2d}{D}\beta$$

Thus the wavelength $\lambda$ of the monochromatic source can be determined by measuring $D$, $2d$ and $\beta$.

**(b) *Determination of the Distance between Two Virtual Sources by Fresnel's Biprism***

The distance between the two virtual sources by Fresnel's biprism is determined by any of the following *two* methods :

(*i*) Deviation method (*ii*) Displacement method.

(*i*) *Deviation Method.* Biprism is constituted by two prisms as already shown in Fig. 4.16. Since the refractive angle $\alpha$ of each prism is about 30′, therefore, deviation produced by each of the prism is quite small. If $\varphi$ be the deviation produced by a single prism then using prism formula, we get

$$\mu = \frac{\sin(\alpha+\varphi)/2}{\sin\alpha/2}$$

$\left[\because \frac{\alpha+\varphi}{2} \text{ and } \frac{\alpha}{2} \text{ are very small then } \sin\frac{\alpha+\varphi}{2} \text{ and } \sin\frac{\alpha}{2} \text{ are equal to } \frac{\alpha+\varphi}{2} \text{ and } \frac{\alpha}{2} \text{ respectively}\right]$

$$\Rightarrow \quad \mu = \frac{(\alpha+\varphi)/2}{\alpha/2} = \frac{(\alpha+\varphi)}{\alpha} \qquad \text{...(4.26)}$$

where $\mu$ is the refractive index of biprism glass

$$\varphi = \mu\alpha - \alpha$$

$$\varphi = \alpha(\mu - 1) \qquad \text{...(4.27)}$$

From Fig. 4.16, it is clear that total deviation produced is equal to $2\varphi$. If distance between the prism $ABC$ and source $S$ be $a$, as shown, then we get

$$SS_1 = SS_2 = a \tan\varphi = a\varphi$$

$\therefore$
$$2d = S_1S_2 = 2a\varphi \qquad ...(4.28)$$

Substituting the value of $\varphi$ from Eq. (4.27) in Eq. (4.28)

$$2d = 2a(\mu - 1)\alpha \qquad ...(4.29)$$

*(ii) Displacement Method.* To determine (2*d*), a convex lens is chosen such that its focal length is less than one-fourth of the distance between the slit and the focal plane of the eyepiece. The lens is mounted on a stand which is kept between the stands holding the Fresnel's biprism and the eyepiece (Fig. 4.19). The lens is so adjusted that for two of its positions the real images of two virtual sources $S_1$ and $S_2$ are focused on the plane of eyepiece. If $x$ and $y$ are the separation between real images of $S_1$ and $S_2$ for two positions of the lens.

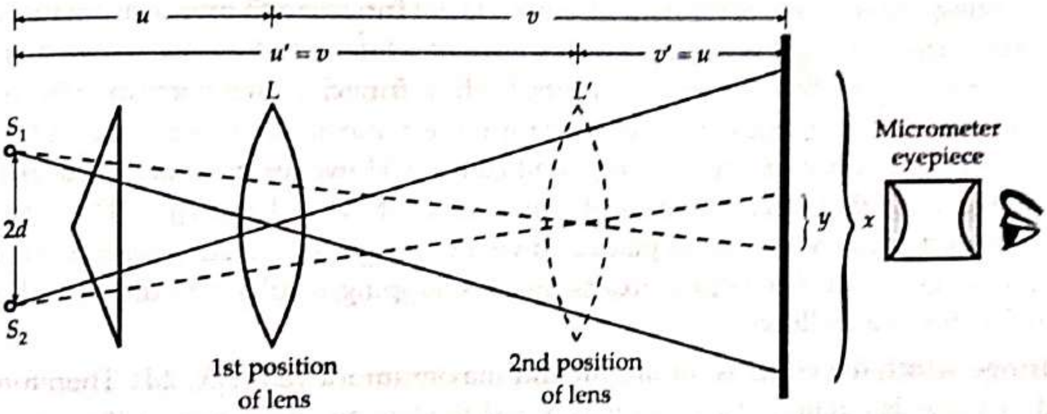


**Fig. 4.19** Measurement of 2*d* (displacement method).

In these positions the magnifications are :

$$m_1 = \frac{x}{2d} = \frac{v}{-u} \quad \text{and} \quad m_2 = \frac{y}{2d} = \frac{v'}{u'} = \frac{u}{-v}.$$

where $u$ and $v$ are the distances of the object and image respectively from the lens in the first conjugate positions.

$\therefore$
$$\frac{x}{2d} \times \frac{y}{2d} = \frac{v}{-u} \times \frac{u}{-v} = 1$$

or
$$xy = (2d)^2$$

or
$$2d = \sqrt{(xy)} \qquad ...(4.30)$$

### 4.9.2 Effect of Increasing the Angle of Biprism on Fringes

If the angle $\alpha$ of the biprism be increased, the distance $2d$ between the virtual sources would increase because $2d = 2a(\mu - 1)\alpha$. This, in turn, would reduce the fringe width $\left(\beta = \frac{D\lambda}{2d}\right)$. The fringes will not be separately visible and may disappear ultimately.

### 4.9.3 Effect of Increasing the Slit Width on Fresnel Fringes

When in the biprism experiment, the width of the slit is gradually increased, the visibility between the bright and dark fringes becomes poorer and poorer. Ultimately, the fringes disappear, leaving a uniform illumination everywhere.

*Explanation.* On increasing the slit width, the two virtual source slits are correspondingly widened. They are then equivalent to a large number of pairs of narrow slits. All pairs produce their fringe patterns, which are relatively shifted. This causes partial overlapping of maxima and minima, due to different pairs, resulting in indistinctness. Greater the width of the source slits, greater the overlapping when the slit width equals half the fringe width, there is complete overlapping of maxima and minima and the fringes disappear.

### 4.9.4 Interference Fringes with White Light

After making adjustments of biprism assembly to get fringes with monochromatic light ; if the source of monochromatic light is replaced by a source of white light then *few coloured fringes with a central white appear in the field of view*. For central white fringe, which corresponds to zero path difference, all constituent colours of white light form their maxima at this place and therefore, we get central white as a result of overlapping of all colours. However, for initial orders of maxima and minima, we know that since fringe width $\beta$ increases with wavelength $[\beta = (D\lambda/2d)]$, therefore overlapping of different colours takes place and we get fringes of mixed colours with inner edge red and outer edge violet. For higher orders this overlapping results into uniform illumination. The reason for this is as follows :

The fringe width for violet is minimum and maximum for red $(D\lambda/2d)$. Therefore, the first dark band of violet is obtained first and that of red the last on either side of the zero order. The inner edge of first minimum of violet receives sufficient intensity from red because the maximum of red falls in its vicinity and the edge is reddish. The first maximum of violet falls close to the inner edge of minimum of red and the edge appears violet. For points at large distances from the centre, the maxima and minima due to large number of wavelengths overlap and it results in uniform illumination. For example, at a point on screen, we may have

$$\text{Path difference} \begin{cases} = 10\lambda_1 = 11\lambda_2 = 12\lambda_3 \ldots \text{(For maxima)} \\ = \left(10+\frac{1}{2}\right)\lambda'_1 = \left(11+\frac{1}{2}\right)\lambda'_2 = \left(12+\frac{1}{2}\right)\lambda'_3 \ldots \text{(For minima)} \end{cases}$$

Thus at that point we will have 10th, 11th, 12th... bright fringes of $\lambda_1, \lambda_2, \lambda_3 \ldots$ and 10th, 11th, 12th,... dark fringes of $\lambda'_1, \lambda'_2, \lambda'_3, \ldots$ Therefore, that point and actually all other points in a similar way, will show resultant white colour leading to uniform illumination.

### 4.9.5 Location of Zero Order Fringe in Biprism Experiment

When monochromatic light is used in biprism, alternate bright and dark fringes are obtained in which all the bright fringes are exactly similar in appearance. Hence it is not possible to locate the zero order fringe.

To locate zero order fringe, monochromatic light is replaced by a source of white light. Now the zero order (central) fringe is white and all other fringes are coloured. Now the vertical crosswire is adjusted on zero order (white) fringe and white light is again replaced by monochromatic light. The vertical crosswire will still be on zero order fringes and thus zero order fringe is located.

## 4.10 DISPLACEMENT OF FRINGES

The determination of the thickness of thin transparent plate (glass, mica, soap solution) by biprism can be understood with the help of Fig. 4.20. $S_1$ and $S_2$ be two virtual coherent sources derived from a source $S$ by biprism. $SO$ is the principal axis such that $S_1O = S_2O$ and therefore, $O$ is the place of zero order maximum and its place is located by using white light fringes giving central white fringe. On interposing a film of thickness $t$ and of refractive index $\mu$ in the path of light from one of the sources, say $S_1P$, the path difference occurs and results in the shift of central maximum from $O$ to $O'$. This shift of central white fringe from $O$ to $O'$ is measured by means of micrometer screw provided with the eyepiece. Let the shift $OO' = x_0$. Now we can relate the shift $x_0$ to thickness of plate $t$ and other known physical quantities in the following way :

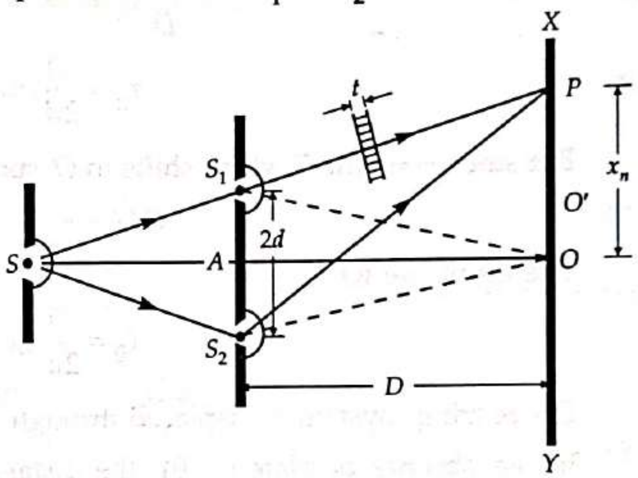


**Fig. 4.20** Thickness of plate of biprism.

Consider a point $P$ on the screen distant $x_n$ from $O$ to be the $n$th order maxima. The path difference for this point from $S_1$ and $S_2$ with plate of thickness $t$ in the path of $S_1$ may be calculated as follows :

Light from $S_1$ travels a path $(S_1P - t)$ in air and $t$ is plate of refractive index $\mu$. If $c$ and $v$ be the velocities of light in air and in the glass respectively, then time taken by light from $S_1$ to $P$.

$$= \frac{S_1P - t}{c} + \frac{t}{v}$$

$$= \frac{S_1P - t}{c} + \frac{\mu t}{c} \qquad \left[\because \mu = \frac{c}{v}\right]$$

$$= \frac{S_1P + (\mu - 1)t}{c}$$

Thus the air path $S_1P$ has been increased by $(\mu - 1)t$ as a result of introduction of the plate.

Therefore, the effective path difference at the point $P$

$$\Delta' = S_2P - [S_1P + (\mu - 1)t]$$

$$= S_2P - S_1P - (\mu - 1)t \qquad ...(4.31)$$

However, if thin plate was not there then the path change would have been $\Delta = (S_2P - S_1P)$, as already given in Eq. (4.24), in which it has been shown that

$$\Delta = S_2P - S_1P = \frac{2d}{D}x_n \qquad ...(4.32)$$

Substituting this value in Eq. (4.31), we get

$$\Delta' = \frac{2d}{D}x_n - (\mu - 1)t \qquad ...(4.33)$$

Now since the condition of path difference ($\Delta$) corresponding to $n$th bright fringe at $P$ in presence of plate, therefore,

$$\frac{2d}{D}x_n - (\mu-1)t = 2n\frac{\lambda}{2} = n\lambda \quad ...(4.34)$$

or

$$\frac{2d}{D}x_n = n\lambda + (\mu-1)t$$

or

$$x_n = \frac{D}{2d}[n\lambda + (\mu-1)t] \quad ...(4.35)$$

But since $n=0$, for $O$ which shifts to $O'$ such that

$$OO' = x_0$$

Therefore, we write

$$x_0 = \frac{D}{2d}(\mu-1)t \quad ...(4.36)$$

Entire fringe system is displaced through this distance as may be seen below.

In the absence of plate ($t=0$), the distance of $n$th maximum from $P_0$ [by putting $t=0$ in Eq. (4.35)].

$$= \frac{D}{2d}n\lambda \quad ...(4.37)$$

Therefore, displacement of $n$th bright fringe by subtracting Eq. (4.37) from Eq. (4.35)

$$= \frac{D}{2d}[n\lambda + (\mu-1)t] - \frac{D}{2d}n\lambda$$

$$= \frac{D}{2d}(\mu-1)t = x_0 \quad ...(4.38)$$

Since the expression is independent of $n$, it indicates that all the bright fringes are displaced through the same amount. It may be shown that dark fringes are also displaced through a distance $\frac{D}{2d}(\mu-1)t$. It means that *introduction of the plate in the path of one of the interfering beams displaces the entire fringe system through a distance* $\frac{D}{2d}(\mu-1)t$ *towards the beam in the path of which the plate is introduced.*

Now writing Eq. (4.35) for $(n+1)$th fringe, we get

$$x_{n+1} = \frac{D}{2d}[(n+1)\lambda + (\mu-1)t] \quad ...(4.39)$$

Subtracting Eq. (4.35) from Eq. (4.39), we get fringe width $\beta$, given by

$$\beta = x_{n+1} - x_n = \frac{D}{2d}[(n+1)\lambda + (\mu-1)t - n\lambda - (\mu-1)t]$$

or

$$\beta = \frac{D}{2d}\lambda \quad ...(4.40)$$

which is the same as before the introduction of the plate and shows that the *presence of plate does not change the fringe width.*

Substituting $\frac{D}{2d}$ from Eq. (4.40) in Eq. (4.38), we get displacement of *n*th bright fringe,

$$x_0 = \frac{D}{2d}(\mu - 1)t = \frac{\beta}{\lambda}(\mu - 1)t \quad \text{...(4.41)}$$

If the central bright fringe moves through a distance formerly occupied by *nth* bright fringe, then

$$x_0 = n\beta$$

$$\therefore \quad \frac{\beta}{\lambda}(\mu - 1)t = n\beta$$

or $$n = \frac{(\mu - 1)t}{\lambda} \quad \Rightarrow \quad \mu = \frac{n\lambda}{t} + 1 \quad \text{...(4.42)}$$

It gives us the number of order through which fringe system is displaced and refractive index of the material of the plate.

Equation (4.42) may be written to give thickness of the plate also

$$t = \frac{n\lambda}{\mu - 1} \quad \text{...(4.43)}$$

Thickness of plate is also given by Eq. (4.38) to be

$$t = \frac{x_0(2d)}{D(\mu - 1)} \quad \text{...(4.44)}$$

**Example 4.6** *In an experiment with Fresnel's biprism fringes for light of wavelength* $5 \times 10^{-7}$ *m are observed* $0.2 \times 10^{-3}$ *m apart at a distance of 1.75 m from the prism. The prism is made of glass of refractive index 1.50 and it is at a distance of 0.25 m from the illuminated slit. Calculate the angle of the vertex of the biprism.*

***Solution.*** In Fresnel's biprism,

Given that : $\lambda = 5 \times 10^{-7}$ m, $\beta = 0.2 \times 10^{-3}$ m, $a = 0.25$ m, $\mu = 1.50$ and $b = 1.75$ m, $\alpha = ?$

We know that distance between virtual sources in Fresnel's biprism

$$2d = 2a(\mu - 1)\alpha \quad \text{...(i)}$$

and fringe width $$\beta = \frac{\lambda D}{2d}$$

or $$2d = \frac{\lambda D}{\beta} \quad \text{...(ii)}$$

where $D = (a + b) = (1.75 + 0.25)\text{ m} = 2.00\text{ m}$

From Eqs. (*i*) and (*ii*),

$$\frac{\lambda D}{\beta} = 2a(\mu - 1)\alpha \quad \text{...(iii)}$$

or $$\alpha = \frac{\lambda D}{\beta[2a(\mu - 1)]} = \frac{5 \times 10^{-7} \times 2.00}{0.2 \times 10^{-3} \times 2 \times 0.25 \times (1.5 - 1)}$$

$$= 0.02 \text{ radian}$$

Vertex angle $\varphi = (\pi - 2\alpha) = (\pi - 0.04) = 177°42'$.

**Example 4.7** *A thin film ($\mu = 1.6$) is introduced in one of the beams in a biprism experiment and central fringe is found to be displaced to the position of 20th dark fringe for light of $\lambda = 6000$ Å. Calculate thickness of the film.*

**Solution.** In a biprism experiment,

Given that : $\mu = 1.6, \quad n = 20, \quad \lambda = 6000 \times 10^{-10}$ m

We know that the thickness of the film is

$$t = \frac{n\lambda}{(\mu - 1)}$$

$$= \frac{20 \times 6000 \times 10^{-10}}{(1.6 - 1)} = \frac{12 \times 10^{-6}}{0.6} = 20 \times 10^{-6} \text{ m} = 20 \ \mu\text{m}.$$

**Example 4.8** *In an interference pattern, at a point we observe the 12th order maxima for wavelength $\lambda_1 = 6000$ Å. What order will be visible here if source is replaced by light of wavelength $\lambda_2 = 4800$ Å ?*

**Solution.** In an interference pattern,

Given that : $n_1 = 12, \quad \lambda_1 = 6000$ Å, $\quad n_2 = ?, \quad \lambda_2 = 4800$ Å

Suppose $\beta_1$ and $\beta_2$ are the widths for wavelengths $\lambda_1$ and $\lambda_2$ then

$$\beta_1 = \frac{\lambda_1 D}{(2d)} \qquad ...(i)$$

and

$$\beta_2 = \frac{\lambda_2 D}{(2d)} \qquad ...(ii)$$

Dividing Eq. (*i*) by Eq. (*ii*)

$$\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} \qquad ....(iii)$$

*In field view*, $n_1$ fringes are for wavelength $\lambda_1$ and $n_2$ fringes are for wavelength $\lambda_2$.

Then width of field of view

$$n_1\beta_1 = n_2\beta_2 \qquad ...(iv)$$

or

$$n_2 = \frac{n_1\beta_1}{\beta_2} \qquad ...(v)$$

Putting the value of $\frac{\beta_1}{\beta_2}$ from Eq. (*iii*) in Eq. (*v*), we get

$$n_2 = \frac{n_1\lambda_1}{\lambda_2} \qquad ...(vi)$$

Putting all given values in Eq. (*vi*)

$$n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{12 \times 6000}{4800}$$

$$\Rightarrow \qquad n_2 = 15.$$

## Division of Amplitude

### 4.11 CHANGE OF PHASE BY REFLECTION AND DIVISION OF AMPLITUDE : STOKE'S LAW

To investigate the phase change in the reflection of light at an interface between two media. Sir G.C. Stoke used the principle of optical reversibility. *The principle states that a light ray, that is reflected or refracted, will retrace its original path, if its direction is reversed, provided there is no absorption of light.*

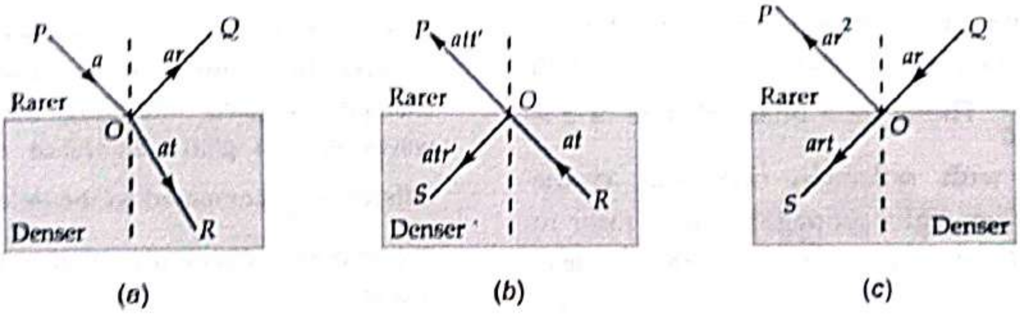


**Fig. 4.21** (*a*) Reflection and refraction of light wave from rarer to denser medium ; (*b*) Reflection and refraction of light wave with amplitude of refracted wave of (*a*) and with incidence angle equal to angle of refraction of (*a*) from denser to rarer medium ; (*c*) Reflection and refraction of light wave with amplitude of reflected wave of (*a*) and with incidence angle equal to angle fo reflection of (*a*) from rarer to denser medium.

Consider a light wave *PO* with amplitude *a* falls on the interface of a denser medium from a rarer medium as shown in Fig. 4.21. Now we can define coefficient of reflection by *r* as

$$r = \frac{\text{Amplitude of reflected wave}}{\text{Amplitude of incident wave}} \qquad ...(4.45)$$

and coefficient of refraction by *t* as

$$t = \frac{\text{Amplitude of refracted wave}}{\text{Amplitude of incident wave}} \qquad ...(4.46)$$

Therefore, the amplitude of the reflected wave *OQ* is *ar* and that of refracted wave *OR* is *at*.

Now consider the situation when the directions of reflected and refracted waves are reversed. To do so, first it is considered that a light wave of amplitude *at* is allowed to fall an interface from denser to rarer medium along *RO*. Then one has a reflected ray along *OS* with amplitude $atr'$ and a refracted wave with amplitude $att'$ along *OP*, where $r'$ and $t'$ are the coefficient of reflection and coefficient of refraction from denser to rarer medium respectively. Thereafter, it is allowed to fall the light wave of amplitude of *ar* on the interface from rarer to denser medium along *QO*. Now, there is a reflected along *OP* with amplitude $ar^2$ and a refracted wave with amplitude *art* along *OS*.

Now superposition of these two cases of propagation of light waves gives a light wave with amplitude $(ar^2 + att')$ along *OP* and another one with amplitude $(art + atr')$ along *OS*. The reversal of reflected (with amplitude $ar$) and refracted (with amplitude $at$) light wave must produce a light wave with amplitude along $OP$, and no wave along $OS$ because when we have considered propagation of light wave from rarer to denser medium along $PO$, there is no wave along $OS$.

Therefore, $$ar^2 + att' = a \quad ...(4.47)$$

and $$art + ar't = 0 \quad ...(4.48)$$

From Eq. (1.47), we have $$tt' = 1 - r^2 \quad ...(4.49)$$

and from Eq. (4.48), we get $$r' = -r \quad ...(4.50)$$

The negative sign in Eq. (4.50) indicates a displacement in opposite direction that is equivalent to a phase change of $\pi$ or a path difference $\frac{\lambda}{2}$. Therefore a phase change of $\pi$ is associated with reflection occurring at the interface when light propagates from rarer to denser medium. This is known as *Stoke's law of reflection*.

**STATEMENT**

Stoke's law states that if waves are reflected at a rarer to denser medium interface (for example, air-glass interface), the reflected waves have a phase difference $\pi$ (or path difference $\frac{\lambda}{2}$) compared to the incident wave. This also occurs in elastic waves such as sound waves.

## 4.12 INTERFERENCE FROM PARALLEL THIN FILMS OR COLOUR OF THIN FILMS

Colour of thin films can be explained by interference, which is exposed to composite light. Young explained the phenomenon on the basis of interference between light reflected from upper and bottom surface of thin film. It has been observed that in this case of thin film takes place due to (*a*) reflected light and (*b*) transmitted light.

### 4.12.1 Interference due to Reflected Light

Let us consider a transparent film $GHH'G'G$ (Fig. 4.22) of thickness $t$ and refractive index $\mu$. A ray $AB$ incidents on the upper surface of the film is partly reflected along $BR$ and partly refracted along $BC$. At $C$, part of it is internally reflected along $CD$ and finally emerges out $DR_1$, parallel to $BR$. This will continue further in the same way.

**Aim : *To get path difference between two reflected rays.***

Draw a normal $DE$ on $BR$ and other normal $BF$ on $CD$.

One also produces $DC$ in backward direction which meets at $P$ on $BQ$ line.

In Fig. 4.22,

$$\angle ABN = \angle i = \text{angle of incidence} \quad \text{and} \quad \angle QBC = \angle r = \text{angle of refraction.}$$

From geometry of Fig. 4.22,

$$\angle BDE = \angle i \quad \text{and} \quad \angle QPC = \angle r$$

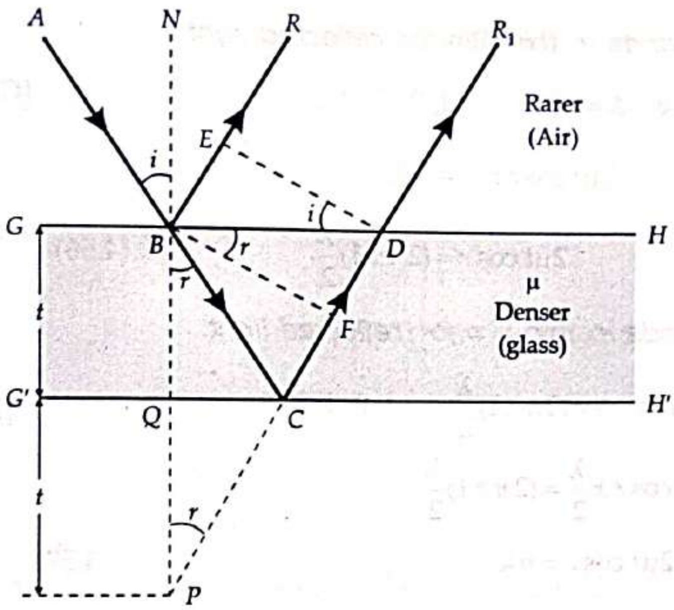


**Fig. 4.22** Interference in light reflected from a thin film is due to a combination of rays $BR$ and $CR_1$ from lower and upper surface of the film.

The optical path difference between two reflected rays ($BR$ and $DR_1$) is given by

$$\Delta = \text{Path } (BC + CD) \text{ in film} - \text{Path } BE \text{ in air}$$

$$= \mu(BC + CD) - BE \quad \text{...(4.51)}$$

We know that, $\mu = \dfrac{\sin i}{\sin r} = \dfrac{BE/BD}{FD/BD} = \dfrac{BE}{FD}$.

or $$BE = \mu(FD) \quad \text{...(4.52)}$$

From Eqs. (4.51) and (4.52),

$$\Delta = \mu(BC + CD - FD)$$

$$= \mu(BC + CF + FD - FD) = \mu(PC + CF)$$

$$= \mu(PF) \qquad [\because PC = BC] \quad \text{...(4.53)}$$

From $\Delta\, BPF$, $\cos r = \dfrac{PF}{BP}$

or $$PF = BP \cos r = 2t \cos r \quad \text{...(4.54)}$$

Substituting Eq. (4.53) in Eq. (4.54)

$$\Delta = \mu \times 2t \cos r = 2\mu t \cos r \quad \text{...(4.55)}$$

***It should be remembered that a ray reflected at a surface backed by a denser medium suffers an abrupt phase change of $\pi$, which is equivalent to a path difference of $\frac{\lambda}{2}$.[3]***

Thus the effective path difference between the two reflected rays $= 2\mu t \cos r \pm \dfrac{\lambda}{2}$

3. It is on the basis of Stoke's treatment.

*Condition for bright bands in thin film for reflected light.*

The path difference $\Delta = n\lambda$, $n = 0, 1, 2, 3, 4, ...$ [Constructive interference]

then $$2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

or $$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \quad ...(4.56)$$ [Film will appear bright]

*Condition for dark bands in thin film for reflected light.*

If the path difference $\Delta = (2n \pm 1)\frac{\lambda}{2}$, $n = 0, 1, 2, 3, ...$ [Destructive interference]

then $$2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

or $$2\mu t \cos r = n\lambda \quad ...(4.57)$$ [Film will appear dark]

### 4.12.2 Interference due to Transmitted Light

Let us consider a transparent parallel film $GHH'G'G$ (Fig. 4.23) of thickness $t$ and refractive index $(\mu)$. A ray $AB$ incidents on the upper surface. This ray $AB$ refracted as $BC$. The ray $BC$ is partly internally reflected as $CD$ and partly transmitted as $CT$. The ray $CD$ also partly internally reflected as $DE$ and finally emerges as transmitted ray $ET_1$, which is parallel to $CT$. This will continue further in the same way.

**Aim : *To get path difference between two transmitted rays.***

We also produce $ED$ in backward direction which meets produced $CF$ at $I$.

Draw a normal $EP$ on $CT$ and other normal $CQ$ on $DE$, one also produces $ED$ in backward direction, which meets at $I$ on $CF$ and angle of incidence $\angle ABN = \angle i$ and angle of refraction $\angle CBN' = \angle r$. From the geometry of Fig. 4.23,

$$\angle ECQ = \angle r, \quad \angle PEC = \angle i \text{ and } \angle CIQ = \angle r.$$

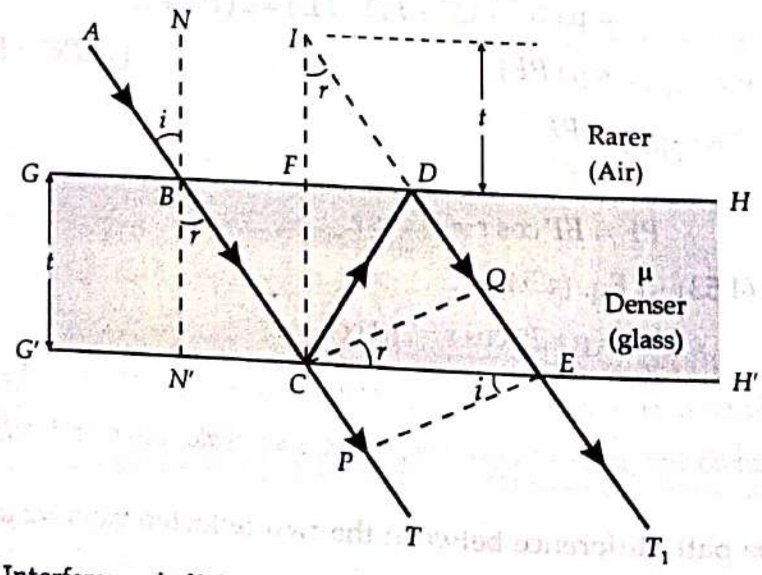


**Fig. 4.23** Interference in light transmitted from a thin film is due to combination of rays $CT$ and $DT$ from the lower and upper surfaces of the film.

The effective path difference

$$\Delta = \mu(CD + DE) - CP \quad \text{...(4.58)}$$

Also,

$$\mu = \frac{\sin i}{\sin r} = \frac{CP/CE}{QE/CE} = \frac{CP}{QE}$$

$$CP = \mu(QE) \quad \text{...(4.59)}$$

From Eqs. (4.58) and (4.59),

$$\Delta = \mu(CD + DQ + QE) - QE(\mu) = \mu(CD + DQ)$$

$$= \mu(ID + DQ) \quad [\because CD = ID]$$

$$= \mu(IQ)$$

$$= \mu(2t\cos r) \quad \text{[From Fig. 4.23]}$$

$$\Rightarrow \quad \Delta = 2\mu t \cos r \quad \text{...(4.60)}$$

***Hence it should be remembered that inside the film, reflection at different points takes place at the surface backed by rarer medium (air), thus no abrupt change of π takes place in this case.***

*Condition for bright bands in transmitted light (Constructive interference)*

$$\Delta = 2\mu t \cos r = n\lambda \quad \text{...(4.61)} \quad \text{(Film will appear bright)}$$

*Condition for dark bands in transmitted light (Destructive interference)*

$$\Delta = 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \quad \text{...(4.62)} \quad \text{(Film will appear dark)}$$

These conditions (4.61) and (4.62) are just reverse by reflected light as given by Eqs. (4.56) and (4.57).

*Hence the interference patterns in reflected and transmitted lights are complimentary.*

**Example 4.9** *A uniform water film has thickness $3.0 \times 10^{-5}$ cm. What colour does it show when seen in reflected white light along the normal ? ($\mu_{water} = \frac{4}{3}$).*

**Solution.** The wavelengths for destructive interference in reflected light (condition for minima) are given by

$$2\mu t = n\lambda$$

$$\Rightarrow \quad 2 \times \frac{4}{3} \times 3.0 \times 10^{-5} \text{ cm} = n\lambda \quad (n = 1, 2, ...)$$

$$\therefore \quad \lambda = 8 \times 10^{-5} \text{ cm}, \ 4 \times 10^{-5} \text{ cm}, \ 2.7 \times 10^{-5} \text{ cm}, \ ...$$

The possible λ values in the visible range are $8 \times 10^{-5}$ cm (red), $4 \times 10^{-5}$ cm (violet). Hence these parts of the spectral colours are absent in reflected light. Intermediate wavelengths correspond to yellow green, that colour will be seen.

**Example 4.10** *A thin film of soap solution is illuminated by white light at an angle of incidence $i = \sin^{-1}(4/5)$. In reflected light, two dark consecutive overlapping fringes are observed corresponding to the wavelength $6.1 \times 10^{-7}$ m and $6.0 \times 10^{-7}$ m. Calculate the thickness of the film ($\mu = 4/3$).*

***Solution.*** In colour of thin film (soap film)

Given : $i = \sin^{-1}(4/5)$ or $\sin i = 4/5$, $\lambda_n = 6.1 \times 10^{-5}$cm,

$\lambda_{n+1} = 6.0 \times 10^{-5}$cm, $\mu = 4/3 = 1.33$

The condition for dark bands in this film for reflected light

$$2\mu t \cos r = n\lambda$$

Now according to questions ; consecutive dark bands for wavelength $\lambda_n$ and $\lambda_{n+1}$ ; above condition will be as :

$$2\mu t \cos r = n\lambda_n \quad ...(i)$$

and

$$2\mu t \cos r = (n+1)\lambda_{n+1} \quad ...(ii)$$

From Eqs. (*i*) and (*ii*),

$$n\lambda_n = (n+1)\lambda_{n+1}$$

or

$$n(\lambda_n - \lambda_{n+1}) = \lambda_{n+1}$$

$$\Rightarrow \quad n = \frac{\lambda_{n+1}}{(\lambda_n - \lambda_{n+1})} \quad ...(iii)$$

Putting this value of $n$ in Eq. (*i*), we get

$$2\mu t \cos r = \frac{\lambda_{n+1}\lambda_n}{(\lambda_n - \lambda_{n+1})}$$

$$\Rightarrow \quad t = \frac{\lambda_n \lambda_{n+1}}{(\lambda_n - \lambda_{n+1})(2\mu \cos r)} \quad ...(iv)$$

But $\cos r = \sqrt{(1 - \sin^2 r)}$ ; $\frac{\sin i}{\sin r} = \mu$ or $\sin r = \frac{\sin i}{\mu}$

$$\cos r = \sqrt{\left(1 - \frac{\sin^2 i}{\mu^2}\right)} = \frac{1}{\mu}\sqrt{(\mu^2 - \sin^2 i)} \quad ...(v)$$

Putting the value of $\cos r$ from Eq. (*v*) in Eq. (*iv*)

$$t = \frac{\lambda_n \lambda_{n+1}}{2 \times (\lambda_n - \lambda_{n+1})\sqrt{(\mu^2 - \sin^2 i)}}$$

$$= \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5}}{2 \times (6.1 - 6.0) \times 10^{-5}\sqrt{(1.33)^2 - (0.8)^2}}$$

$$= 1.715 \times 10^{-5} \text{ m.}$$

**Example 4.11** *A soap film, suspended in air has thickness $5 \times 10^{-5}$ cm and viewed at an angle 35° to the normal. Find the wavelength of light in visible spectrum, which will be absent for a reflected light. The μ for the soap film as 1.33 and the visible spectrum is 4000 to 7800 Å.* **[GGSIPU, Dec. 2009 (4 marks)]**

***Solution.*** In colour thin film :

Given that : $t = 500 \text{ nm} = 5.0 \times 10^{-7}$ m, $i = 35°$, $\mu = 1.33$

We know that :

$$2\mu t \cos r = n\lambda \quad ...(i)$$

and

$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \sin r = \frac{\sin 35°}{1.33}$$

then

$$\cos r = \sqrt{(1 - \sin^2 r)} = \left[1 - \left(\frac{\sin 35°}{1.33}\right)^2\right]^{1/2} = \sqrt{(1 - 0.186)} = 0.902$$

For *first order i.e.,* $n = 1$.

$$\lambda_1 = 2\mu t \cos r = 2 \times 1.33 \times 5.0 \times 10^{-7} \times 0.902$$

$$= 1.199 \times 10^{-6} \text{ m} = 12000 \text{ Å (approx.)}$$

For *second order i.e.,* $n = 2$

$$\lambda_2 = \frac{2\mu t \cos r}{2} = \mu t \cos r = 1.33 \times 5.0 \times 10^{-7} \times 0.902 = 6000 \text{ Å (approx.)}$$

For *third order i.e.,* $n = 3$

$$\lambda_3 = \frac{2\mu t \cos r}{3} = \frac{2 \times 1.33 \times 5.0 \times 10^{-7} \times 0.902}{3} = 4000 \text{ Å (approx.)}$$

For *fourth order i.e.,* $n = 4$

$$\lambda_4 = \frac{2\mu t \cos r}{4} = \frac{2 \times 1.33 \times 5.0 \times 10^{-7} \times 0.902}{4} = 3000 \text{ Å (approx.)}$$

Hence $\lambda_2$ and $\lambda_3$ wavelengths of light in visible spectrum will be absent.

### 4.12.3 Colours in Reflected and Transmitted Light be Complementary

The colours observed in thin film in case of reflected light will be complementary of those observed in transmitted light. This is because the conditions for maxima and minima in the reflected light is just the reverse of those in the transmitted light.

### 4.12.4 Production of Colours in Thin Films

When a thin film of oil on water, or a soap bubble, exposed to an extended source of "white" light (such as sky) is observed under reflected light, brilliant colours are seen in the film or the bubble. These colours arise due to the interference of the light waves reflected from the top and bottom surfaces of the film.

The eye looking the film receives rays of light reflected from both the surfaces of the film. The path difference between these interfering rays depends upon $t$ (thickness of the film) and upon $\angle r$ and hence upon the inclination of the incident rays (the inclination is determined by the position of the eye relative to the region of the film which is being looked). Now white light consists of a continuous range of wavelengths (colours). At a particular point of the film, and for a particular position of the eye (*i.e.*, for a particular $t$ and a particular $\angle r$) the rays of only certain wavelengths will have a path difference satisfying the condition of maxima. Hence only those wavelengths (colours) will be present with maximum intensity, other neighbouring wavelengths will be present with less intensity, while some others which satisfy the condition of minima will be missing. Hence the point of the film appears coloured.

The colouration will, clearly, vary with the thickness of the film as well as with the position of the eye with respect to the point of the film (*i.e.*, with the inclination of the rays). Therefore if the same point of the film is observed with eye in different positions, or different points of the film are observed with eye in the same position, a different set of colours will be observed.

If the film is of uniform thickness everywhere and the incident light is parallel, the path difference at each point of the film will be same and the entire film will have uniform colouration.

### 4.12.5 Colour in Thick Films

When the thickness of the film is large compared to the wavelength of light, the path difference at any point of the film will be large. Then the same point will have maximum intensity for a large number of wavelengths, and minimum intensity for another large number of wavelengths and the number of wavelengths sending maximum intensity at a point will be almost equal to the number of wavelengths sending minimum intensity. These wavelengths sending maximum and minimum intensity will be distributed equally over all the colours in white light. Hence if a certain number of wavelengths, say in red colour, is sending maximum intensity at a point, the same number of wavelengths in red is sending minimum intensity at the same point. Consequently that point will receive average intensity due to red. The same holds for all colours. Hence the resultant effect at any point will be the sum of all colours, *i.e.*, white.

### 4.12.6 An "Extended" Source is Necessary to Observe Colours in Thin Films

An 'extended' source is necessary to enable the eye to see a large area of the film simultaneously. When a thin film is illuminated by a point source [Fig. 4.24(*a*)] then, for different incident rays, different pairs of interfering rays are obtained along widely different angles. All these pairs cannot

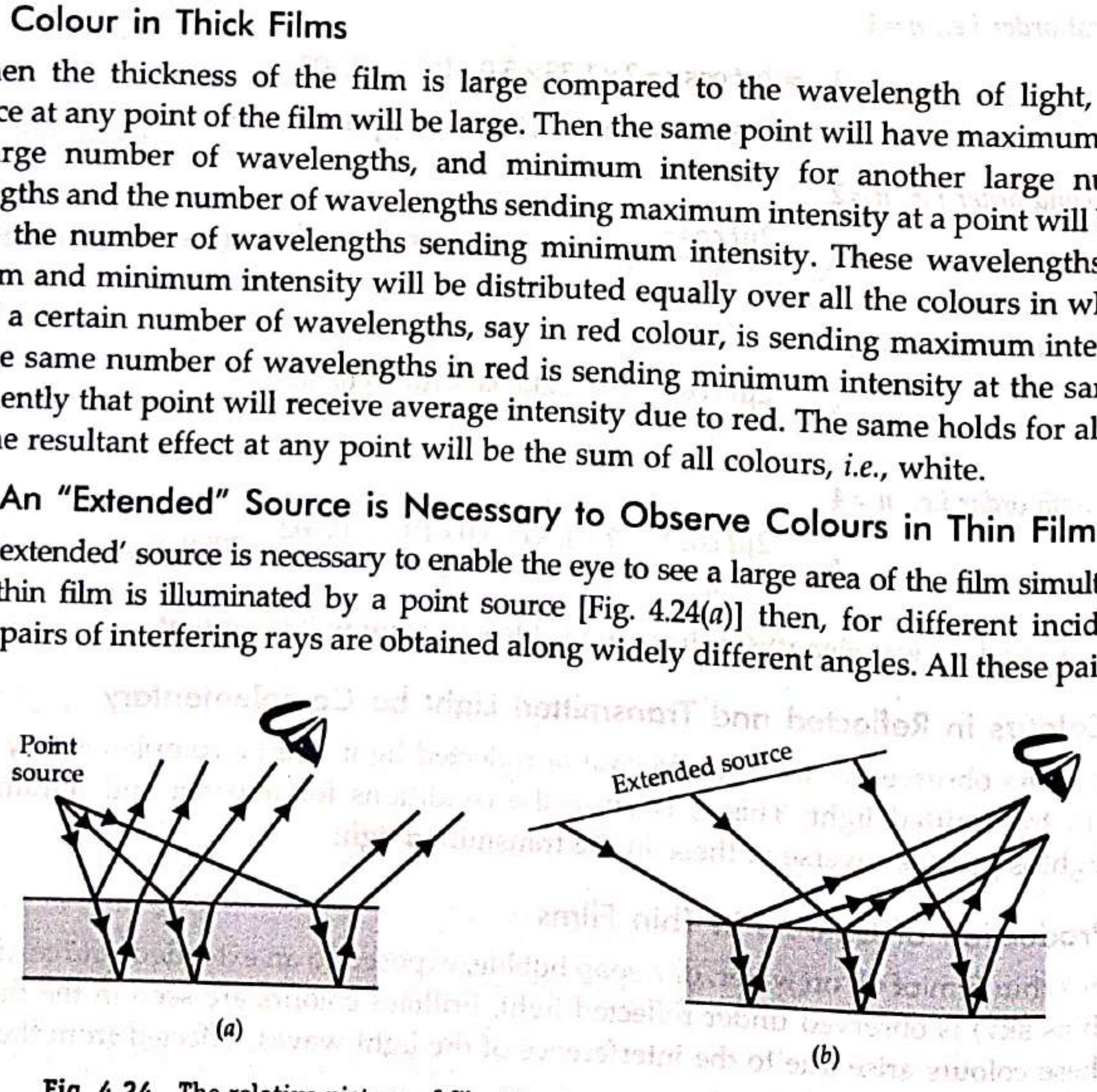


**Fig. 4.24** The relative picture of film illuminated by a point and extended sources.

be received by eye. The ray only from a small portion of the film can enter the eye. Hence the entire film cannot be seen by the eye placed in a fixed position.

When the film is illuminated by an "extended" source [Fig. 4.24(*b*)], the rays from different points of the source are reflected from different parts of the film so as to enter the eye placed in a fixed position. Hence one can see the entire film simultaneously.

### 4.12.7 How Thin Must be a Thin Film ?

We do not see interference colour, when thick layer of oil is illuminated in day light. In fact, even when a thin film is illuminated by so-called monochromatic light, the interference pattern disappears as the thickness of the film is increased beyond a certain limit.

The necessary condition for observing interference in thin films is that the path difference between two interfering beams must be less than longitudinal coherence length $l_c$ otherwise they would be incoherent. Hence, for interference to be visible, we find that

$$2\mu\, t \cos r \le \frac{\lambda^2}{\Delta\lambda}$$

or

$$t = \frac{(\lambda^2 / \Delta\lambda)}{2\mu \cos r} \qquad ...(4.63)$$

Human eyes can distinguish between colours corresponding to difference $\Delta\lambda \cong 100$ Å. Hence assuming $\lambda \sim 5000$ Å, and taking $\cos r = 1$, we get

$$t \le 8\,\mu\text{m}$$

For glass ($\mu = 1.5$) ; the glass film therefore, must be of the order of a few μm.

### 4.12.8 Explanation of Colour Effect

When a thin film is exposed to white light, *e.g.*, sunlight colours appear in the reflected light. The cause of this could be understood from the following explanation :

#### Soap Bubble

Let the thickness $t$ of indices the film of the soap bubble is constant. White light has different wavelengths and refractive indices. Due to the spherical nature of bubble the angle of refraction $r$ varies from point to point even for parallel incident beam. Hence varying values of $\mu$ and $r$ can satisfy the condition of constructive interference *i.e.*, $2\mu\, t \cos r = (2n \pm 1)\frac{\lambda}{2}$ for a particular wavelength $\lambda$ *i.e.*, for particular colour. So that point will be maximum in that particular colour.

In the similar way the other points may satisfy the condition of *constructive interference* and may appear bright in other colour.

#### Thin Layer of Oil Film

A thin layer of oil film may be obtained by powering a little oil on water surface. When parallel sun rays are incident, the angle of refraction $r$ will remain constant. For different values of $\mu$ due to different wavelength $\lambda$, the thickness of the film $t$ may not be constant for different points of the film. Thus the different points of the film satisfy, the condition of *construction interference* for different colours depending on $\mu t$ values. That is why the film appears multicoloured.

If a *monochromatic light* of wavelength $\lambda$ and refractive index $\mu$ be incident on a film where thickness $t$ is not constant. Depending on the values of $\lambda, \mu$ and $t$, certain points of the film may satisfy the condition of darkness and this points will appear dark. The other points may appear bright. So the film will have alternate dark and bright bands.

If $t$ be larger for a film, then the condition of brightness, *i.e.*, $2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ may be satisfied for large number of wavelengths. The film under this condition will exhibit no colour effect.

### 4.12.9 Classification of Fringes Exhibited by Thin Films

We have the path difference between two interfering rays of light from a transparent film of thickness $t$ and refractive index $\mu$ is

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$$

So, the phase difference

$$\varphi = \frac{2\pi}{\lambda} \times \left[2\mu t \cos r \pm \frac{\lambda}{2}\right] \qquad \text{...(4.64)}$$

Clearly $\varphi$ depends on (*i*) $\lambda$, (*ii*) $\mu t$ and (*iii*) $r$ for a particular film.

**(*i*) Fringes of Equal Inclination (FEI) [or Haidinger Fringes]**

We have
$$\varphi = \frac{4\mu t \cos r}{\lambda} = (2n \pm 1)\pi, \ldots \qquad \text{...(4.65)}$$

for bright fringes.

If $\mu t$ and $\lambda$ are constant the equation $2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ shows that a particular order number $n$ of bright fringe is governed by the particular angle $r$, *i.e.*, the *inclination of the rays with normal to the film*. Such fringes are called the *fringes of equal inclination* (FEI).

Circular fringes in Michelson interferometer are the examples of fringes of equal inclination (FEI).

**(*ii*) Fringes of Equal Thickness (FET)/Fizeau Fringes**

If $\lambda$ and $r$ are constant then from equation $2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$, we note that a particular order $n$ of the bright fringe is dependent on particular thickness and value of $\mu t$, *i.e.*, the thickness of film. Such fringes are called the *fringes of equal thickness* (FET).

Circular fringes as Newton's rings are the example of this class of fringes.

**(*iii*) Fringes of Equal Chromatic Order (FECO)**

If $\mu t$ and $r$ are constant, then a particular order number $n$ of a bright fringe is governed by $\lambda$, *i.e.*, particular colour. A particular colour fringe satisfies the condition for particular wavelength. These fringes are called *fringes of equal chromatic order* (FECO).

## 4.13 INTERFERENCE PRODUCED BY WEDGE SHAPED FILMS

Let us consider $OX$ and $OY$ are two planes, which are inclined at angle $\alpha$. A medium of refractive index $\mu$ is enclosed between these two planes. When a light ray incidents on the inclined plane, then it is reflected from top and bottom surface of the film in the form of $AR_1$ and $CR_2$. Let the angle of incidence and refraction are $\angle i$ and $\angle r$ respectively. If refractive index of medium is greater than refractive index of medium of incident ray, then $AR_1$ suffers an extra path difference of $\frac{\lambda}{2}$ [Fig. 4.25].

Since the time taken by first light ray to go from $AN$ is same as for the second ray to go from $A$ to $B$, $B$ to $C$, thus a path difference ($\Delta$) between reflected rays $AR_1$ and $CR_2$ can be written as

$$\Delta = (AB + BC)_{\text{med.}} - \left(AN \pm \frac{\lambda}{2}\right)_{\text{air}}$$

$$= (AM + MB + BC)_{\text{med.}} - \left(AN \pm \frac{\lambda}{2}\right)_{\text{air}} \qquad ...(4.66)$$

$$= \mu(AM + MB + BC) - AN \pm \frac{\lambda}{2} \qquad ...(4.67)$$

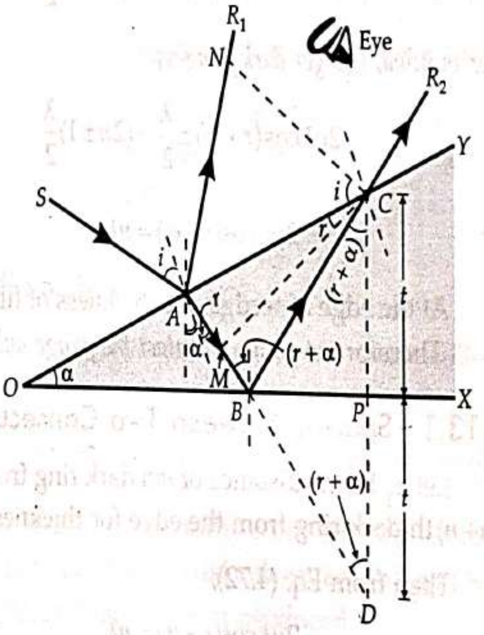


Fig. 4.25 Interference produced by wedge shaped film.

From Snell's law, $\mu = \frac{\sin i}{\sin r}$

From $\Delta$'s $ANC$ and $AMC$,

$$\mu = \frac{AN/AC}{AM/AC} \quad \Rightarrow \quad AN = \mu AM$$

Putting the value of $AN$ in Eq. (4.67), we get

$$\Delta = \mu(AM + MB + BC - AM) \pm \frac{\lambda}{2} = \mu(MB + BC) \pm \frac{\lambda}{2} \qquad ...(4.68)$$

In $\Delta$'s $CPB$ and $DPB$,

$$\angle BCP = \angle BDP = (r + \alpha)$$

$$\angle CPB = \angle DPB = 90° \text{ and } BP \text{ is common.}$$

Thus $\Delta CPB$ and $\Delta DPB$ are equilateral triangles.

$$BC = BD \quad \text{and} \quad CP = PD = t = \text{thickness of the film.}$$

Equation (4.68) becomes

$$\Delta = \mu(MB + BD) \pm \frac{\lambda}{2} = \mu \cdot MD \pm \frac{\lambda}{2} \qquad ...(4.69)$$

From $\Delta CMD$, $\quad MD = 2t\cos(r + \alpha)$

Then Eq. (4.69) becomes

$$\Delta = 2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2} \quad ...(4.70)$$

*For maxima, (to get bright fringes)*

$$2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2} = n\lambda$$

or

$$2\mu t \cos(r+\alpha) = (2n \pm 1)\frac{\lambda}{2} \quad ...(4.71)$$

*For minima, (To get dark fringes)*

$$2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

or

$$2\mu t \cos(r+\alpha) = n\lambda \quad ...(4.72)$$

**NOTE**

Since locus of constant thickness from point of contact is line, thus line fringes are formed in wedges shaped thin film. The fringes are localised fringes, because they are formed within the film due to diverging nature of reflected light.

At the edge of wedge, the thickness of film is zero, hence it satisfies the minima condition for $n=0$. Therefore *at point of contact the fringe will be dark.*

### 4.13.1 Spacing between Two Consecutive Dark Bands

Let $x_1$ be the distance of $n$th dark ring from the edge for thickness $t_1$ and $x_2$ be the distance of $(m+n)$ th dark ring from the edge for thickness $t_2$ as shown in Fig. 4.26.

Then from Eq. (4.72),

$$2\mu t \cos(r+\alpha) = n\lambda$$

$$t_1 = \frac{n\lambda}{2\mu \cos(r+\alpha)} \quad ....(4.73) \quad \text{[for } n\text{th dark band]}$$

and

$$t_2 = \frac{(m+n)\lambda}{2\mu \cos(r+\alpha)} \quad ...(4.74) \quad \text{[for } (m+n)\text{th dark band]}$$

From Fig. 4.26, we have

$$t_1 = x_1 \tan\alpha \quad ...(4.75)$$

and

$$t_2 = x_2 \tan\alpha \quad ...(4.76)$$

Then from Eqs. (4.73) and (4.75)

$$x_1 = \frac{n\lambda}{2\mu \tan\alpha \cos(r+\alpha)} \quad ...(4.77)$$

Similarly from Eqs. (4.74) and (4.76)

$$x_2 = \frac{(m+n)\lambda}{2\mu \tan\alpha \cos(r+\alpha)} \quad ...(4.78)$$

From Eqs. (4.77) and (4.78), we get

$$x_2 - x_1 = \frac{m\lambda}{2\mu \tan\alpha \cos(r+\alpha)}$$

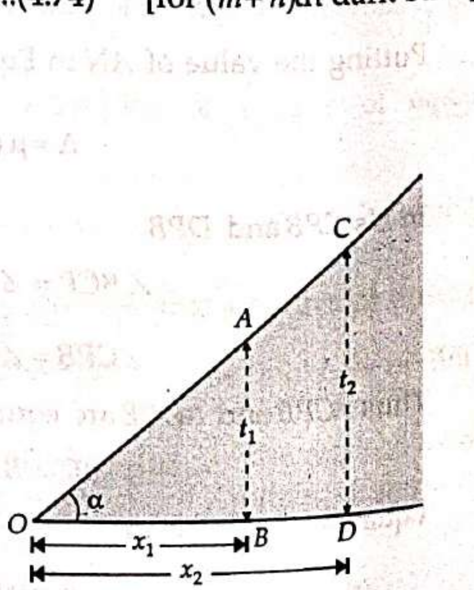


**Fig. 4.26** Wedge shaped film.

The width of single band (Fringe width)

$$\beta = \frac{(x_2 - x_1)}{m} = \frac{\lambda}{2\mu \tan\alpha \cos(r+\alpha)} \quad ...(4.79)$$

If $\alpha$ is very small, then

$$\tan\alpha = \alpha,\ r >> \alpha \quad \Rightarrow r + \alpha = r$$

$$\beta = \frac{\lambda}{2\mu\alpha \cos r} \quad ...(4.80)$$

For normal incidence $r = 0$, therefore,

$$\beta = \frac{\lambda}{2\mu\alpha} \quad ...(4.81)$$

**NOTE**

A wedge-shaped air film ($\mu = 1$) may be obtained by inserting a thin piece of paper or hair the plane parallel glass plates. Then $\alpha = \frac{t}{x}$, where $t$ is the thickness of air and $x$ its distance from the edge, where the two plates touch each other.

### 4.13.2 If White Light is Substituted for a Sodium Light

When the film is seen in white light, each colour (wavelength) produces its own interference fringes. The separation between two consecutive fringes will be least for violet, and greatest for red. At the edge of the film $t = 0$ and hence $m = \left(\frac{\lambda}{2}\right)$. Hence each wavelength gives minimum intensity at the edge. The edge will therefore be dark. As we move away from the edge in the direction of thickness increasing, we obtain a few coloured bands of mixed colour. For still greater thickness, the overlapping increases so much that uniform illumination is produced.

### 4.13.3 Testing of Optical Flatness of Surfaces

The important application of the phenomenon of interference produced by a wedge-shaped film is to measure the flatness of a glass plate.

If two surfaces $OA$ and $OB$ are perfectly plane the air film between them gradually varies in thickness from $O$ to $A$ as shown in Fig. 4.27. The fringes are of equal thickness as each fringe is the locus of the points at which the thickness of the film has a constant value.

If the fringes are not of equal thickness, it means that the surface is not flat (plane).

To test the optical flatness of a surface, the specimen surface to be tested ($OB$) is placed over an optically plane surface ($OA$). The fringes are observed in the field of view. If they are of equal thickness the surface $OB$ is plane. If not then surface $OB$ is not plane. The surface $OB$ is polished and the process is repeated. When the fringes observed are of equal width, it means the surface $OB$ is plane.

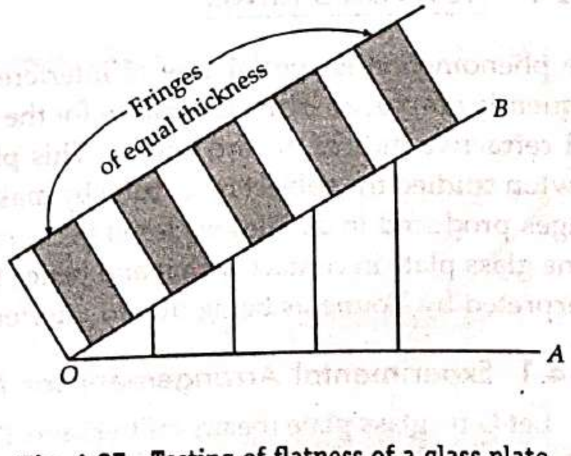


**Fig. 4.27** Testing of flatness of a glass plate.

**Example 4.12** *A beam of monochromatic light of wavelength $5.82 \times 10^{-7}$ m falls normally on a glass wedge with the wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of dark interference fringes per cm of the wedge length.*

**[GGSIPU, Sept. 2013 reappear (4 marks) ; Sept. 2012 (4 marks)]**

***Solution.*** Given $\lambda = 5.82 \times 10^{-7}$ m, $\alpha = 20''$, $\mu = 1.5$

The fringe width $\beta = \dfrac{\lambda}{2\mu\alpha}$

$$\alpha = \frac{20 \times \pi}{60 \times 60 \times 180} \text{ radian}$$

$$\beta = \frac{5.82 \times 10^{-7} \times 60 \times 60 \times 180}{2 \times 1.5 \times 20 \times \pi} = \frac{5.82 \times 6 \times 6 \times 18 \times 10^{-5}}{2 \times 1.5 \times 2 \times \pi}$$

$$= 2.0 \times 10^{-3} \text{ m} = 0.2 \text{ cm.}$$

Number of dark interference fringes ($m$) per cm of the wedge length *i.e.*,

$$\frac{m}{x_2 - x_1} = \frac{1}{\beta} = \frac{1}{0.2 \text{ cm}} = \frac{10}{2} \text{ per cm} = 5 \text{ fringes per cm.}$$

**Example 4.13** *A glass wedge of angle 0.01 radian is illuminated by monochromatic light of wavelength 6000 Å falling normally on it. At what distances from the edge of the wedge will the 10th fringe be observed by reflected light ?*

**[GGSIPU, Nov. 2006, Sept. 2005 (4 marks)]**

***Solution.*** Given that $\alpha = 0.01$ radian, $n = 10$

$$\lambda = 6000 \text{ Å} = 6000 \times 10^{-10} \text{ m}$$

The condition for dark fringe $2t = n\lambda$

The angle of wedge $\alpha = \dfrac{t}{x}$ or $t = \alpha x$

$$\therefore \quad 2x\alpha = n\lambda$$

$$x = \frac{n\lambda}{2\alpha} = \frac{10 \times 6000 \times 10^{-10}}{2 \times 0.01} = 3 \text{ mm.}$$

## 4.14 NEWTON'S RINGS

The phenomenon is special case of interference in a thin film of slowly varying thickness and frequently employed in the laboratory for the measurement of wavelength of monochromatic light and refractive indices of rare liquids. This phenomenon was first mentioned by Hooke in 1665. Newton studied the subject more fully by making careful measurement of diameters of the circular fringes produced in air film enclosed between the slightly curved surface of a convex lens and a plane glass plate in contact with it and hence they are known as *Newton's rings*. These results were interpreted by Young as being due to interference.

### 4.14.1 Experimental Arrangement for Newton's Rings

Let G be glass plate (beam splitter) and $T$ is low power microscope. $L$ is a planoconvex lens of large radius of curvature. This lens with its convex surface is placed on a plane glass plate at $C$.

Light from an extended monochromatic source such as sodium lamp falls on glass plate (using a convex lens $L_1$ (collimator lens) to do parallel, the rays coming from $S$). $G$ is held at 45° with the vertical. The glass plate G reflects normally, a part of incident light towards the air film enclosed by the lens $L$ and the glass plate $P$.

A part of the incident light is reflected by a curved surface of the lens $L$ and a part is transmitted which is reflected back from the plane surface of the plate. These two reflected rays interfere and give rise to an interference pattern in the form of *circular rings*. These rings are localised in the air film and can be seen with a low power microscope $T$ focussed on the film [Fig. 4.28].

**NOTE**

Newton's rings are foci of constant thickness of the air film these foci are concentric circles hence fringes are circular.

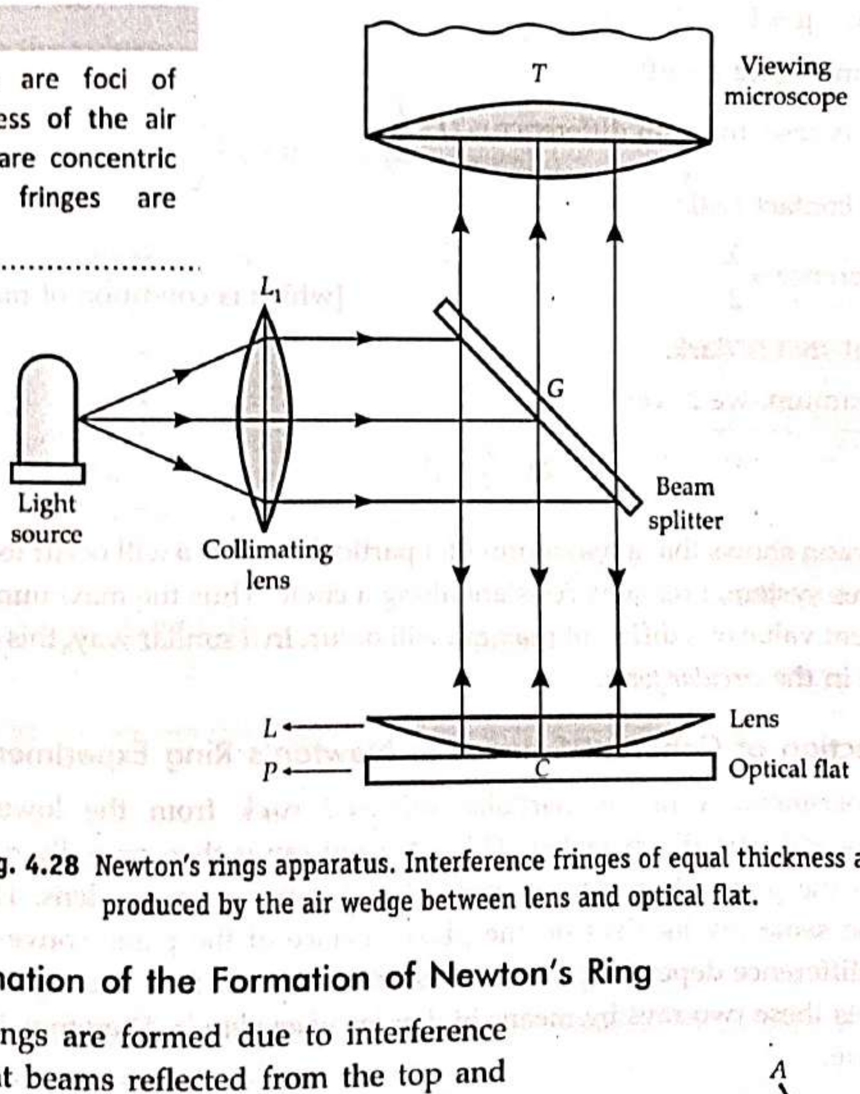


**Fig. 4.28** Newton's rings apparatus. Interference fringes of equal thickness are produced by the air wedge between lens and optical flat.

## 4.14.2 Explanation of the Formation of Newton's Ring

Newton's rings are formed due to interference between the light beams reflected from the top and bottom surfaces of air film formed between the plates. The formation of Newton's rings can be explained with the help of Fig. 4.29. $AB$ is a monochromatic light which falls on the system. A part is reflected at $C$ (glass-air interface) which goes out in the form of ray 1 without any reversal. The other part is refracted along $CD$. At point $D$, it is again reflected and goes out in the

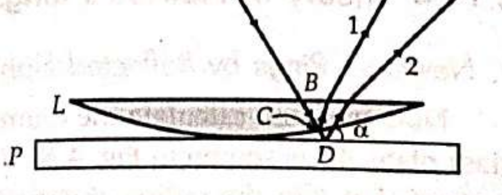


**Fig. 4.29** Formation of Newton's ring.

form of ray 2 with a phase reversal of $\pi$. The reflected rays 1 and 2 are in a position to produce interference fringes as they have been derived from the same ray $AB$ and hence fulfill the condition of interference. As the rings are observed in the reflected light, the path difference between them is

$$\text{Actual path difference} = 2\mu t\cos(r+\alpha) \pm \frac{\lambda}{2};$$

$\alpha$ is the angle of wedge, due to large radius of curvature of planoconvex lens, $\alpha$ is extremely small and can be neglected.

So, path difference $= 2\mu t\cos r \pm \dfrac{\lambda}{2}$

For air film, $\mu = 1$

and for normal incidence $\angle r = 0$

Hence in this case, the path difference $= 2t \pm \dfrac{\lambda}{2}$.

At point of contact $t = 0$.

$\therefore$ Path difference $= \dfrac{\lambda}{2}$ [which is condition of minimum intensity]

Thus central spot is dark.

For $n$th maximum, we have

$$2t \pm \frac{\lambda}{2} = n\lambda \qquad ...(4.82)$$

This expression shows that a maximum of a particular order $n$ will occur for a constant value of $t$. In case of this system, $t$ remains constant along a circle. Thus the maximum is in the form of circle. For different value of $t$, different maxima will occur. In a similar way, this can be shown that minima are also in the *circular form*.

### 4.14.3 Production of Coherent Sources in Newton's Ring Experiment

In this experiment, a ray is partially reflected back from the lower surface of the planoconvex lens and partially refracted. This refracted ray is then partially reflected back from upper surface of the plane glass plate placed below the plano-convex lens. These two rays are derived from the same ray incident on the plane surface of the plano-convex lens and have a constant phase difference depending on the thickness of the air film at the point of reflection. In this way, one gets these two rays by means of *division of amplitude*. Therefore, these two rays are coherent in nature.

### 4.14.4 Theory of Newton's Rings

*1. Newton's Rings by Reflected Light*

Now we shall calculate the diameters of dark and bright rings. Let $LOL'$ be lens placed on a glass plate $AB$ as shown in Fig. 4.30. The curved surface $LOL'$ is the part of spherical surface with centre $C$. Let $R$ be the radius of curvature and $r$ radius of corresponding ring to the constant film thickness $t$. As discussed above.

$$2t \pm \frac{\lambda}{2} = n\lambda$$

$$2t = (2n \pm 1)\frac{\lambda}{2} \qquad ...(4.83) \qquad \text{[For bright ring]}$$

where $n = 1, 2, 3, \ldots$ etc.

and

$$2t = n\lambda \qquad ...(4.84) \qquad \text{[For dark ring]}$$

where $n = 0, 1, 2, 3, 4, \ldots$ etc.

From the property of the circle, (Fig. 4.30)

$$NP \times NQ = NO \times ND$$

Substituting the values,

$$r \times r = t(2R - t)$$

$$= 2Rt - t^2 = 2Rt \quad \text{(approximately)}$$

$$\therefore \quad r^2 = 2Rt$$

or

$$t = \frac{r^2}{2R}$$

or

$$2t = \frac{r^2}{R} \qquad ...(4.85)$$

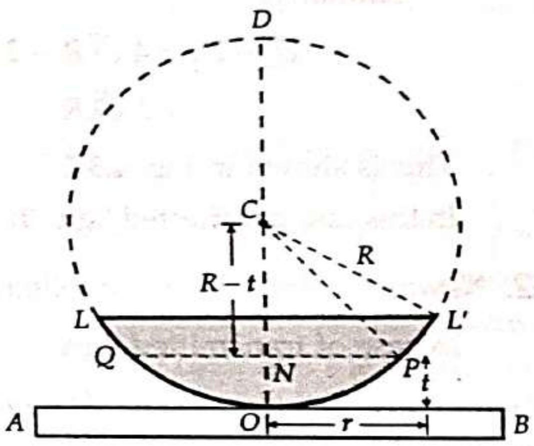


**Fig. 4.30** Essential geometry for calculating diameter of Newton's rings.

Thus for bright ring,

$$\frac{r^2}{R} = \frac{(2n \pm 1)\lambda}{2}$$

or

$$r^2 = \frac{(2n \pm 1)\lambda R}{2} \qquad ...(4.86)$$

Replacing $r$ by $\frac{D}{2}$; we get the diameter of $n$th bright ring is

$$\frac{D^2}{4} = \frac{(2n \pm 1)\lambda R}{2}$$

or

$$D = \sqrt{(2\lambda R)}\sqrt{(2n \pm 1)} \qquad ...(4.87)$$

or

$$D \propto \sqrt{(2n \pm 1)}$$

Thus the diameters of the bright rings are proportional to the square root of odd natural numbers, as $(2n \pm 1)$ is an odd number.

Similarly, for a dark ring,
$$\frac{r^2}{R} = n\lambda$$

or

$$r^2 = n\lambda R \quad \text{or} \quad D^2 = 4n\lambda R$$

$$D = 2\sqrt{n\lambda R}$$

$$D \propto \sqrt{n} \qquad ...(4.88)$$

Thus diameters of dark rings are proportional to the square roots of natural numbers.

It can be shown that fringe width decreases with the order of the fringe and fringes get closer with increase in their order.

The diameters of 16th and 9th dark rings are :

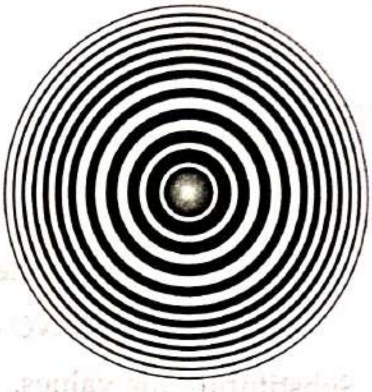

Fig. 4.31 Newton's rings [reflected case].

$$D_{16} = 2\sqrt{16\lambda R} = 8\sqrt{\lambda R}$$

$$D_9 = 2\sqrt{9\lambda R} = 6\sqrt{\lambda R}$$

$$\therefore \quad D_{16} - D_9 = 8\sqrt{\lambda}\,R - 6\sqrt{\lambda R}$$

$$= 2\sqrt{\lambda}R$$

Similarly,

$$D_4 - D_1 = 4\sqrt{\lambda}R - 2\sqrt{\lambda}R$$

$$= 2\sqrt{\lambda}R$$

This is shown in Fig. 4.31.

In this case of reflected light the central ring will be *dark*.

*2. Newton's Rings by Transmitted Light [Fig. 4.32]*

In case of transmitted light

$$2t = n\lambda \qquad \text{...(4.89)} \quad \textit{[For bright rings]}$$

and

$$2t = (2n \pm 1)\frac{\lambda}{2} \qquad \text{...(4.90)} \quad \textit{[For dark rings]}$$

*For bright rings,*

$$\therefore \quad 2 \times \frac{r^2}{2R} = n\lambda \quad \text{or} \quad r^2 = n\lambda R$$

or

$$D = 2\sqrt{n\lambda R}$$

or

$$D \propto \sqrt{n} \qquad \text{...(4.91)}$$

Fig. 4.32 Newton's rings [transmitted case].

*For dark rings,*

$$\therefore \quad 2 \times \frac{r^2}{2R} = (2n \pm 1)\frac{\lambda}{2}$$

or

$$r^2 = \frac{(2n \pm 1)\lambda R}{2}$$

$$\therefore \quad D = \sqrt{2\lambda R} \times \sqrt{(2n \pm 1)}$$

$$\therefore \quad D \propto \sqrt{(2n \pm 1)} \qquad \text{...(4.92)}$$

Thus in case of transmitted light, the central ring is *bright*. *The rings are just opposite to the rings in reflected light.*

### 4.14.5 Determination of Wavelength of Sodium Light using Newton's Ring

Experimental arrangement is shown in Fig. 4.28

*Theory.* Let $R$ be the radius of curvature of the surface in contact with the plate ; $\lambda$ be the wavelength of the light used and $D_n$ and $D_{m+n}$ the diameters of $n$th and $(m+n)$th dark rings respectively, then

$$D_n^2 = 4n\lambda R \quad \text{and} \quad D_{m+n}^2 = 4(m+n)\lambda R$$

or
$$D_{m+n}^2 - D_n^2 = 4mR\lambda$$

or
$$\lambda = \frac{D_{m+n}^2 - D_n^2}{4mR} \qquad ...(4.93)$$

Using Eq. (4.93) ; $\lambda$ can be determined.

*Procedure.* First of all the eyepiece of the microscope is adjusted on its cross-wires. Now the distance of the microscope from the film is adjusted at the rings with dark centre in well focus. The centre of cross-wires is adjusted at the centre of fringe pattern. By counting the number of fringes, the microscope is moved to the extreme left of the pattern and the cross-wire is adjusted tangentially in the middle of $n$th (say 20th) bright or dark fringes. The reading of micrometer screw (attached with eyepiece) is noted. The microscope is now moved to the right and the readings of micrometer screw are noted successively at $(n-2)$th (say 18th), $(n-4)$th (say 16th)... rings etc. till we are very near to the central dark spot. Again crossing the central dark spot in the same direction the reading corresponding to ... $(n-4)$th (16th), $(n-2)$th (18th) ... $n$th (20th) rings are noted on other side.

Now a graph is plotted between a number of rings ($n$) and square of the corresponding diameter. The graph is shown in Fig. 4.33.

From the graph, $$\frac{D_{m+n}^2 - D_n^2}{m} = \frac{AB}{CD}$$

The radius ($R$) of the planoconvex lens can be obtained with the help of spherometer using following formula,

$$R = \frac{l^2}{6h} + \frac{h}{2} \qquad ...(4.94)$$

where, $l$ is the distance between two legs of the spherometer and $h$ is the difference of the reading of spherometer. [When it is placed on the lens as well as when placed on the plane surface]

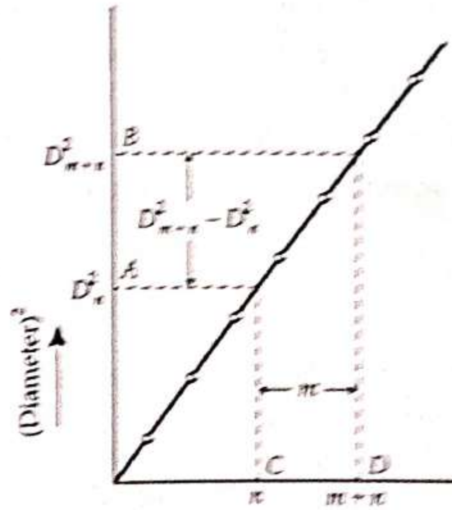


Fig. 4.33 Number of rings vs. square of the diameter of Newton's rings.

### 4.14.6 Determination of Refractive Index of a Liquid

First of all experiment is performed when there is an air film between the glass plate and planoconvex lens. The system is placed in a metal container. The diameters of $n$th and $(m+n)$th rings are determined with the help of traveling microscope as discussed in previous section 4.14.5 at page 225. So, when there is air film between glass plate and planoconvex lens, we have

$$D_n^2 = 4n\lambda R \quad \text{and} \quad D_{m+n}^2 = 4(m+n)\lambda R$$

$$D_{m+n}^2 - D_n^2 = 4m\lambda R \qquad ...(4.95)$$

Now the liquid [whose refractive index (μ) is to be determined] is poured in the container without disturbing the whole arrangement. Again the diameters of $n$th ring and $(m+n)$th ring are determined. So when there is a liquid film between glass plate and plano-convex lens, we have

$$D_n'^2 = \frac{4n\lambda R}{\mu}$$

and

$$D_{m+n}'^2 = \frac{4(m+n)\lambda R}{\mu}$$

$\therefore$

$$D_{m+n}'^2 - D_n'^2 = \frac{4m\lambda R}{\mu} \qquad ...(4.96)$$

From Eqs. (4.95) and (4.96)

$$\mu = \frac{D_{m+n}^2 - D_n^2}{D_{m+m}'^2 - D_n'^2} \qquad ...(4.97)$$

Using Eq. (4.97), the refractive index (μ) can be computed.

### 4.14.7 (*a*) Newton's Rings by Contact of Concave and Convex Surfaces

Let $R_1$ and $R_2$ be the radii of curvature of the convex and concave surfaces. It is essential that $R_2 > R_1$. In Fig. 4.34, a convex surface of radius of curvature $R_1$ is in contact with a concave surface $R_2$ at point $O$. Let us consider a Newton's ring of radius $r_n$, where the thickness of the air film is $t$.

From Fig. 4.34, it is evident that

$$t = PQ - QR = \left[\frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2}\right] \qquad ...(4.98)$$

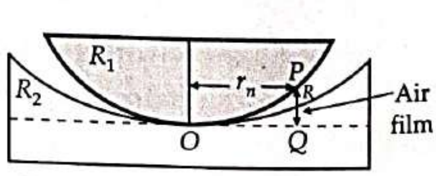


Fig. 4.34 Newton's rings by contact of concave and convex surfaces.

For the thin film and normal incidence, the path difference is given by

$$\Delta = 2\mu t = 2\mu\left[\frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2}\right] \qquad ...(4.99)$$

*For maxima or bright rings*

$$\Delta = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \quad \mu r_n^2\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (2n+1)\frac{\lambda}{2} \quad \Rightarrow \quad \mu D_n^2\left[\frac{R_2 - R_1}{2R_1R_2}\right] = (2n+1)\lambda$$

where $D_n = 2r_n$ = diameter of $n$th ring or

$$D_n^2 = \frac{2(2n+1)\lambda R_1 R_2}{\mu(R_2 - R_1)} \quad ...(4.100)$$

For air $\mu = 1$ and hence Eq. (4.100) becomes

$$D_n^2 = \frac{2(2n+1)\lambda R_1 R_2}{(R_2 - R_1)} \quad ...(4.101)$$

*For minima or dark fringes*

$$\Delta = n\lambda$$

$$\mu r_n^2 \left[\frac{1}{R_1} - \frac{1}{R_2}\right] = n\lambda$$

$$\Rightarrow \quad \mu D_n^2 \left[\frac{R_2 - R_1}{4R_1 R_2}\right] = n\lambda$$

$$\Rightarrow \quad D_n^2 = \frac{4n\lambda R_1 R_2}{\mu(R_2 - R_1)} \quad ...(4.102)$$

For air $\mu = 1$, Eq. (4.102) is as

$$D_n^2 = \frac{4n\lambda R_1 R_2}{(R_2 - R_1)} \quad ...(4.103)$$

### 4.14.7 (b) Newton's Rings by Contact of Two Convex Surfaces

Let the radii of curvature of two convex surfaces in contact (Fig. 4.35) be $R_1$ and $R_2$, then thickness of air film for convex-concave surfaces as shown in Fig. 4.35 is given by

$$t = t_1 + t_2 = \frac{r_n^2}{2R_1} + \frac{r_n^2}{2R_2} \quad ...(4.104)$$

Proceeding, exactly as for the previous case, one obtains the expression for diameter of $n$th bright ring

$$D_n^2 = \frac{2(2n+1)\lambda R_1 R_2}{\mu(R_1 + R_2)} \quad ...(4.105)$$

For air, put $\mu = 1$ in Eq. (4.105),

$$D_n^2 = \frac{2(2n+1)\lambda R_1 R_2}{(R_1 + R_2)} \quad ...(4.106)$$

The diameter of $n$th dark ring is given by

$$D_n^2 = \frac{4n\lambda R_1 R_2}{\mu(R_1 + R_2)} \quad ...(4.107)$$

For air, $\mu = 1$, the Eq. (4.107) becomes

$$D_n^2 = \frac{4n\lambda R_1 R_2}{(R_1 + R_2)} \quad ...(4.108)$$

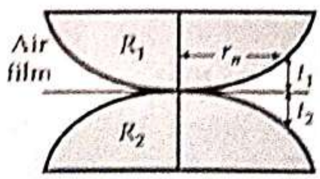


**Fig. 4.35** Newton's rings by contact of two convex surfaces.

Wavelength relation can be expressed as

$$D^2_{m+n} - D^2_n = \frac{4m\lambda R_1 R_2}{\mu(R_2 + R_1)} \quad ...(4.109)$$

Here $(R_2 - R_1)$ is the radius of curvature for concave-convex surface and $(R_2 + R_1)$ for convex-convex surface. Similarly, one can obtain expressions for transmitted patterns.

***Example 4.14*** *In a Newton's ring experiment the diameters of 4th and 12th dark rings are 0.4 cm and 0.8 cm respectively. Deduce the diameter of 20th dark ring.* **[GGSIPU, Dec. 2011 ; Dec. 2012 (2.5 marks)]**

***Solution.*** In Newton's ring experiment,

Given that : $n = 4$ ; $(m+n) = 12$, $m = 8$

$$D_n = 0.4 \text{ cm and } D_{m+n} = 0.8 \text{ cm}$$

The wavelength of sodium light using Newton's ring is

$$\lambda = \frac{D^2_{m+n} - D^2_n}{4mR}$$

or

$$4\lambda R = \frac{D^2_{m+n} - D^2_n}{m}$$

$\Rightarrow$

$$4\lambda R = \frac{(0.8)^2 - (0.4)^2}{m} \quad ...(i)$$

We know that the diameter of $n$th dark ring in presence of air is

$$D^2_n = 4n\lambda R$$

$\Rightarrow$

$$D^2_{20} = 20 \times (4\lambda R) \quad ...(ii)$$

Putting the value of $4\lambda R$ from Eq. (*i*) in Eq. (*ii*)

$$D^2_{20} = \frac{20 \times [(0.8)^2 - (0.4)^2]}{8} = \frac{20}{8} \times 1.2 \times 0.4 \quad \Rightarrow \quad D_{20} = 1.2 \text{ cm}$$

***Example 4.15*** *In a Newton's ring set up, diameter of 20th dark ring is found to be 7.25 mm. The space between spherical surface and the flat slab is then filled with water* $(\mu = 1.33)$. *Calculate the diameter of the 16th dark ring in new set up.*

***Solution.*** In Newton's ring experiment,

*1st set up :*

Given that : $D_{20} = 7.25$ mm

We know that the diameter of $n$ th dark ring in presence of air is

$$(D_n)^2 = 4n\lambda R$$

*i.e.,*

$$D^2_{20} = 4 \times 20 \times \lambda R$$

or

$$4\lambda R = \frac{(7.25)^2}{20} \quad ...(i)$$

*New set up :*

Now liquid is introduced, then diameter of $n$th ring is

$$(D'_n)^2 = \frac{4n\lambda R}{\mu}$$

$$\Rightarrow \quad (D'_{16})^2 = \frac{4\times16\times\lambda R}{1.33} \qquad [\text{as } \mu = 1.33]$$

$$= \frac{16\times(4\lambda R)}{1.33} \qquad ...(ii)$$

Putting the value of $4\lambda R$ from Eq. (*i*) in Eq. (*ii*), we get

$$(D'_{16})^2 = \frac{16\times(7.25)^2}{20\times1.33}$$

or

$$D'_{16} = \frac{4\times7.25}{\sqrt{20\times1.33}} \quad \Rightarrow \quad D'_{16} = 5.62 \text{ mm}$$

**Example 4.16** *If the diameter of nth dark ring in an arrangement giving Newton's rings changes from 3 mm to 2.5 mm as a liquid is introduced between the lenses and plate, what is the value of refractive index of the liquid ?*

***Solution.*** In Newton's ring arrangement,

Given that : $D_n = 3 \text{ mm} = 3\times10^{-3}$ m, $D'_n = 2.5 \text{ mm} = 2.5\times10^{-3}$ m, $\mu = ?$

We know that the diameter of $n$th ring in presence of liquid is

$$(D'_n)^2 = \frac{4n\lambda R}{\mu} \qquad ...(i)$$

and the diameter of $n$th ring in air is

$$(D_n)^2 = 4n\lambda R \qquad ....(ii) \qquad [\because \mu = 1 \text{ for air}]$$

Dividing Eq. (*ii*) by Eq. (*i*), we get

$$\mu = \frac{(D_n)^2}{(D'_n)^2} = \frac{(3.0)^2}{(2.5)^2} = (1.2)^2 = 1.44.$$

### 4.14.8 The Perfect Blackness of the Central Spot in Newton's Rings System

Newton's rings in reflected light are formed by interference between the ray (1) reflected directly from the upper surface of the air film and the ray (2), (3), ... etc. which are obtained after one, three, five etc. internal reflections. These rays are shown in Fig. 4.36. Near about the point of contact the thickness of the air film is almost zero and hence no path difference is introduced between the interfering rays. But ray (2)

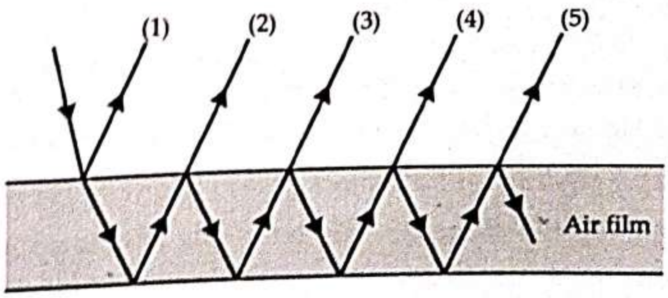


**Fig. 4.36** Illustration for perfect blackness of the central spot in Newton's rings system.

reflected from the lower surface of the film suffers a phase change of $\pi$, while the ray (1) reflected from the upper does not suffer such change. Thus the two interfering waves at the centre are opposite in phase and destroy each other. The destruction is however, not complete, since the amplitude of (2) is less than that of ray (1). But sum of the amplitudes of (2), (3), (4), ... etc., which are all in phase[4] is exactly equal to amplitude of (1) as shown by the Stoke's treatment. Hence complete destructive interference is produced and the centre of the ring system is 'perfectly' dark.

**Bright Centre**

If Newton's rings are obtained by using a crown glass lens placed on a flint glass plate with a small quantity of oil of sassafras between them, the centre of the ring system is 'bright'. This is because the oil of sassafras is optically denser than the crown glass, but rarer than the flint glass. Therefore the reflections at both the upper and lower surfaces of the film take place under similar conditions *i.e.*, in going from a rarer to a denser medium. Thus there is a phase change of $\pi$ at both reflections. Hence relative phase difference between the interfering rays at the point of contact is zero and the central spot appears bright.

### 4.14.9 Newton's Rings are Circular but Air-wedge Fringe are Straight

In both the Newton's rings arrangement and the air wedge fringes arrangement, each fringe is the locus of points of equal thickness of the film. In Newton's ring arrangement, the point of equal thickness of film lie on circles with the point of contact of the lens and plate as centre. Hence the fringes are concentric circles. In case of wedge-shaped air-film the loci of equal thickness are straight lines parallel to the edge of wedge. Hence the fringes are straight and parallel.

### 4.14.10 Expected Changes in Newton's Ring

(*a*) *If lens placed on silver glass plate in Newton's rings arrangement*

If, in Newton's rings arrangement, the top surface of the glass plate on which the lens is kept highly silvered, the rings would disappear. This is because the transmitted rays will then be reflected at silvered surface and the two complementary systems of rings would superimpose on each other, resulting in uniform illumination.

(*b*) *If white light is used*

With white light only a few coloured rings are visible, fading into general illumination. This is because the white light is composed of a number of colours (wavelengths). Each produces its own ring system having a different spacing. Therefore at a point near the point of contact, the condition for a bright ring will be satisfied by some colours, while that for a dark ring by some others. Hence the ring passing through that point will be coloured. But as we move away from the point of contact, the thickness of the film increases and therefore the number of colours at a point and the closeness of rings of each colour increases. This results in a greater overlapping and hence in general illumination.

(*c*) *If the lens is lifted slowly off the plate.* (*Effect of increasing the distance between lens and plate*).

As the distance between the lens and the plate is increased, the order of the ring at a given point increases. The rings, therefore, come closer and closer until they can no longer separately observed.

4. The rays (2), (3), (4) etc. suffer one, three, five etc. internal reflections and hence a change of $\pi$, $3\pi$, $5\pi$ etc. in phase. Thus any two consecutive rays have a phase difference of $2\pi$, they are all in the same phase.

## 4.15 NEED OF NARROW SOURCE FOR BIPRISM BUT EXTENDED SOURCE FOR NEWTON'S RINGS

In case of the biprism experiment, the wavefront emerging from the narrow slit $S$ is divided in width by the biprism. After refraction a part of the wavefront appears to diverge from $S_1$ and the other part from $S_2$. The two coherent sources $S_1$ and $S_2$ have a definite relative position and the interference fringes can be obtained anywhere in the region which permits both the coherent sources to be seen. These *non-localised fringes* have a good contrast provided the source slits is narrow. A wide slit is equivalent to a number of adjacent narrow slits, each producing its own set of fringes. These sets would be relatively displaced and result in a poor contrast due to overlapping.

Newton's rings are formed due to interference taking place between the waves reflected from the top and bottom surfaces of an airfilm. In this case the width of the incident wavefront remains intact but the amplitude is divided and fringes are *localised* in a particular plane whose position is determined by the amplitude dividing film. With a point source, entire film cannot be seen by the eye placed in a fixed position because of the limited size of the pupil of the eye. When an extended source of light is used, different points of the source, so that the entire film can be seen. Thus, there is a need of an extended source to see localised fringes.

## 4.16 APPLICATIONS OF INTERFERENCE OF LIGHT WAVES

The phenomenon of interference of light waves is quite important and widely used in different applications where precision measurements results are required. Some of important applications of interference are given below :

1. The wavelength of light can be determined upto about eight significant figure accurately, and even small wavelength difference of bi-chromatic light can be precisely determined.
2. Interference phenomenon is used to do standardization of standard meter. The standard meter is a length which contains exactly 1650763.73 wavelength of orange red light emitted by krypton-86.
3. Interference phenomenon is used in astronomic observations like determination of angular separation two stars and planets, to determine complete information regarding the position of astronomic object etc.
4. Interference of light waves is also used to determine small displacements, which are causes due to thermal expansion or compression of crystal or elongation of metal red.
5. Interference phenomenon is used for testing quality of surface finish of optical components like lens, mirror etc., during their fabrication to employ in microscope and telescope etc.
6. Interference phenomenon is used to determine the thickness, refractive index of dielectric or metallic thin film used in optical components.
7. Interference phenomena is used in producing antireflection coatings of optical instrument's component such as cameras and telescopes etc.
8. Interference phenomenon is used in interference filter to obtained monochromatic beam of light.

## 4.17 THE MICHELSON INTERFEROMETER

The Michelson interferometer, first introduced by Albert Abraham Michelson in 1881, has played a vital role in the development of Optics and Modern Physics. This simple and versatile instrument was used, for example, to establish experimental evidence for the validity of the special theory of relativity, to detect and measure hyperfine structure in line spectra, to measure the tidal effect of the moon on the earth, and to provide a substitute standard for the metre in terms of wavelengths of light. Michelson himself pioneered much of this work.

**The amplitude of light beam is divided into two parts of nearly equal intensity by partial reflection and refraction. They are sent in two directions at right angles and meet together by after reflection by plane mirror to produce interference.**

### Construction

The Michelson interferometer consists of two highly polished front silvered plane mirrors $M_1$ and $M_2$. The mirror $M_1$ is mounted on a carriage and can be moved parallel to itself with the help of a micrometer screw. The planes of the mirrors can be slightly adjusted with the fine screw attached at their backs as shown in Fig. 4.37. The mirror $M_2$ is fixed.

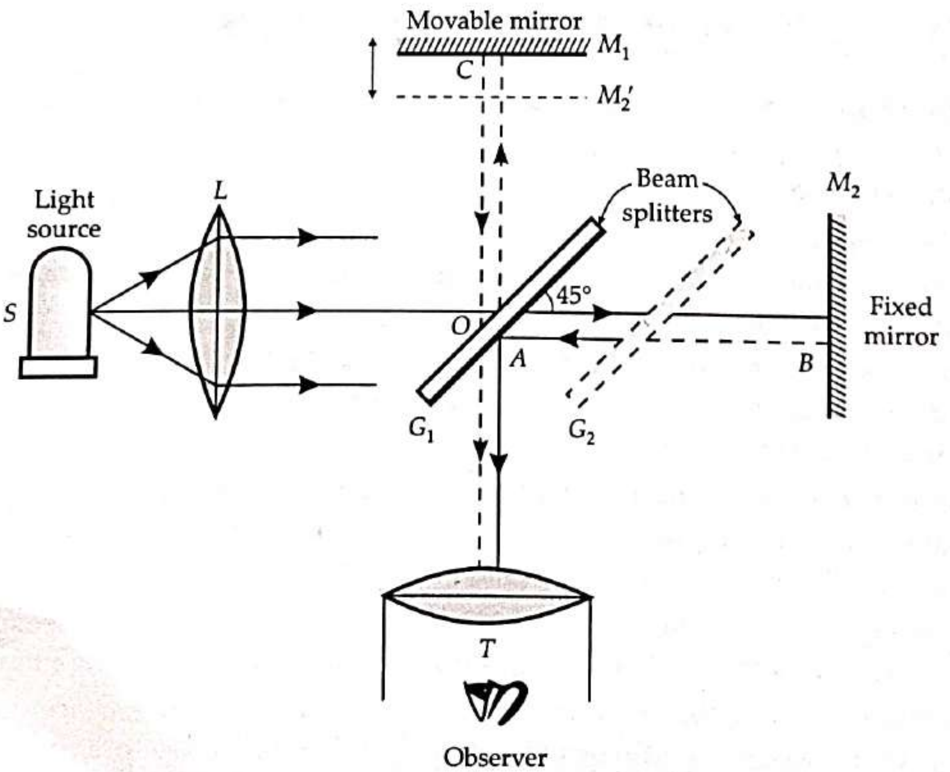


**Fig. 4.37** The experimental arrangement for Michelson Interferometer.

There are two parallel plates $G_1$ and $G_2$ of same thickness. The glass plate $G_1$ is semisilvered on the backside and functions as a beam splitter *i.e.*, a beam incident on $G_1$ is partially reflected and partially transmitted. This glass plate is inclined at an angle of 45° to the incident beam. The glass plate $G_2$ is called the compensating plate.

A telescope is positioned normal to $M_1$ to receive the reflected rays from mirrors $M_1$ and $M_2$.

**Working**

Light from an extended monochromatic source S, rendered nearly parallel by a lens L, falls on $G_1$, which is inclined at angle of 45° to the incident beam. A ray of light incident on $G_1$, is partially reflected and partially transmitted as rays AC and AB respectively. The reflected ray moves towards $M_1$ and falls normally on it. It is reflected back along the same path and emerge out along AT. The transmitted ray AB falls normally on the mirror $M_2$. It is reflected along the same path. After reflection at the back surface of $G_1$, it travels along AT. Since the two rays entering the telescope are derived from the same incident ray, they are coherent and hence in position to interfere. These two beams produce interference under suitable conditions.

*Function of Compensating Glass Plate $G_2$*

After partial reflection and transmission at O, the ray AC travels through the glass plate $G_1$ twice, while ray AB does not so even once. Thus in absence of $G_2$, the paths of rays AC and AB in glass are not equal. To equalise these paths a glass plate $G_2$, which has the same thickness as $G_1$, is placed parallel to $G_1$. $G_2$ is called the 'compensating plate'.

### 4.17.1 Types of Fringes

The simplest way of explaining the effects of interference in this case is to consider the image of $M_2$ in plate $G_1$. In case when $M_2$ and $M_1$ are at right angles and $G_1$ is inclined at 45° to either $M_2$ or $M_1$; the mirror $M_1$ and the image $M_2'$ of $M_2$ in plate $G_1$ are parallel to each other. Now treat the planes of $M_1$ and $M_2'$ as two surfaces of a thin film giving reflected beams to interfere. The path differences and hence the interference pattern depends upon :

(i) The separation between $M_1$ and $M_2'$.

(ii) The angle θ subtended on the eye.

(iii) The inclination between the two surfaces of wedge shaped film.

(a) *Circular Fringes.* Circular fringes are produced when $M_1$ and $M_2'$ are parallel. If distances of mirrors $M_2$ and $M_1$ from the plate $G_1$ differ by distance t, then separation between $M_1$ and image $M_2'$ will be 2t as shown in Fig. 4.38. The path difference along a circle with centre C as the foot of the perpendicular from the eye is constant. The circumference of this circle subtends an angle θ on the eye. Therefore, the path differences for different values of θ, shown θ and θ′ in Fig. 1.24 will be different ; but same for one value of θ. Hence a ring which satisfies the condition for constructive interference appears bright and one which satisfies the condition for destructive interference appear black with monochromatic

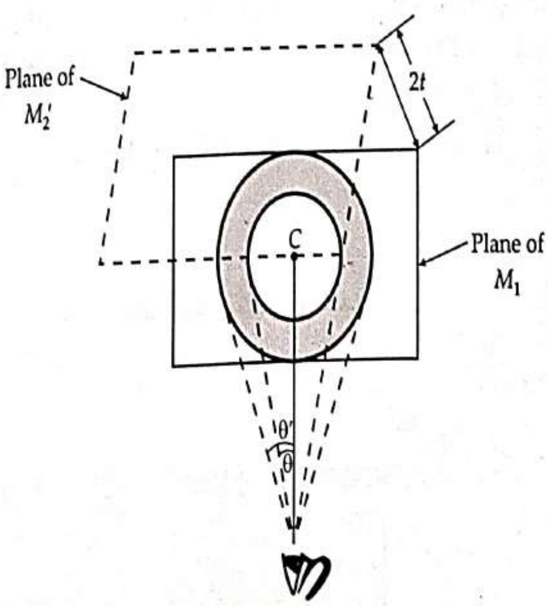


**Fig. 4.38** Formation of circular fringes in Michelson's interferometer

light and a pattern bright and dark rings resembling Newton's rings is seen. However, these rings differ from Newton's rings in origin and are called *Haidinger type of fringes*. These are fringes of equal inclination while Newton's rings are fringes of equal optical thickness (*Fizeau's fringes*). The optical path difference for bright and dark rings are given by

$$2t\cos\theta = n\lambda \qquad \text{[for bright rings]}$$

$$2t\cos\theta = (2n\pm1)\frac{\lambda}{2} \qquad \text{[for dark rings]}$$

Let the radius of the $n$th ring be $x_n$ and the distance of the foot of the perpendicular from eye be $L$, then for small value of $\theta$, it may be shown that

$$\tan\theta = \frac{x_n}{L} = \theta$$

The optical path difference for bright rings can be written as

$$2t\cos\theta = 2t\left(1-\frac{\theta^2}{2}\right) = 2t\left(1-\frac{x_n^2}{2L^2}\right) = n\lambda$$

Obviously, the order of fringes decreases as $\theta$ increases *i.e.*, as we move away from the centre. The fringes are formed at infinitely because the interfering rays are parallel.

For successive rings, we may write

$$x_{n+1}^2 - x_n^2 = \frac{\lambda L^2}{t} \qquad ...(4.110)$$

(*b*) ***Straight Bands and Curved Fringes.*** When mirror $M_1$ is not perpendicular to $M_2$ *i.e.*, the mirror $M_1$ and the virtual mirror $M_2'$ (image of $M_2$) are inclined, the air film between $M_1$ and $M_2'$ is wedge shaped. The shape of fringes for various values of path difference are shown in Fig. 4.39. In general the fringes are curved and always convex towards the thin edge of the wedge as shown in Fig. 4.39(*a*) and (*c*). The fringes are straight when $M_1$ actually intersects $M_2'$ in the middle. Hence, the essential condition for formation of straight bands is that the distance of $M_2$ and $M_1$ from plate $G_1$ are optically equal.

These fringes are formed near the film and are observed for small path differences only.

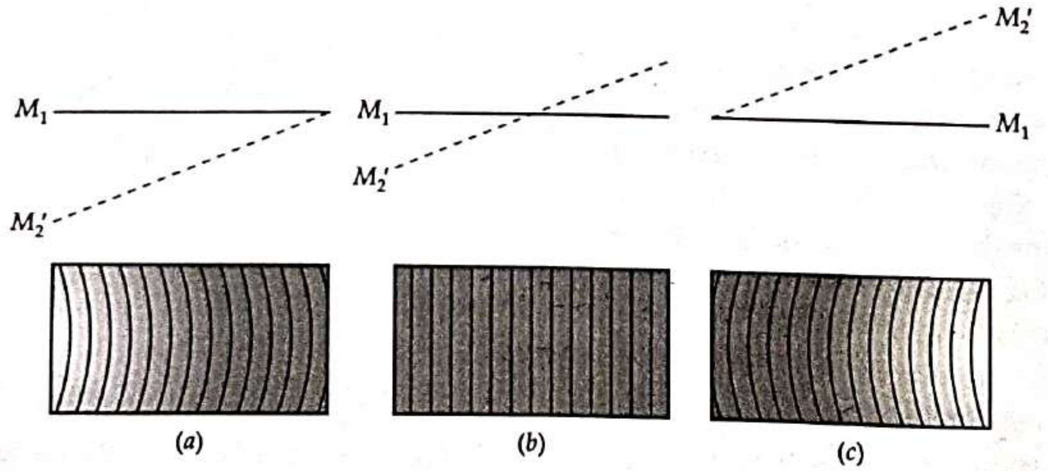


**Fig. 4.39** Straight and curved bands in Michelson's interferometer.

(*c*) ***White Light Straight Fringes.*** As shown in Fig. 4.39(*b*), the optical path difference for the plane of intersection of $M_1$ and $M_2'$ is the same for all colours. Therefore, if monochromatic light is replaced by white light, a few straight colour fringes with central white or dark appear in the field of view. The central white or dark depends upon whether the plate $G_1$ is unpolished or polished at the back. If the thickness of the film is large, a uniform illumination is obtained.

## Uses of White Light Straight Fringes

The white light straight fringes are used to find the thickness of transparent films and the procedure adopted is as given below :

- Obtain straight fringes with monochromatic light. Replace monochromatic light by white light.
- Note the position of the central achromatic fringes which is perfectly straight , with the help of micrometer screw.
- Insert the thin film in the path of beam towards the mirror $M_2$. The centre fringe shifts.
- Move the micrometer and note the shift of the central fringe by setting the cross wires on it.

If $t$ is the thickness of the film and $x$ the displacement of mirror $M$ to bring the fringe back to their initial position, then

$$2x = 2(\mu - 1)t$$

If correspond to a shift of $n$ bands of monochromatic light, then

$$x = n\lambda = (\mu - 1)t \qquad ...(4.111)$$

This relation may be used to find

(*i*) the thickness of the film or sheet

(*ii*) the refractive index of the sheet material, if the thickness is known.

## 4.17.2 Applications of Michelson Interferometer

(*a*) ***Determination of Unknown Wavelength of a Monochromatic Source.*** Using the given monochromatic source, the mirrors $M_1$ and $M_2$ are adjusted to get circular fringes in the field of view of microscope. The vertical crosswire of telescope is made to coincide with a bright fringe and the position of mirror $M_1$ is noted on the scale as $x_1$.

Now the mirror $M_1$ is moved with the help of the handle of the micrometer screw to a new position $x_2$ and the number of bright fringes, say $n$, passing through the vertical crosswire is counted. Now the path difference introduced between the rays due to displacement of $(x_2 - x_1)$ of $M_1 = 2(x_2 - x_1)$ ...(4.112)

The path difference between the rays in terms of wavelength $= n\lambda$ ...(4.113)

From Eqs. (4.112) and (4.113), we have

$$2(x_2 - x_1) = n\lambda$$

or

$$\lambda = \frac{2(x_2 - x_1)}{n} \qquad ...(4.114)$$

Equation (4.114) is used to determine the wavelength of monochromatic light.

(b) ***Determination of Thickness of a Transparent Medium.*** Initially set the Michelson interferometer to obtain circular fringes in the field of view of microscope. Coincide the vertical cross-wire of the eyepiece with any one of the bright fringes. Note the position of mirror $M_1$ as $x_1$. Now introduce the given transparent object of thickness $t$ of refractive index $\mu$ on the path of ray $AB$ in Fig. 4.37. It is seen that the bright fringe gets shifted. Now move the mirror $M_1$ such that the same bright fringe is made to coincide with the vertical cross-wire. Note the position of mirror $M_2$ as $x_2$.

The path difference introduced between the rays due to displacement $(x_2 - x_1)$ of $M_1$

$$= 2(x_2 - x_1) \quad \text{...(4.115)}$$

Also the path difference introduced between the rays due to introduction of transparent material

$$= 2t(\mu - 1) \quad \text{...(4.116)}$$

From Eqs. (4.115) and (4.116), we get

$$2(x_2 - x_1) = 2t(\mu - 1) \quad \text{...(4.117)}$$

or

$$t = \frac{(x_2 - x_1)}{(\mu - 1)} \quad \text{...(4.118)}$$

We also know that $2(x_2 - x_1) = n\lambda$, then

$$2t(\mu - 1) = n\lambda \quad \text{...(4.119)}$$

or

$$\mu = \frac{n\lambda}{2t} + 1 \quad \text{...(4.120)}$$

(c) ***Wavelength Separation between Closely Spaced Spectral Lines.*** Michelson interferometer is extremely sensitive and versatile instrument. Its least count is as low as few micron and hence it can be used to measure the wavelength separation between closed spectral lines for example $D_1$ and $D_2$ lines of sodium.

Let two closely spaced spectral lines have wavelengths $\lambda_1$ and $\lambda_2$ and $\lambda_1 > \lambda_2$. When interferometer is adjusted for circular fringes both wavelengths $\lambda_1$ and $\lambda_2$ produce their own rings. The mirror is moved so that best contrast circular fringes are obtained. This shall happen when path difference is such that maximum due to $\lambda_1$, coincides with maximum due to $\lambda_2$. Under this condition, say $n_1$ order of $\lambda_1$ coincides with $n_2$ order of $\lambda_2$,

*i.e.,*

$$\Delta = n_1\lambda_1 = n_2\lambda_2 \quad \text{...(4.121)}$$

As the mirror separation is increased gradually the contrast decreases it becomes worst and then again increases and becomes best again.

Suppose the mirror $M_1$ is displaced by $x$ while moving from one best contrast to next best contrast. The condition occurs when $(n_1 + m)$ order due to $\lambda_1$ coincides with $(n_2 + m + 1)$ order due to $\lambda_2$. The new position implies that

$$\Delta + 2x = (n_1 + m)\lambda_1 = (n_2 + m + 1)\lambda_2 \quad \text{...(4.122)}$$

Subtracting Eq. (4.121) from Eq. (4.122), we get

$$2x = m\lambda_1 = (m+1)\lambda_2$$

$$\Rightarrow \quad m\lambda_1 = (m+1)\lambda_2 \qquad ...(4.123)$$

$$\Rightarrow \quad m = \frac{\lambda_2}{(\lambda_1 - \lambda_2)} \qquad ...(4.124)$$

Substituting the value of $m$ in Eq. (4.124), we get

$$2x = m\lambda_1 = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

or

$$\lambda_1 - \lambda_2 = \Delta\lambda = \frac{\lambda_1\lambda_2}{2x} \qquad ...(4.125)$$

Therefore, $\lambda_1$ and $\lambda_2$ are quite close so $\sqrt{\lambda_1\lambda_2} = \lambda$

or

$$\Delta\lambda = \frac{\lambda^2}{2x} \qquad ...(4.126)$$

Thus, measuring mirror movement between two best contrast positions of interference fringes, $\Delta\lambda$ can be obtained.

***Example 4.17.*** *Michelson interferometer is set for straight fringes using light of $\lambda = 5000$ Å. Calculate the number of fringes that move across the field of view, when one of the mirrors is moved back by a distance of 0.1 mm.*

***Solution.*** In Michelson interferometer,

Given that : $\lambda = 5000$ Å, $(x_2 - x_1) = 0.1$ mm, $n = ?$

We know that, $2(x_2 - x_1) = n\lambda$

or

$$n = \frac{2(x_2 - x_1)}{\lambda}$$

$$= \frac{2 \times 0.1 \times 10^{-3}}{5000 \times 10^{-10}} = \frac{2000}{5} = 400$$

***Example 4.18.*** *A thin transparent sheet of refractive index $\mu = 1.6$ is introduced in one of the beams of Michelson interferometer and a shift of 24 fringes for $\lambda = 6000$ Å is obtained. Calculate the thickness of the sheet.*

***Solution.*** In Michelson interferometer,

Given that : $\lambda = 6000$ Å $= 6.0 \times 10^{-7}$ m, $\mu = 1.6$, $n = 24$ ; $t = ?$

We know that, $2t(\mu - 1) = n\lambda$

or

$$t = \frac{n\lambda}{2(\mu - 1)}$$

$$= \frac{24 \times 6.0 \times 10^{-7}}{2 \times (1.6 - 1)} = \frac{24 \times 6.0 \times 10^{-7}}{2 \times 0.6} = 120 \times 10^{-7} \text{ m}$$

$$t = 1.2 \times 10^{-5} \text{ m.}$$

## Formulae at a Glance

**4.1** Phase difference $(\varphi) = \frac{2\pi}{\lambda} \times$ path difference$(x)$ ;

$$x = (S_2P - S_1P)$$

**4.2** In Young's double slits experiment

(*a*) Resultant intensity

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\varphi$$

$I_1$ and $I_2$ be intensities of two waves

where $a_1, a_2$ = amplitudes of two light waves

$\varphi$ = phase difference

or $$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\varphi$$

(*b*) At maxima,

$$\varphi = 2n\pi, \; x = n\lambda$$

$$I_{max} = (a_1 + a_2)^2 = I_1 + I_2 + 2\sqrt{I_1I_2}$$

$$I_{max} > (I_1 + I_2)$$

(*c*) At minima, $\varphi = (2n+1)\pi$,

$$x = (2n \pm 1)\frac{\lambda}{2}$$

$$I_{min} = (a_1 - a_2)^2 = I_1 + I_2 - 2\sqrt{I_1I_2}$$

$$I_{min} < (I_1 + I_2)$$

If $a_1 = a_2$ then $I_{min} = 0$

(*d*) Average intensity $I_{av} = I_1 + I_2$

**4.3** Theory of interference fringes

(*a*) $(S_2P - S_1P) = \frac{2xd}{D}$

where

$S_2P - S_1P$ = path difference

$(2d)$ = separation between two slits

$D$ = distance between slits and screen

$x$ = distance of the observed fringe from central fringe or displacement

(*b*) Position of bright fringes

$$\frac{2xd}{D} = n\lambda \quad \text{or} \quad x = \frac{n\lambda D}{2d}$$

(*c*) Position of dark fringes

$$\frac{(2n \pm 1)\lambda}{2} = \frac{2xd}{D}$$

or $$x = \frac{(2n \pm 1)\lambda D}{4d}$$

(*d*) Fringe width $(x_2 - x_1) = \beta = \frac{\lambda D}{2d}$

**4.4** Fersnel's biprism

(*a*) The wavelength of monochromatic light

$$\lambda = \frac{(2d)}{D}\beta = \frac{2d}{(a+b)}\beta$$

where $a$ = distance between slits and biprism,

$b$ = distance between biprism and screen.

(*b*) Distance between 2 virtual sources

(*i*) deviation Method : $2d = 2a(\mu - 1)\alpha$

where $\alpha$ = refractive angle of biprism.

(*ii*) displacement Method : $2d = \sqrt{xy}$

where $x$ = size of image formed by first position of lens

$y$ = size of image formed by second position of lens.

**4.5** Interference due to thin sheet

(*a*) $n = \frac{(\mu - 1)t}{\lambda}$

(*b*) The thickness of the plate

(*i*) $t = \frac{n\lambda}{(\mu - 1)}$ (*ii*) $t = \frac{x \times (2d)}{D(\mu - 1)}$

where $n$ = number of fringes,

$x$ = displacement,

$\mu$ = refractive index of sheet.

**4.6** Interference from parallel thin film

(*a*) Interference due to reflected light

The path difference $\Delta = 2\mu t\cos r$

where $\mu$ = refractive index of film,

$t$ = thickness of film,

$r$ = angle of refraction.

(*i*) For maxima, $2\mu t\cos r = (2n \pm 1)\frac{\lambda}{2}$

[Film will be bright]

(*ii*) For minima, $2\mu t\cos r = n\lambda$

[Film will be dark]

(*b*) Interference due to transmitted light

(*i*) For maxima, $\Delta = 2\mu t \cos r = n\lambda$ [Bright film]

(*ii*) For minima, $\Delta = 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ [Dark film]

**4.7** Wedge-shaped film [Reflected case]

(*a*) For Bright fringes

$$2\mu t \cos(r + \alpha) = (2n \pm 1)\frac{\lambda}{2}$$

(*b*) For dark fringes

$$2\mu t \cos(r + \alpha) = n\lambda$$

$\beta$ = width of single band= fringe width

$$= \frac{(x_2 - x_1)}{m} = \frac{\lambda}{2\mu\alpha}$$

where $x_1$ = distance of $n$th dark band from the edge of the wedge

$x_2$ = distance of $(m+n)$ th dark band from the edge of the wedge

$\alpha$ = angle formed by wedge film.

**4.8** Newton's ring $2t = \frac{r^2}{R}$

where $t$ = thickness of the film,

$r$ = radius of the ring and

$R$ = radius of curvature.

(*a*) Newton's ring by reflected light ($D$ = Diameter of ring)

(*i*) For bright ring

$$2\frac{r^2}{2R} = (2n \pm 1)\frac{\lambda}{2}$$

or $$r^2 = \frac{(2n \pm 1)\lambda R}{2}$$

or $$D = \sqrt{(2\lambda R)}\sqrt{(2n \pm 1)}$$

(*ii*) For dark ring $2\frac{r^2}{2R} = n\lambda$

or $r^2 = n\lambda R$ or $D = 2\sqrt{n\lambda R}$

(*b*) Newton's ring by transmitted light

(*i*) For bright ring

$$2\frac{r^2}{2R} = n\lambda \quad \text{or} \quad r^2 = n\lambda R$$

or $D = 2\sqrt{(n\lambda R)}$

(*ii*) For dark ring

$$2 \times \frac{r^2}{2R} = (2n \pm 1)\frac{\lambda}{2}$$

or $$r^2 = \frac{(2n \pm 1)\lambda R}{2}$$

or $$D = \sqrt{2\lambda R} \times \sqrt{(2n \pm 1)}$$

(*c*) Wavelength of sodium light

$$\lambda = \frac{D_{m+n}^2 - D_n^2}{4mR}$$

where $D_{m+n}$ = Diameter of $(m+n)$th ring in air.

$D_n$ = Diameter of $n$th ring in air.

(*d*) Refractive index of a liquid :

$$\mu = \frac{D_{m+n}^2 - D_n^2}{D_{m+n}'^2 - D_n'^2}$$

where

$D_{m+n}$ = Diameter of $(m+n)$th ring in medium of refractive index ($\mu$)

$D_n$ = Diameter of $n$th ring in medium of refractive index ($\mu$)

**4.9** (*a*) Newton's rings by contact of concave and convex surfaces.

For maxima, $$D_n^2 = \frac{2(2n+1)\lambda R_1 R_2}{(R_2 - R_1)}$$

For minima, $$D_n^2 = \frac{4n\lambda R_1 R_2}{(R_2 - R_1)}$$

(*b*) Newton's rings by contact of two convex surfaces

For maxima, $$D_n^2 = \frac{2(2n+1)\lambda R_1 R_2}{(R_1 + R_2)}$$

For minima, $$D_n^2 = \frac{4n\lambda R_1 R_2}{(R_1 + R_2)}$$

(*c*) $$D_{m+n}^2 - D_n^2 = \frac{4m\lambda R_1 R_2}{R_1 + R_2}$$

**4.10** Michelson interferometer

(*a*) Shape of fringes

(*i*) Circular fringes

For bright rings → $2t\cos\theta = n\lambda$

[$t$ = distances differ from $M_1$ and $M_2$]

$\theta$ = angle subtended by circle

For dark fringe → $2t\cos\theta = (2n \pm 1)\frac{\lambda}{2}$

If $x_n$ = radius of $n$th ring

$L$ = distance of the fool ⊥ from eye

then $\tan\theta = \frac{x_n}{L} = \theta$

then $2t\cos\theta = 2t\left(1 - \frac{\theta^2}{2}\right)$

$$= 2t\left(1 - \frac{x_n^2}{2L^2}\right) = n\lambda$$

$$(x_{n+1}^2 - x_n^2) = \frac{\lambda L^2}{t}$$

(*b*) White light straight fringes

$t$ = thickness of the film

$x$ = displacement of mirror $M$ to bring the image back their position

$2x = 2(\mu - 1)t$

$\mu$ = refractive index of film

$x = n\lambda = (\mu - 1)t$

(*c*) Wavelength of monochromatic light

$$\lambda = \frac{2l}{m}$$

where $l$ = displacement

$m$ = fringes are produced.

(*d*) Refractive index of thin plane sheet

$$\mu = 1 + \frac{m\lambda}{2t}$$

$\mu$ = refractive index of sheet.

(*e*) Small difference in two wavelengths from the same source

$$\Delta\lambda = \frac{\lambda^2}{2l}$$

$\Delta\lambda = (\lambda_1 - \lambda_2)$ $[\lambda^2 = \lambda_1\lambda_2]$

## Miscellaneous Solved Numerical Problems

**Problem 4.1** *Two waves of same frequency have amplitudes 1.00 and 2.00. They interference at a point, where the phase difference is 60°. What is the resultant amplitude ?* **[GGSIPU, Dec. 2009 (3 marks)]**

***Solution.*** Given that $a_1 = 1.00,\ a_2 = 2.00$ and $\varphi = 60°$

We know that, the resultant amplitude

$$R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\varphi}$$

$$= \sqrt{1^2 + 2^2 + 2(1)(2)\cos 60°}$$

$$= \sqrt{1+4+2} = \sqrt{7} = 2.65 \text{ unit.}$$

**Problem 4.2** *Superimpose the following waves*

$$y_1 = 20\sin\omega t;\quad y_2 = 20\sin(\omega t + 60°)$$

*Show also the superimposition diagrammatically.* **[GGSIPU, Dec. 2013 reappear (3 marks)]**

***Solution.*** Given $a_1 = 20, a_2 = 20$ and $\varphi = 60°$

The resultant amplitude $R = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\varphi}$

$$= \sqrt{(20)^2 + (20)^2 + 2 \times 20 \times 20 \times \cos 60^\circ}$$

$$= \sqrt{400 + 400 + 400} = 20\sqrt{3} = 20 \times 1.732 = 34.64 = \mathbf{35}$$

Direction $\tan\theta = \dfrac{a_2 \sin\varphi}{a_1 + a_2 \cos\varphi}$

$$= \frac{20 \times \sin 60^\circ}{20 + 20\cos 60^\circ} = \frac{20 \times \frac{\sqrt{3}}{2}}{20 + 20 \times \frac{1}{2}} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Resultant displacement $Y = 20\sqrt{3}\sin(\omega t + 30^\circ)$ for $Y = 20\sqrt{3}\sin(\omega t + 30^\circ)$

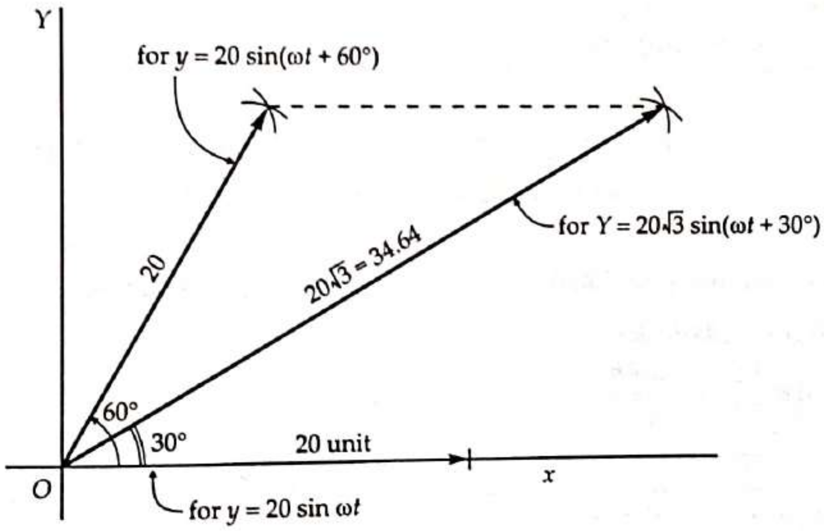


**Fig. 4.40** Superimposition of two light waves.

**Problem 4.3** *A light source emits light of two wavelengths $\lambda_1 = 4300$ Å and $\lambda_2 = 5100$ Å. The source is used in a double slit interference experiment. The distance between the source and the screen is 1.5 m and the distance between the slits is 0.025 mm. Calculate the separation between the third order bright fringes due the these two wavelengths.*

***Solution.*** In Young's double slit experiment,

Given $\lambda_1 = 4300 \text{ Å} = 4.3 \times 10^{-7}$ m and $\lambda_2 = 5100 \text{ Å} = 5.1 \times 10^{-7}$ m.

$D = 1.5$ m, $2d = 0.025$ mm $= 2.5 \times 10^{-5}$ m

$n = 3$ [for third bright fringes]

$$\Delta x = (x_3)_{\lambda_2} - (x_3)_{\lambda_1} = (x_2 - x_1) = ?$$

$$x_2 - x_1 = \frac{n\lambda_2 D}{2d} - \frac{n\lambda_1 D}{2d} = \frac{nD}{2d}[\lambda_2 - \lambda_1]$$

Putting the values : $(x_2 - x_1) = \dfrac{3 \times 1.5 \times (5.1 - 4.3) \times 10^{-7}}{2.5 \times 10^{-5}} = 0.0144 \text{ m} = 1.44 \text{ cm}.$

**Problem 4.4** *The inclined faces of a biprism of refractive index 1.5 make an angle of 2° with the base. A slit illuminated by monochromatic light is placed at a distance of 10 cm from the biprism. If the distance between two dark fringes observed at a distance of 1 cm from the prism is 0.18 mm, find the wavelength of light used.*

***Solution.*** In biprism experiment,

Given $\mu = 1.5$, $\alpha$ = angle of prism $= 2° = \frac{\pi}{90}$ radian

$$a = 10 \text{ cm} = 0.10 \text{ m}, \quad b = 1 \text{ m}, \quad D = a + b = 1.1 \text{ m}$$

$$(2d) = 2(\mu - 1)\alpha a = 2 \times (1.5 - 1) \times \frac{\pi}{90} \times 0.10 = 3.49 \times 10^{-3} \text{ m.}$$

$\lambda$ = wavelength of monochromatic source = ?

For which fringe width ($\beta$) = 0.18 mm $= 1.8 \times 10^{-4}$ m

$$\because \quad \beta = \frac{\lambda D}{2d} \quad \Rightarrow \quad \lambda = \frac{\beta(2d)}{D}$$

$$= \frac{1.8 \times 10^{-4} \times 3.49 \times 10^{-3}}{1.1} = 5.711 \times 10^{-7} \text{ m} = 5711 \text{ Å}$$

**Problem 4.5** *A biprism is placed at a distance of 5 cm from slit illuminated by sodium light of wavelength 5890 Å. Find the width of fringes observed in eyepiece at a distance of 75 cm from biprism, given the distance between virtual sources is 0.005 cm.* **[GGSIPU, Oct. 2013 (2 marks)]**

***Solution.*** Given $a = 5$ cm, $\lambda = 5890$ Å, $\beta = ?$, $b = 75$ cm, $2d = 0.005$ cm

The fringe width ($\beta$) is given as

$$\beta = \frac{\lambda D}{2d} = \frac{\lambda(a+b)}{2d}$$

$$= \frac{5890 \times 10^{-8} \text{ cm}(5+75)\text{ cm}}{0.005 \text{ cm}} = \frac{5890 \times 10^{-8} \times 80}{0.005} \text{ cm} = 589 \times 8 \times 10^{-3} \text{ cm}$$

$$= 4.712 \text{ cm.}$$

**Problem 4.6** *A beam of parallel rays is incident at an angle of 30° with the normal on a plane parallel film of thickness $4 \times 10^{-5}$ cm and refractive index 1.50. Show that the refracted light whose wavelength is $7.539 \times 10^{-5}$ cm will be strengthened by reinforcement.*

***Solution.*** For plane parallel film :

Given : $t = 4 \times 10^{-5}$ cm, angle of incidence ($i$) = 30°, $\mu = 1.5$

Show that : $\lambda = 7.549 \times 10^{-5}$ cm will be strengthened by reinforcement in reflected light.

$$\mu = \frac{\sin i}{\sin r} \quad \Rightarrow \quad \sin r = \frac{\sin i}{\mu} \quad \Rightarrow \quad \sin r = \frac{0.5}{1.5} = 0.33$$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.33)^2} = 0.9432.$$

$\because$ Thin film in reflected region.

$$2\mu t \cos r = (2n - 1)\frac{\lambda}{2}\ ; \quad n = 1, 2, 3, 4$$

$$\lambda = \frac{4\mu t \cos r}{(2n-1)} = \frac{4\times 1.5\times 4\times 10^{-5}\times 0.9437}{(2n-1)} = \frac{22.6488\times 10^{-5}}{(2n-1)} \text{ cm.}$$

$n=1$, $$\lambda_1 = \frac{22.6488\times 10^{-5}}{1} = 22.6488\times 10^{-5} \text{ cm.}$$

$n=2$, $$\lambda_2 = \frac{22.6488\times 10^{-5}}{3} = 7.5496\times 10^{-5} \text{ cm.}$$

$n=3$, $$\lambda_3 = \frac{22.6488\times 10^{-5}}{5} = 4.5297\times 10^{-5} \text{ cm.}$$

and $n=4$, $$\lambda_4 = \frac{22.6488\times 10^{-5}}{7} = 3.2355\times 10^{-5} \text{ cm.}$$

So the wavelength of $7.5496\times 10^{-5}$ cm and $4.5297\times 10^{-5}$ cm will be strengthened by reinforcement.

**Problem 4.7** *Interference fringes are produced by monochromatic light falling normally on wedge shaped film of cellophane whose refractive index is 1.4. The angle of wedge is 40" and distance between successive fringes is 1.25 mm. Calculate the wavelength of light used.*

***Solution.*** Given $\mu = 1.4$, $\alpha = 40'' = \frac{40}{60\times 60}\times\frac{3.14}{180}$ radian, $\beta = 1.25$ mm $= 1.25\times 10^{-3}$ m.

For normal incidence, the fringe width $\beta$ is

$$\beta = \frac{\lambda}{2\mu\alpha} \qquad \text{[here } \lambda = \text{wavelength of monochromatic light]}$$

Then $$\lambda = 2\mu\alpha\beta$$

$$= 2\times 1.4\times\frac{40\times 3.14}{60\times 60\times 180}\times 1.25\times 10^{-3} = 6.784\times 10^{-7} \text{ m} = 6784 \text{ Å}$$

**Problem 4.8.** *In Newton's ring experiment an air film is formed between two convex surfaces each of radius of curvature 1 m. Newton's rings are generated by using a light of wavelength 5000 Å. Find the distance between 16th and 9th dark rings.*

***Solution.*** Given $n = 16$ or 9, $\lambda = 5000$, Å $= 5.0\times 10^{-7}$ m, $R_1 = R_2 = 1$.

We know that the diameter of $n$th dark ring (if there are two convex surfaces).

$$D_n^2 = \frac{4n\lambda R_1 R_2}{(R_1 + R_2)} \qquad \text{[if system is in air]}$$

Then radius $$r_n = \sqrt{\frac{n\lambda R_1 R_2}{(R_1 + R_2)}}$$

For $n=16$ $$r_{16} = \sqrt{\frac{16\times 5.0\times 10^{-7}\times 1\times 1}{1+1}} = 2\times 10^{-3} \text{ m} = 2 \text{ mm}$$

and For $n=9$ $$r_9 = \sqrt{\frac{9\times 5.0\times 10^{-7}\times 1\times 1}{1+1}} = 1.5\times 10^{-3} \text{ m} = 1.5 \text{ mm.}$$

Then distance between 16th and 9th dark rings $= r_{16} - r_9 = 2.0 - 1.5$ mm $= 0.5$ mm.

**Problem 4.9** *Newton's rings are observed normally in reflected light of wavelength $5.9\times10^{-5}$ cm. The diameter of the 10th dark ring is 0.50 cm. Find the radius of curvature of the lens and thickness of the film.*

***Solution.*** Given $\lambda=5.9\times10^{-5}\text{ cm}=5.9\times10^{-7}\text{ m}$, $n=10$, $D_{10}=0.50\text{ cm}=5\times10^{-3}\text{m}$

We know that diameter of $n$th dark ring is

$$D_n^2=4n\lambda R$$

Then radius of curvature $R$ will be

$$R=\frac{D_n^2}{4n\lambda}=\frac{(5.0\times10^{-3})^2}{4\times10\times5.9\times10^{-7}}=1.06\text{ m}=106\text{ cm}.$$

The thickness of the film is

$$2t=\frac{r^2}{R}\quad\text{or}\quad t=\frac{r^2}{2R}$$

$$=\frac{D_n^2}{8R}=\frac{(5.0\times10^{-3})^2}{8\times1.06}=2.95\times10^{-6}\text{m}\qquad\left[\text{as } r=\frac{D_n}{2}\right]$$

**Problem 4.10** *In a Newton's ring experiment, the wavelength of light used is $6.0\times10^{-5}$ cm and the difference of square of diameters of successive rings are 0.125 cm$^2$. What will happen to this quantity if:*

*(i) The wavelength of light is changed to $4.5\times10^{-5}$ cm?*

*(ii) The liquid of refractive index 1.33 is introduced between the lens and the glass plate?*

*(iii) The radius of curvature of convex surface of the plano-convex lens is doubled?*

***Solution.*** Given $\lambda=6.0\times10^{-7}\text{ cm}=6.0\times10^{-7}\text{m}$,

$$(D_{m+n}^2-D_n^2)=0.125\text{ cm}^2=1.25\times10^{-5}\text{m}^2\quad[\text{in } I \text{ medium}]$$

(*i*) Since we know that

$$D_{m+n}^2-D_n^2=\frac{4m\lambda R}{\mu}\qquad\text{...(i)}$$

Here $m=1$ [because as per question, difference is for two successive rings]

$$D_{n+1}^2-D_n^2=\frac{4\lambda R}{\mu}\qquad\text{...(ii)}$$

From Eq. (*ii*), we see that

$$(D_{n+1}^2-D_n^2)\propto\lambda$$

Then
$$\frac{(D_{n+1}^2-D_n^2)_{\lambda_1}}{(D_{n+1}^2+D_n^2)_{\lambda_2}}=\frac{\lambda_1}{\lambda_2}$$

or
$$(D_{n+1}^2-D_n^2)_{\lambda_2}=\frac{\lambda_2}{\lambda_1}(D_{n+1}^2-D_n^2)_{\lambda_1}\qquad[\lambda_1=6.0\times10^{-7}\text{m and }\lambda_2=4.5\times10^{-7}\text{ m}]$$

Then
$$(D_{n+1}^2-D_n^2)_{\lambda_2}=\frac{4.5\times10^{-7}}{6.0\times10^{-7}}\times1.25\times10^{-5}\text{m}^2=9.38\times10^{-6}\text{m}^2$$

(*ii*) From the Eq. (*i*), it can be seen that

$$(D^2_{m+n} - D^2_n) \propto \frac{1}{\mu}$$

Then $$\frac{[D^2_{m+n} - D^2_n]_{I\text{ medium}}}{(D^2_{m+n} - D^2_n)_{II\text{ medium}}} = \frac{\mu_2}{\mu_1}$$

[where $\mu_1$ and $\mu_2$ are refractive indices of media $I$ and $II$ respectively]

or $$[D^2_{m+n} - D^2_n)_{II\text{ medium}} = \left(\frac{\mu_1}{\mu_2}\right)[D^2_{m+n} - D^2_n)_{I\text{ medium}}$$

$$= \frac{1 \times 1.25 \times 10^{-5}}{1.33} \qquad [\text{as } \mu_2 = 1.33]$$

$$= 9.40 \times 10^{-6}\,\text{m}^2$$

(*iii*) From Eq. (*i*), we see that

$$(D^2_{m+n} - D^2_n) \propto R$$

$$\frac{(D^2_{m+n} - D^2_n)_{I\text{ arrangement}}}{(D^2_{m+n} - D^2_n)_{II\text{ arrangement}}} = \frac{R_1}{R_2} \qquad [\text{Given } R_2 = 2R_1 = 2R]$$

or $$(D^2_{m+n} - D^2_n)_{II\text{ arrangement}} = \frac{2R}{R} \times [D^2_{m+n} - D^2_n)_{I\text{ arrangement}}$$

$$= 2 \times 1.25 \times 10^{-5}\,\text{m}^2 \qquad [\text{as } \mu_2 = 1.33]$$

$$= 2.50 \times 10^{-5}\,\text{m}^2$$

**Problem 4.11** *A Newton ring arrangement is used with a light sources of wavelength $\lambda_1 = 6000$ Å and $\lambda_2 = 5000$ Å and it is found that the nth dark ring due to $\lambda_1$ coincide with $(n+1)$th dark ring due to $\lambda_2$. If the radius of curvature of curved surface of the lens is 90 cm, then find the diameter for the nth, dark ring for $\lambda_1$.*
**[GGSIPU, Sept. 2009 (3 marks)]**

***Solution.*** Given $\lambda_1 = 6000$ Å for $n$th ring

$$\lambda_2 = 5000 \text{ Å for } (n+1)\text{th ring}, \; R = 90 \text{ cm} = 0.9 \text{ m}$$

$$[\because D_n = \sqrt{4n\lambda R}]$$

and $$(D_n)_{\lambda_1} = (D_{n+1})_{\lambda_2}$$

or $$\sqrt{4n\lambda_1 R} = \sqrt{4(n+1)\lambda_2 R}$$

or $$n\lambda_1 = (n+1)\lambda_2$$

$\Rightarrow$ $$6000 \times n = (n+1) \times 5000$$

$\Rightarrow$ $$n = 5$$

$$D_n = \sqrt{4n\lambda_1 R}$$

$$D_5 = \sqrt{4 \times 5 \times 6000 \times 10^{-10} \times 0.9} = 3.286 \times 10^{-3}\,\text{m}$$

**Problem 4.12** *Light containing two wavelengths $\lambda_1$ and $\lambda_2$ falls normally on a plano-convex lens of radius of curvature R resting on a glass plate. If the nth dark ring due to $\lambda_1$, coincides with $(n+1)$th dark ring due to $\lambda_2$. Prove that the radius of the nth dark ring of $\lambda_1$ is $\sqrt{\frac{\lambda_1\lambda_2 R}{(\lambda_1-\lambda_2)}}$.*

***Solution.*** We know the radius of $n$th dark ring due to $\lambda_1$

$$= \sqrt{n\lambda_1 R} \qquad ...(i)$$

The radius of $(n+1)$th dark ring due to $\lambda_2$

$$= \sqrt{(n+1)\lambda_2 R} \qquad ...(ii)$$

According to problem, both are equal, hence

$$r = \sqrt{n\lambda_1 R} = \sqrt{(n+1)\lambda_2 R}$$

or
$$n\lambda_1 R = (n+1)\lambda_2 R$$

or
$$n = \frac{\lambda_2}{(\lambda_1 - \lambda_2)} \qquad ...(iii)$$

From Eq. (*iii*), substituting the value of $n$ in Eq. (*i*), we have

$$r = \sqrt{(n\lambda_1 R)} = \sqrt{\frac{\lambda_2\lambda_1 R}{(\lambda_1 - \lambda_2)}} \quad \text{Hence proved.}$$

**Problem 4.13** *In Newton's ring experiment the diameters of the fourth and twelfth dark rings are respectively 0.420 cm and 0.726 cm. If the radius of curvature of the convex surface forming the air film is 225 cm, what is the wavelength of the radiation ? What is the radius of the fourth ring at this wavelength if the medium between the convex lens and glass plate is water of refractive index 1.33 ?*

***Solution.*** For Newton's ring experiment,

Given $D_4 = 0.420$ cm, $D_{12} = 0.726$ cm, $n = 4$, $(m+n) = 12$, $m = 8$

$R = 225$ cm, $D$ = diameter of dark ring.

$\lambda$ for air film = ? and radius of 4th ring $(r_n)$ = ? if $\mu = 1.33$

$$\lambda = \frac{D_{m+n}^2 - D_n^2}{4mR} = \frac{(0.726)^2 - (0.420)^2}{4\times 8\times 225} \text{ cm}$$

$$= \frac{0.527076 - 0.1764}{4\times 8\times 225} = 4.871\times 10^{-5} \text{ cm} = 4871 \text{ Å}$$

$$\because \; 2\mu t = n\lambda \text{ and } 2t = \frac{r_n^2}{R}$$

Hence
$$\frac{r_n^2}{R}\mu = n\lambda$$

$$\Rightarrow \quad r_n^2 = \frac{n\lambda R}{\mu} \qquad [\mu = 1.33]$$

$$\Rightarrow \quad r_4 = \sqrt{\frac{4\times 4.871\times 10^{-5}\times 225}{1.33}} = 0.182 \text{ cm.}$$

**Problem 4.14** *Newton's rings are observed normally in reflected light of wavelength 5893 Å. The diameter of the 10th dark ring is 0.005 m. Find the radius of curvature of the lens and thickness of the air film.*

**[GGSIPU, Sept. 2005 (3 marks) ; Dec. 2017 (3.5 marks)]**

***Solution.*** Given that, wavelength ($\lambda$) of reflected light $= 5893 \times 10^{-10}$ m, $n = 10$ (dark), $R = ?$, Diameter $(D) = 2r = 0.005$ m, $t = ?$

The condition for dark ring (in case of reflected light)

$$D^2 = 4n\lambda R$$

$$R = \frac{D^2}{4n\lambda} = \frac{0.005 \times 0.005}{4 \times 10 \times 5893 \times 10^{-10}} \text{ m} = 1.061 \text{ m}$$

The thickness of the air film will be

$$t = \frac{r^2}{2R} = \frac{D^2}{8R} = \frac{D^2 \times 4n\lambda}{8D^2} = \frac{n\lambda}{2}$$

$$= \frac{10 \times 5893 \times 10^{-10}}{2} \text{ m} = 2.947 \times 10^{-6} \text{ m.}$$

**Problem 4.15** *A thin planoconvex lens of focal length 1.8 m and of refractive index 1.6 is used to obtain Newton's ring. The wavelength of the light used is 5890 Å. Calculate the radius of 10th dark ring by (i) reflection and (ii) transmission.*

***Solution.*** In Newton's ring,

Given : $f = 1.8 \text{ m} = 180 \text{ cm}$, $\mu = 1.6$, $\lambda = 5890 \text{ Å} = 5.89 \times 10^{-7}$ m.

Here we use the lens formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$R_1 = R \text{ and } R_2 = \infty \text{ then } \frac{1}{f} = (\mu - 1)\frac{1}{R}$$

or

$$R = (\mu - 1) f$$

$$= (1.6 - 1) \times 180 \text{ cm} = 0.6 \times 180 \text{ cm} = 108 \text{ cm} = 1.08 \text{ m.}$$

(*i*) Radius of 10th dark ring (in case of reflection)

$$r_n^2 = n\lambda R$$

$$r_n = \sqrt{(n\lambda R)} = \sqrt{10 \times 5.89 \times 10^{-7} \times 1.08} = 0.252 \text{ cm.}$$

(*ii*) Radius of 10th dark ring (in case of transmission)

$$r_n^2 = \frac{(2n-1)\lambda R}{2}$$

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}$$

$$= \sqrt{\frac{(2 \times 10 - 1)}{2} \times 5.89 \times 10^{-7} \times 1.08} = 0.245 \text{ cm.}$$

**Problem 4.16.** *Two planoconvex lenses each of radius of curvature 1.0 m are used to observe Newton's rings with their curved surfaces in contact with each other in light of wavelength 600 nm. Find distance between 10th and 20th rings.*

**Solution.** Fig. 4.41

$$t = t_1 + t_2 = \frac{D^2}{8R_1} + \frac{D^2}{8R_2}$$

and for $n$th dark ring

$$2t = n\lambda$$

$$2\left[\frac{D^2}{8R_1} + \frac{D^2}{8R_2}\right] = n\lambda$$

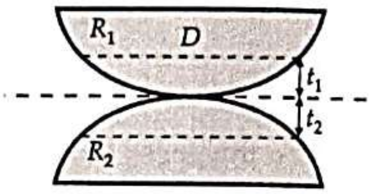


Fig. 4.41

Writing $D = 2r_n$

where $r_n$ = radius of $n$th ring

$$r_n = \left[\frac{n\lambda R_1 R_2}{(R_1 + R_2)}\right]^{1/2}$$

The desired separation,

$$r_{20} - r_{10} = \left[\frac{\lambda R_1 R_2}{R_1 + R_2}\right]^{1/2} [(20)^{1/2} - (10)^{1/2}]$$

$$= \left[\frac{6 \times 10^{-7} \times 1 \times 1}{1+1}\right]^{1/2} [(20)^{1/2} - (10)^{1/2}]$$

$$= 0.717 \times 10^{-3} \text{ m} = 0.717 \text{ mm.}$$

**Problem 4.17** *In a Newton's ring arrangement with air film observed with light of wavelength* $6 \times 10^{-5}$ *cm, the difference of squares of diameter of given successive rings is 0.125* $cm^2$.
*What will happen to this quantity if :*

(*a*) *wavelength of light is changed to* $4.5 \times 10^{-5}$ *cm ?*

(*b*) *liquid of refractive index 1.33 is introduced between the lens and the plate ?*

(*c*) *the plane glass plate is replaced by a planoconcave lens of radius of curvature twice that of planoconvex lens ?*

(*d*) *the plane glass plate is replaced by planoconvex lens identical to one and put on the top of it ?*

**Solution.** (*a*) We know that,

$$D_{m+n}^2 - D_n^2 = \frac{4m\lambda R}{\mu}$$

In the present case, $m = 1$

$$D_{n+1}^2 - D_n^2 = \frac{4\lambda R}{\mu}$$

When the wavelength changes from $\lambda_1$ to $\lambda_2$, we have

$$\frac{(D_{n+1}^2 - D_n^2)_{\lambda_1}}{(D_{n+1}^2 - D_n^2)_{\lambda_2}} = \frac{\lambda_1}{\lambda_2}$$

Thus, $$(D_{n+1}^2 - D_n^2)_{\lambda_2} = (D_{n+1}^2 - D_n^2)_{\lambda_1} \frac{\lambda_2}{\lambda_1}$$

$$= \frac{0.125 \times 4.5 \times 10^{-5}}{6 \times 10^{-5}} = 0.094 \text{ cm}^2$$

(b) When the liquid of refractive index $\mu$ is introduced

$$\frac{(D_{n+1}^2 - D_n^2)_{\text{air}}}{(D_{n+1}^2 - D_n^2)_{\text{liquid}}} = \mu$$

and so $$(D_{n+1}^2 - D_n^2)_{\text{liquid}} = \frac{(D_{n+1}^2 - D_n^2)_{\text{air}}}{\mu} = \frac{0.125}{1.33} = 0.094 \text{ cm}^2$$

(c) The air film causing interference in present case shall have thickness $t_n$ corresponding to $n$th dark ring

$$t_n = \frac{r_n^2}{2R} - \frac{r_n^2}{2(2R)} = \frac{r_n^2}{4R} = \frac{n\lambda}{2}$$

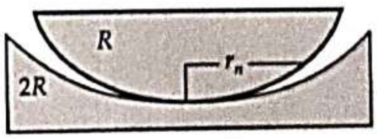


**Fig. 4.42** Planoconcave and planoconvex lens.

which gives us,

$$D'^2_{n+1} + D'^2_n = 4\lambda(2R) = 2 \times 4\lambda R$$

$$= 2 \times 0.125 = 0.250 \text{ cm}^2$$

(d) In this case, the thickness of air film corresponding to $n$th dark ring shall be

$$t_n = \frac{r_n^2}{2R} + \frac{r_n^2}{2R} = \frac{n\lambda}{2}$$

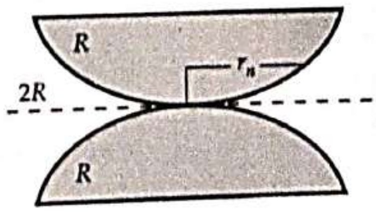


**Fig. 4.43** Two planoconvex lens.

which gives us $$D'^2_{n+1} - D'^2_n = 2\lambda R = \frac{1}{2}(4\lambda R)$$

$$= \frac{1}{2} \times 0.125 = 0.0625 \text{ cm}^2$$

**Problem 4.18.** *Calculate the distance between successive positions of the movable mirror of Michelson interferometer giving best fringes in case of sodium source having wavelengths 5896 Å and 5890 Å.*

**Solution.** In Michelson interferometer,

Given that :

$$\lambda_1 = 5896 \text{ Å} = 5.896 \times 10^{-7} \text{ m}; \quad \lambda_2 = 5896 \text{ Å} = 5.89 \times 10^{-7} \text{ m}.$$

We know that small difference in two wavelengths :

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2(x_2 - x_1)}$$

or

$$(x_2 - x_1) = \frac{\lambda_1 \lambda_2}{2\Delta\lambda} = \frac{5.896 \times 10^{-7} \times 5.89 \times 10^{-7}}{2 \times (5.896 - 5.890) \times 10^{-7}}$$

$$(x_2 - x_1) = 28.94 \text{ Å}$$

**Problem 4.19** *A shift of 100 circular fringes is observed, when movable mirror of Michelson interferometer is shifted by 0.295 mm. Calculate wavelength of light.*

**Solution.** In Michelson interferometer,

Given that : $(x_2 - x_1) = 0.295$ mm, $n = 200$, $\lambda = ?$

We know that,

$$2(x_2 - x_1) = n\lambda$$

$$\Rightarrow \quad \lambda = \frac{2(x_2 - x_1)}{n}$$

$$= \frac{2 \times 0.295 \times 10^{-3}}{100} = 5900 \times 10^{-9} \text{ m}$$

$$\lambda = 5900 \text{ nm.}$$

## Conceptual Questions

**4.1** *"Any monochromatic light is necessarily coherent," true or false ? Justify your answer.*

**[GGSIPU, Dec. 2013-reappear (2 marks)]**

**Ans.** Monochromatic light is coherent, if the waves of the monochromatic light oscillate in the same direction and have the same frequency and the phase. In other words, the monochromatic light must be collimated. This requirement applies to a laser as Nuclear Ghost mentioned it. In contrast to that the waves of a light which come from the light bulb are incoherent, as the wave oscillates in different directions which have different frequencies and phases.

**4.2** *What is meant by coherent sources of light ? Can two identical and independent sodium lamps act as coherent sources ? Justify.*

**Ans.** Two light sources are said to be coherent if they continuously emit light waves of same frequency (or wavelength) with zero or constant phase different between them. Two independent sources cannot act as coherent sources. The emission of light in them is due to millions of atoms in which electrons jumps from higher to lower orbit. The process is occurs in $10^{-8}$s. Thus phase difference can remain constant for about $10^{-8}$s only *i.e.*, phase changes $10^8$ times in one second. Such rapid changes in the positions of maxima and minima cannot be detected by our eyes. The interference pattern is lost and almost a uniform illumination is seen on the screen.

**4.3** *What are bright and dark fringes in case of Young double slit experiment ?*

**Ans.** The intensity maxima and minima in the interference are called bright and dark fringes. The fringes are neither image nor shadow of slit, but a locus of a point, which moves in such a way that point the path difference between the waves from the two sources remain constant, in case of bright fringes, it is integral multiple of the wavelength and in case of dark fringes, it is odd multiple of half of the wavelength. An array of fringes is called the interference pattern.

**4.4** *Can two independent point sources of light operating under similar conditions produce sustained interference ?* **[GGSIPU, Nov. 2012 (2 marks) ; Dec. 2009 (1 mark)]**

***Or***

*Can non-coherent sources produce interference ? Justify your answer.*

**[GGSIPU, Dec. 2017 (2.5 mark)]**

**Ans.** No, the two independent point sources of light operating under similar conditions may produce light of the same wavelength and amplitude, but they will not able to satisfy the most essential requirement of coherence for sustained interference *i.e.*, constancy in phase relationship. The two independent sources emit light the phase difference between the two interfering beams goes on varying randomly. As the phase difference between the two interfering beams goes on varying randomly with time, it will not be possible to obtain sustained interference pattern.

**4.5** *What is difference between fringes obtained by Fresnel's biprism and those obtained by Newton's rings ?*

**[GGSIPU, Sept. 2008 (2 marks)]**

**Ans.** (*i*) The biprism fringes are straight and equally spaced whereas the fringes in Newton's rings are circular and not equally spaced.

(*ii*) In biprism fringes are obtained by division of wavelength whereas in Newton's rings, they are obtained by division of amplitudes.

(*iii*) In biprism, fringes are non-localised while in Newton's rings they are localised.

**4.6** *Why two independent sources of light of the same wavelength cannot produce interference fringes ?*

**[GGSIPU, Sept. 2009 (2 marks)]**

***Or***

*Why two independent sources cannot produce observable interference pattern ?*

**[GGSIPU, Jan. 2015 (2.5 marks)]**

**Ans.** If two independent sources of light of same wavelength are placed side by side, no interference fringes or effect are observed because the light waves from one source are emitted independently of those from the other source. The emissions from the two independently do not maintain constant phase relationship with each other over time. Light waves from an ordinary source such as light bulb undergo random phase changes in time intervals less than a nanosecond. Therefore, the conditions for constructive interference, destructive interference or some intermediate state are maintained only for such time intervals. Because the eye cannot follow such rapid changes, no interference fringes/effects are observed. Such light sources are said to be incoherent.

**4.7** *Why the colours of thin films in reflected and transmitted light are complementary ?*

**[GGSIPU, Nov. 2012 (2 marks)]**

**Ans.** In the reflected system, there is an additional path difference of $\lambda/2$ between the two rays producing interference as one of the rays suffers reflection at a denser medium, while in the transmitted system it is not so. Thus for the same path difference in the reflected system, a bright band will correspond to a dark band in transmitted system and vice versa. Thus, the two systems are complementary.

**4.8** *Why does the colour of the oil film on the surface of water continuously changes ?*

**Ans.** The position of the bright and dark fringes produced by thin oil films depends upon the thickness of the film. The thickness of oil film on the surface of water continuously varies and as a result, the position of the coloured fringes also varies. This appears as a variation in the colour of the oil films.

**4.9** *Explain why excessively thin film seen in reflected light appears dark.* **[GGSIPU, Sept. 2009 (2 marks)]**

**Ans.** Excessively thin film is dark in reflected system. The effective path difference between the interfering reflected rays is $2\mu t \cos r - \lambda / 2$. When the film is excessively thin, so that $t$ is practically zero, the effective path difference is $\lambda / 2$. This is the condition for minimum intensity. Hence, the film appears dark.

**4.10** *The central part in Newton's rings seen in reflected light appears dark. Why ?*

**[GGSIPU, Dec. 2015 (Reappear) (2 marks)]**

**Ans.** In Newton's rings experimental arrangement at the point of the contact between the lens and the glass plate the thickness of the air film is zero. Therefore, there is no path difference between the interfering rays due to difference in the path lengths. But one of the rays suffers a phase change of $\pi$ on reflection at the surface of the glass plate, *i.e.*, denser medium. This is why the rays suffer destructive interference and centre appears dark.

**4.11** *What are Newton's rings ? Why the central ring is dark when observed in reflected light ?*

**[GGSIPU, Sept. 2010 (2 marks)]**

**Ans.** When a plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, on air film is formed between the lower surface of the lens and the upper surface of the plate. If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings with their centre dark concentric rings with their centre dark is formed. Since these rings were discovered by Newton, so these are called Newton's rings. The central ring is dark when observed in reflected light because effective path difference $\Delta = 2t + \frac{\lambda}{2}$. At the point of contact $t = 0$, then $\Delta = \frac{\lambda}{2}$. This is the condition for minimum intensity. Hence the central ring is dark.

**4.12** *Explain, why interference fringes are circular in Newton's rings.*

**[GGSIPU, Sep. 2013 (reappear) (2 marks) ; Dec. 2008 (2 marks)]**

*Or*

*Why Newton's rings are circular ?* **[GGSIPU, Sept. 2009 (Reappear), 2 marks ; Dec. 2018 (3 marks)]**

**Ans.** In Newton's ring experiment, the path difference of two interfering waves is dependent on the thickness of the air film. In the set up of this experiment, the locus of same thickness forms a circle with centre at the point of contact between the lens and plane glass plate. Therefore, the condition of constructive interference or destructive interference is satisfied over a circle and the fringe system becomes circular.

**4.13** *What happens to the ring system if a plane polished mirror is used instead of a glass plate in Newton's ring arrangement ?* **[GGSIPU, Dec. 2013 (2.5 marks)]**

**Ans.** If a plane polished mirror is used instead of a glass plate in Newton's ring arrangement, the interference pattern produced due to reflected and transmitted light will be superimposed and we get these two patterns, which are complimentary their superposition. Hence we get uniform illumination.

**4.14** *The interference fringes produced in the Newton's ring experiment are real or virtual ? Justify.*

**[GGSIPU, Dec. 2009 (1 mark)]**

**Ans.** The interference fringes produced in the Newton's ring experiment are real because as we are observing the reflected geometry, the centre of the ring system is dark spot. These rings are formed in the plane of the film and there are observed by a microscope. So these rings are real.

**4.15** *Why do we prefer a convex lens of large radius of curvature for producing Newton's rings ?*
**[GGSIPU, Nov. 2012 (2 marks)]**

**Ans.** In Newton's ring arrangement, it is preferred to be large radius of curvature Lens because (*i*) It results in increase the diameter of Newton's rings which increases the accuracy in the measurement of their diameters ; (*ii*) Large value of radius of curvature in the decrease of the thickness of the air film at any point and hence it is justified to neglect $t^2$ as compared to $2Rt$. (*iii*) The angle of wedge-shaped film enclosed between the glass plate and the lower surface of lens is very small and hence can be neglected.

## EXERCISES

### Theoretical Questions

**4.1** What is the interference of light ? **[GGSIPU, Nov. 2012 reappear (2 marks)]**

**4.2** What is interference of light waves ? What are the conditions necessary for obtaining interference fringes ? **[GGSIPU, Sept 2004 (3 marks)]**

**4.3** What are the necessary conditions for obtaining interference fringes ? **[GGSIPU, Sept. 2013 reappear (2 marks)]**

**4.4** Give conditions of sustained interference. **[GGSIPU, October 2013 (2 marks)]**

**4.5** Illustrate with neat scientific, well-labeled diagrams, the classification of interference in two classes, that is, one due to division of wavefront and another due to division of amplitude. **[GGSIPU, Dec. 2016 (3 marks)]**

**4.6** Derive the mathematical expression for the intensity distribution when two sinusoidal coherent waves with amplitudes $A_1$ and $A_2$ and a phase difference of $\phi$ superpose to produce interference. **[GGSIPU, Dec. 2016 (4 marks)]**

**4.7** Describe the Young's double slit experiment. What is its importance in physics ? **[GGSIPU, Dec. 2009 (5.5 marks)]**

**4.8** Discuss the Young's double slit experiment and find the expression for fringe width. **[GGSIPU, Sept. 2009 (5 marks)]**

**4.9** Find the expression for fringe width in case of Young's double slit experiment. Prove that in this case of interference dark and bright bands are of equal width. **[GGSIPU, Nov. 2012 (5 marks)]**

**4.10** Obtain the relations for constructive and destructive interference due to two slits clearly pointing out the conditions under which these equations are deduced. **[GGSIPU, Dec. 2008 (5.5 marks)]**

**4.11** What are coherent sources ? List down conditions for obtaining a good sustained interference pattern. **[GGSIPU, Sept. 2011 reappear (5 marks)]**

**4.12** Show that the distance between adjacent bright bands is inversely proportional to the distance between the slits. **[GGSIPU, Sept. 2011 (4 marks)]**

**4.13** Give few important conditions for obtaining sustained interference pattern. **[GGSIPU, Sept. 2010 (2 marks)]**

**4.14** What are two general methods for obtaining coherent sources ? **[GGSIPU, Dec. 2018 (2 marks)]**

**4.15** State the superposition principle. Explain the terms : interference of light and coherent sources. **[GGSIPU, Sept. 2010 reappear (3 marks)]**

**4.16** How would you obtain a sustained interference pattern with good contrast ? **[GGSIPU, Dec. 2010 (2 marks)]**

4.17 Give the conditions required to get sustained interference. [GGSIPU, Dec. 2012 (2.5 marks)]

4.18 What are necessary conditions for obtaining interference fringes ? [GGSIPU, Sept. 2012 (2 marks)]

4.19 Draw a labelled ray diagram depicting interference by a biprism. [GGSIPU, Dec. 2009 (3 marks)]

4.20 What is a biprism ? Give schematic diagram showing formation of fringes using Fresnel's biprism. [GGSIPU, Sept. 2012 reappear (3 marks)]

4.21 Draw a labelled ray diagram depicting interference by biprism. [GGSIPU, Sept. 2011 (3 marks) ; Dec. 2017 (3 marks)]

4.22 What is a biprism ? Give the schematic diagram showing formation of fringes using Fresnel biprism. [GGSIPU, Dec. 2012 (3 marks)]

4.23 What do you understand by Fresnel's biprism and explain the formation of fringes by it. How do you determine the wavelength of monochromatic light ? [GGSIPU, Dec. 2012 (4 marks)]

4.24 What is biprism ? Explain the construction and working of it with applications. [GGSIPU, Nov. 2012 reappear (5 marks)]

4.25 Explain the formation of interference fringes by means of Fresnel's biprism when a monochromatic source of light is used and derive the expression for fringe width. How will you measure a wavelength of monochromatic light using biprism method ? [GGSIPU, Jan 2015 (8 marks)]

4.26 Illustrate with neat scientific, well-labeled diagram the formation of fringes due to a Fresnel's Biprism. [GGSIPU, Dec. 2015 (1.5 marks)]

4.27 What will happen to Biprism if,

(*i*) angle of biprism is increased ?

(*ii*) width of slit is increased continuously ? [GGSIPU, Dec. 2019 (3 marks)]

4.28 Discuss the phenomenon of interference of light in thin films and obtain the conditions of maxima and minima for the reflected light.

[GGSIPU, Sept. 2013 reappear (6 marks), Sept. 2011 reappear (5 marks), Dec. 2017 (6 marks)]

4.29 Give the nature of fringes obtained in this parallel films. [GGSIPU, Dec. 2012 (2.5 marks)]

4.30 Explain why interference effects are not observed when light reflected from the two surfaces of a window pane combine. [GGSIPU, Dec. 2019 (3 marks)]

4.31 Explain the term temporal and spatial coherence in context of interference phenomenon. Explain why interference due to division of amplitude is observed in thin films.

[GGSIPU, Dec. 2015 (3 marks)]

4.32 Illustrate with neat scientific, well-labeled diagram the necessity of an extended source to observe fringes in thin film. [GGSIPU, Dec. 2015 (1.5 marks)]

4.33 Derive the relation for path difference and subsequently the width of a single band for a wedge shaped film. [GGSIPU, Dec. 2015 (4 marks)]

4.34 What is the effect of increasing the angle of Biprism on the fringes ? Explain. [GGSIPU, Jan 2015 (2.5 marks)]

4.35 Discuss the interference from parallel thin film. Describe salient features of the fringes formed. Give one of its applications. [GGSIPU, Dec. 2013 reappear (5.5 marks)]

4:36 Discuss the phenomenon of interference of light in thin films and obtain the conditions of maxima and minima. Show that the interference patterns in reflected and transmitted lights are complimentary. [GGSIPU, Nov. 2012, Sept. 2009 (7 marks)]

4.37 Why interference are observed in case of thin films not in case of thick films and why a broad source of light is required to observe interference in thin films ?
**[GGSIPU, 2nd counselling, Nov. 07 (2 marks)]**

4.38 Why interference fringes are observed in case of thin films not in the case of thick films ?
**[GGSIPU, Sept. 2012 (2 marks)]**

4.39 What will happen if a wedge shaped film is placed in white light ?
**[GGSIPU, Dec. 2012 (2 marks)]**

4.40 With the help of a labeled ray diagram discuss the formation of Newton's rings by reflected light. Hence derive an expression for the diameter of $n$ th dark ring.
**[GGSIPU, Sept. 2010 reappear (7 marks)]**

4.41 Why central ring is dark instead of bright some times in reflected system. Give appropriate reason.
**[GGSIPU, Jan 2015 (2.5 marks)]**

4.42 Show that the thickness of successive Newton's ring goes on decreasing.
**[GGSIPU, Dec. 2018 (2 marks)]**

4.43 In case of Newton's rings obtain the relation between the dark ring diameter and air film thickness.
**[GGSIPU, Dec. 2019 (2.5 marks)]**

4.44 Explain the formation of fringes in Newton's ring experiment. Give its application to find out wavelength of light. **[GGSIPU, Sept. 2012 reappear (7 marks)]**

4.45 How are the circular fringes of Michelson's Interferometer differ from Newton's rings ?
**[GGSIPU, Dec. 2010 (4 marks)]**

4.46 What are Newton's rings ? Explain the formation of Newton's rings by reflected system of light. Also show that spacing between rings goes on decreasing with increased order.
**[GGSIPU, Oct. 2013 (8 marks)]**

4.47 Describe and explain formation of Newton's ring in reflected monochromatic light. Prove that in reflected light diameters of the dark rings are proportional to the square root of natural numbers.
**[GGSIPU, Dec. 2013 (8 marks)]**

4.48 Discuss in brief the conditions under which the centre of Newton's ring is bright or dark.
**[GGSIPU, Dec. 2007 (3.5 marks)]**

4.49 What are the conditions for maxima and minima in case of Newton's ring due to reflected light and how the refractive index of any liquid can be determined by Newton's ring method ?
**[GGSIPU, 2nd counselling, Nov. 2007 (3 marks)]**

4.50 Draw a neat ray diagram for Newton's ring interference pattern indicating clearly, how division of amplitude takes place for a given incident beam and the path difference introduced thereby ?
**[GGSIPU, Sept. 2007 (2 marks)]**

4.51 Consider the formation of Newton's ring by a monchromatic light of wavelength ($\lambda$) from an air film (using planoconvex lens with radius '$R$'). Derive an expression for radius of $n$ th dark ring and show that the spacing between the rings decreases as '$n$' increases.
**[GGSIPU, 2nd counselling Nov. 2006 (6 marks)]**

4.52 Explain why in Newton's ring experiment fringes are circular with dark ring at the centre.
**[GGSIPU, Sept. 2005 (2 marks)]**

4.53 The interference fringes produced in the Newton's rings experiment are real or virtual. Justify.
**[GGSIPU, Dec. 2009 (1 mark)]**

4.54 In the formation of Newton's ring ; derive an expression for the radius of $n$th dark ring in the case of reflected light. How are such rings used for the determination of refractive index of a transparent liquid ? **[GGSIPU, Sept. 2005, 2004 (3 marks)]**

4.55 Write a note on Newton's ring. What determines whether the centre shall be bright or dark ? **[GGSIPU, Dec. 2004 (6 marks)]**

4.56 In a Newton's ring experiment the centre is bright instead of dark. What is the reason ? **[GGSIPU, Sept. 2004 (2 marks)]**

4.57 How can Michelson's interferometer be used to be determine the difference between the two D-lines of sodium ? **[GGSIPU, Sept. 2005 (3 marks)]**

4.58 The Michelson interferometer is based upon the interference of light due to division of wavefront or division of amplitude. Justify your answer. **[GGSIPU, Dec. 2007 (4 marks)]**

4.59 How can Michelson interferometer be used to be determine the difference between two D-lines of sodium. **[GGSIPU, Sept. 2005 (3 marks)]**

4.60 Explain why a compensating plate is needed in Michelson's Interferometer? **[GGSIPU, Dec. 2008 (2 marks), Dec. 2010 (2 marks)]**

4.61 Explain the formation of fringes in Michelson's Interferometer with suitable diagram. **[GGSIPU, Dec. 2010 (5 marks)]**

4.62 Explain the formation of fringes in Michelson's Interferometer. Give its application to determine the wavelength of light. **[GGSIPU, Dec. 2011 (6.5 marks)]**

4.63 Give a well labelled diagram of Michelson interferometer. Discuss the use of compensating plate. **[GGSIPU, Dec. 2012 (6.5 marks)]**

4.64 Explain using mathematical derivation the formation of the $n$th bright ring in a Newton's ring set up in the reflected light with a diameter given by two expression : $D = \sqrt{((2\lambda R)(2n-1))}$. **[GGSIPU, Dec. 2016 (4 marks)]**

## Numerical Problems

4.1 Two coherent sources whose intensity ratio is 4:1 produce interference fringes, find the ratio of maximum to minimum intensity in the interference pattern. **[GGSIPU, Dec. 2015 (3 marks)]**

**Hint :** $\dfrac{I_1}{I_2} = \dfrac{4}{1} = \dfrac{a_1^2}{a_2^2} \Rightarrow a_1 = 2a_2$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(3a_2)^2}{a_2^2} = 9 : 1$$

4.2 In Young's two slits experiment, the distance between the slits is 0.2 mm and screen is at a distance 1.0 m. The third bright fringe is at a distance 7.5 mm from the central fringe. Find the wavelength of the light used. **[GGSIPU, Dec. 2008 (3 marks)]**

**Hint :** $x = \dfrac{n\lambda D}{2d} \Rightarrow \lambda = \dfrac{x(2d)}{nD} = 5000$ Å.

4.3 In Young's double slit experiment, fringes are obtained at a screen placed at some distance from the slits. If screen is moved by $5 \times 10^{-2}$ m towards the slits, the change in fringe width is $3 \times 10^{-5}$ m. Calculate the wavelength of light used. Given the distance between the slits is $10^{-3}$ m. **[GGSIPU, Dec. 2011 (3 marks)]**

**Hint :** Fringe with $\beta = \frac{\lambda D}{2d}$; if $\Delta\beta$ is the change in fringe width when the screen is moved by $\Delta D$, then

$$\Delta\beta = \frac{\lambda \Delta D}{2d} \qquad \Rightarrow \quad \lambda = \frac{2d\Delta\beta}{\Delta D} = \frac{10^{-3} \times 3 \times 10^{-5}}{5 \times 10^{-2}} \text{m} = 600 \text{ nm}.$$

4.4 The inclined faces of a biprism ($\mu = 1.5$) make angles of 1° with the base of the prism. The slit is 10 cm from the biprism and is illuminated by the wavelength 590 nm. Find the fringe width observed at a distance of one meter. **[GGSIPU, Nov. 2012 (3 marks)]**

**Hint :** $d = 2a(\mu - 1)\alpha$ ; $D = 10 \text{ cm} + 1\text{m} = 1.1 \text{ m}$, $\alpha = \frac{\pi}{180}$ radian,

$$\lambda = 590\text{nm} = 5.9 \times 10^{-7}\text{m}, \quad a = 0.1 \text{ m}$$

Fringe width $\beta = \frac{\lambda D}{d} = \frac{5.9 \times 10^{-7} \times 1.1 \times 180}{2 \times 0.1 \times 0.5 \times \pi} = 0.000372 \text{ m} = 3.72 \times 10^{-4}\text{m}.$

4.5 Newton's rings are formed between the plane surface of glass and lens. The diameter of third dark ring is $10^{-2}$m. When the light of wavelength $5890 \times 10^{-10}$m is used at such an angle that the light passes through the air film at an angle of 30° to the normal. Find the radius of the lens. **[GGSIPU, Dec. 2018 (4 marks)]**

**Hint :** $2\mu t \cos r = n\lambda \quad \Rightarrow \frac{r^2}{R}\cos r = n\lambda \quad \Rightarrow \frac{D_3^2}{4R}\cos r = 3\lambda$

$$\Rightarrow \qquad R = \frac{D_3^2}{4 \times 3 \times \lambda} = 30° = \frac{(10^{-2})^2}{4 \times 3 \times 5890 \times 10^{-10}} = \frac{\sqrt{3}}{2} = 12.25\text{m}$$

4.6 Light of wavelength 6000 Å falls normally on a wedge shaped film of refractive index 1.4 forming fringes that are 2.0 mm apart. Find the angle of wedge in seconds. **[GGSIPU, Dec. 2013 reappear (4 marks)]**

**Hint :** Given $\lambda = 6000\text{Å} = 6.000 \times 10^{-7}\text{m}, \mu = 1.4, \beta = 2 \text{ mm} = 2 \times 10^{-3}\text{m}$

We know that angle of wedge

$$\alpha = \frac{\lambda}{2\mu\beta} = \frac{6.0 \times 10^{-7}}{2 \times 1.4 \times 2 \times 10^{-3}} = 1.071 \times 10^{-4} \text{ radian} = 22''.$$

4.7 Interference fringes are produced by monochromatic light of wavelength 5460 Å, when a thin sheet of transparent material of thickness $6.3 \times 10^{-4}$cm is introduced in the path of one of the interfering beams, the central fringe shifts of a position occupied by 6th bright fringe. Compute refractive index of the sheet. **[GGSIPU, Sept. 2008 (4 marks)]**

**Hint :** $(\mu - 1)t = n\lambda$

or $\mu = \frac{n\lambda}{t} + 1 = \frac{6 \times 5460 \times 10^{-8}}{6.3 \times 10^{-4} \text{ cm}} + 1 = 1.52$

4.8 Light of wavelength 5893 Å is reflected at nearly normal incidence from a soap film of refractive index, $(m) = 1.42$. What is the least thickness of the film that will disappear (*i*) dark, (*ii*) bright. **[GGSIPU, Dec. 2010 (3.5 marks)]**

**Hint :** Given $(\mu) = 1.42$ and $\lambda = 5893$ Å

(*i*) For the film to appear bright in reflected light at normal incidence :

$$2\mu t = (2n+1)\lambda/2 \quad \Rightarrow \quad t = (2n+1)\lambda/(4\mu)$$

For least thickness , $n = 0$, $t = \lambda/(4\mu) \quad \Rightarrow \quad t = 1037.5$ Å (*ii*) For the film to appear bright in reflected light at normal incidence :

$$2\mu t = n\lambda \quad \Rightarrow t = n\lambda/(2\mu)$$

For least thickness, $n = 1,\ t = \lambda/(2\mu) \ \Rightarrow t = 2075$ Å

**4.9** Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 fringes are observed between these edges in sodium light for normal incidence, find thickness of the wire. **[GGSIPU, Sept. 2010 (2 marks)]**

**Hint :** Given $N = 20$, $\lambda$ = wavelength of sodium = 589 nm = $5.89 \times 10^{-7}$ m.

Let $t$ be the thickness of the wire and $l$, the length of the wedge as shown in Fig. 4.46.

The angle of wedge $\theta = t / l$,

Fringe width in air wedge $\beta = \lambda / 2\theta = \lambda l / 2t$

If $N$ fringes are seen, that $l = N\beta$ ;

$\therefore \quad \beta = \lambda N \beta / 2t$

$\Rightarrow \quad t = N\lambda / 2 = 5.89 \times 10^{-6}$ m.

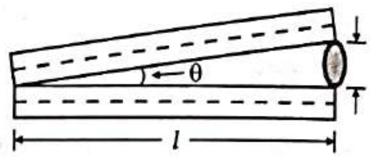


**Fig. 4.46**

**4.10** An interference pattern is first obtained using a bi-prism set up. When a thin sheet of glass ($\mu = 1.5$) of 5 mm thickness is introduced in the path of one of interfering rays, the central fringe is shifted to a position normally occupied by the fifth fringe. Calculate the wavelength of light used.

**Hint :** $\lambda = \dfrac{(\mu - 1)t}{n} = \dfrac{(1.5 - 1) \times (5 \times 10^{-6})}{5}\ 5 \times 10^{-7}\text{m} = 500$ nm.

**4.11** In Newton's ring experiment the diameters of the 4th and 12th bright rings are 0.4 cm and 0.7 cm respectively. Deduce the diameter of 20th bright ring.

**[GGSIPU, Dec. 2011 (3 marks), Dec. 2013 (4.5 marks)]**

**Hint :** $$\lambda = \frac{D^2_{m+n} - D^2_n}{4\text{mR}} \qquad \text{...}(i)$$

and $$D^2_n = 4n\lambda R \qquad \text{...}(ii)$$

From Eq. (*i*) $4\lambda R = \dfrac{D^2_{m+n} - D^2_n}{m}$, then $D^2_n = \dfrac{n}{m}(D^2_{m+n} - D^2_n)$

$$D^2_{20} = \frac{20}{8} \times [(0.7)^2 - (0.4)^2] = \frac{20}{8} \times 1.1 \times 0.3 = 0.91 \text{ cm}.$$

**4.12** In a Newton's ring experiment the diameter of the 13th ring was found to be 0.590 cm and that of the 3rd ring was 0.336 cm. If the focal length of the plano-convex lens is 50 cm, calculate the wavelength of light used. **[GGSIPU, Sept. 2008 (2 marks)]**

**Hint :** $$\left(\frac{1}{f}\right) = (\mu - 1)\left(\frac{1}{R}\right) \qquad \Rightarrow \qquad R = (\mu - 1) f$$

$$\therefore \qquad R = 0.5 \times 50 = 25 \text{ cm}.$$

$$\therefore \qquad \lambda = \frac{D^2_{m+n} - D^2_n}{4mR} = \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 25} = 22520 \text{ Å}$$

Note wavelength of light is too much being wrong data.

4.13 Newton's rings are formed by a light of wavelength 4000 Å.

(i) Between the 3rd and 6th bright fringe, what is the change in thickness of the air film ?

(ii) If the radius of curvature of the curved surface is 5.0 cm, what is the radius of 3rd bright fringe ?

[GGSIPU, Dec. 2019 (4 marks)]

**Hint :** (i) $2t = \frac{r_n^2}{R} \Rightarrow t_n = (2n+1)\frac{\lambda}{4}$

$$t_3 = 7\times 10^{-7}\text{ m}, \qquad t_6 = 13\times 10^{-7}\text{ m}$$

Spacing $\Delta t = t_3 - t_6 = 60\ \mu\text{m}$

(ii) $r_n = \sqrt{\frac{(2n+1)\lambda R}{2}} = 2.65\times 10^{-4}$ cm

4.14 A drop of liquid of volume 0.2 cm$^3$ is dropped on the surface of the tank water of area 1 m$^2$. The drop spreads uniformly over the whole surface. White light is incident normally on the surface. The spectrum contains one dark band whose centre gas wavelength 5500 Å in air. Find the refractive index of the liquid. [GGSIPU, Dec. 2019 (2.5 marks)]

**Hint :** $t = \frac{0.2\text{ cm}}{100\times 100} = 2\times 10^{-5}\text{cm}$

$$2\mu t\cos r = n\lambda \quad \text{and} \quad \mu = \frac{n\lambda}{2t} = \frac{1\times 5.5\times 10^{-5}}{2\times 2\times 10^{-5}\times 1} = 1.375$$

4.15 Two wavelengths of light $\lambda_1$ and $\lambda_2$ are sent through a Young's double slit experiment set up simultaneously. What must be true concerning $\lambda_1$ and $\lambda_2$ if the third order $\lambda_1$ bright fringe is to coincide with the fourth order $\lambda_2$ fringe ? [GGSIPU, Dec. 2016 (3 marks)]

**Hint :** $\beta_1 = \beta_2 \Rightarrow \frac{3\lambda_1 D}{2d} = \frac{4\lambda_2 D}{2d} \Rightarrow \lambda_2 = \frac{3}{4}\lambda_1$

4.16 Newton's rings are obtained by source emitting light of wavelength $\lambda_1 = 6000$ Å and $\lambda_2 = 5000$ Å. It is found that the $n^{th}$ dark ring due to $\lambda_1$ coincides with $(n-1)^{th}$ dark ring due to $\lambda_2$. If the radius of curvature of convex surface is 90 cm, calculate the diameter of $n^{th}$ dark ring of $\lambda_1$.

[GGSIPU, Sept. 2009 (3 marks) ]

**Hint :** $D_n = 4n\lambda_1 R = 4(n+1)\lambda_2 R \Rightarrow n\lambda_1 = (n+1)\lambda_2 \Rightarrow n = 5$

and then $D_n = \sqrt{4n\lambda_1 R} = 3.26$ nm.

4.17 Michelson interferometer experiment is performed with a source which consists of two wavelengths 4882 Å and 4886 Å. Through what distance does the mirror have to be moved between two positions of disappearance of fringes ?

**Hint :** $(\Delta\lambda) = \frac{\lambda^2}{2(x_2 - x_1)} = \frac{\lambda_1\lambda_2}{2(x_2 - x_1)}$

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{(4882 + 4886)\text{ Å}}{2} = 4884\text{ Å} = 4884\times 10^{-10}\text{ m}$$

and $\Delta\lambda = (4886 - 4882)\text{ Å} = 4\times 10^{-10}$ meter.

Then $l = \frac{\lambda^2}{2\Delta\lambda} = \frac{(4884\times 4884)\times 10^{-10}\times 10^{-10}}{2\times 4\times 10^{-10}} = 3.519\times 10^{-4}\text{ m} = 0.352$ mm.

**4.18** In a Michelson interferometer 100 fringes cross the field of view when the movable mirror is displaced by 0.022948 mm. Calculate the wavelength of monochromatic light. **[GGSIPU, Sept. 2009 (3 marks)]**

**Hint :** $\lambda = \frac{2l}{m} = \frac{2 \times 0.022948 \times 10^{-3}}{100}$ m $= 4.459 \times 10^{-7}$ m $= 4459$ Å.

**4.19** The initial and final readings of a Michelson's interferometer screw are 10.7347mm and 10.7051mm respectively when 100 fringes pass through the field of view. Calculate the wavelength of light used.

**Hint :** $x_2 - x_1 = l = n\frac{\lambda}{2}$ or $\lambda = \frac{2l}{n} = \frac{2[10.7347 - 10.7051] \times 10^{-3}}{100} = 5920$ Å.

**4.20** When a thin glass plate ($\mu = 1.5$) is introduced in one of arms of Michelson interferometer using light of wavelengths 5890Å, there is a shift of 10 fringes. Calculate the thickness of the plate.

**Hint :** $t = \frac{n\lambda}{2(\mu - 1)} = \frac{10 \times 5890 \times 10^{-10}}{2 \times 0.5} = 5.89 \times 10^{-6}$ m.

**4.21** In a Michelson interferometer 200 fringes cover the field of view when the movable mirror is displaced through 0.0589 mm. Calculate the wavelength of monochromatic light used.

**Hint :** $2(x_2 - x_1) = n\lambda$

$$\Rightarrow \quad \lambda = \frac{2(x_2 - x_1)}{n}$$

$$= \frac{2 \times 5.89 \times 10^{-5}}{200} = 5.89 \times 10^{-7} \text{ m} = 5890 \text{Å}.$$

**4.22** Sodium light ($\lambda = 5893$ Å) is used first in a Fresnel's Bi-prism set up. A total of 60 fringes are observed in the field of view of the eye-piece. Calculate the number of fringes that would be observed in the same field of view if the sodium light is replaced by mercury vapour lamp ($\lambda = 5460$ Å).

**[GGSIPU, Dec. 2016 (4 marks)]**

**Hint :** $\beta_S = \frac{D\lambda_S}{d}$, $\beta_M = \frac{D\lambda_M}{d}$

$$\frac{\beta_S}{\beta_M} = \frac{\lambda_S}{\lambda_M} = \frac{5893}{5460}$$

Given $60\beta_S = n\beta_M \quad \Rightarrow \quad n = 60 \times \frac{\beta_S}{\beta_M} = 60 \times \frac{5893}{5460} = 65.$

## Multiple Choice Questions

**4.1** If Young's experimental set up is displaced from air and immersed in water, the fringe width will

(*a*) decrease (*b*) increase

(*c*) remain unchanged (*d*) be zero.

**4.2** Oil floating on water surface is seen coloured because of interference of light. The possible thickness of the oil film is

(*a*) 100Å (*b*) 10000Å (*c*) 1 mm (*d*) 1 cm

**4.3.** Fringe of width 1.474 mm is observed at distance of 50 cm inside the geometrical shadow of a wire, when illuminated by light of wavelength 5896Å. The diameter of wire is

(*a*) $4 \times 10^{-4}$ m (*b*) $3 \times 10^{-4}$ m (*c*) $2 \times 10^{-4}$ m (*d*) $1 \times 10^{-4}$ m

4.4 The central fringe of the interference pattern produced by the light of wavelength 6000 Å is found to shift to the position of 4th bright fringe after a glass plate of refractive index 1.5 is introduced. The thickness of the glass plate would be :

(*a*) $4.80 \times 10^{-6}$ m (*b*) $8.23 \times 10^{-6}$ m (*c*) $14.98 \times 10^{-6}$ m (*d*) $3.78 \times 10^{-6}$ m

4.5 The phenomenon which produces colours in a soap bubble is due to

(*a*) diffraction (*b*) dispersion (*c*) interference (*d*) polarisation

4.6 Two light sources are coherent when

(*a*) their amplitudes are same (*b*) their frequencies are same

(*c*) their wavelengths are same

(*d*) their frequencies are same and their phase difference is constant

4.7 In Young's double slit experiment the distance between two slits is 2 mm and the screen is at a distance 120 cm from the slits. The smallest distance from the central maxima where the brightest fringes due to light of wavelength 6500 Å and 5200 Å would coincide is

(*a*) 0.117 cm (*b*) 0.156 cm (*c*) 0.234 cm (*d*) 0.20 cm.

4.8 A thin air film between a plane glass plate and a convex lens is irradiated with a parallel beam of monochromatic light and is observed under a microscope one finds :

(*a*) uniform brightness (*b*) complete darkness

(*c*) field crossed over by concentric bright and dark ring

(*d*) field crossed over by many coloured fringes.

4.9 In Newton's ring experiment the diameter of the bright rings are proportional to the square root of :

(*a*) natural numbers (*b*) odd natural numbers

(*c*) even natural number (*d*) half integral multiples of natural numbers.

4.10 Two waves of equal amplitude and wavelength but differing in phase are superposed. Amplitude of the resultant wave is maximum, when phase difference is

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{3\pi}{2}$ (*c*) $2\pi$ (*d*) $\pi$

4.11 In a Michelson interferometer, when the screw is moved through *d* mm, 100 and 80 circular fringes are observed for lights of wavelength $\lambda_1$ and $\lambda_2$ respectively. The ratio is $\frac{\lambda_1}{\lambda_2}$ equals

(*a*) 0.8 (*b*) 1.25 (*c*) 1.8 (*d*) 4.5

4.12 Newton's rings are fringes of

(*a*) equal inclination (*b*) equal thickness

(*c*) both equal inclination and equal thickness (*d*) equal radii.

4.13 Two ordinary glass plates are placed one over another and illuminated by monochromatic light, the interference fringes will

(*a*) be straight lines (*b*) be circular

(*c*) be irregular shaped (*d*) not to be formed.

4.14 Two coherent sources of intensity ratio 25 : 4 are used in an interference experiment. The ratio of intensities of maxima and minima in the interference pattern is

(*a*) 25 : 16 (*b*) 49 : 4 (*c*) 4 : 9 (*d*) 7 : 3

**4.15** Interference may be seen using two independent

(*a*) sodium lamps (*b*) fluorescent tubes

(*c*) lasers (*d*) mercury vapour lamps.

**4.16** Michelson's interferometer is based on the principle of

(*a*) division of amplitude (*b*) division of wavefront

(*c*) addition of amplitude (*d*) none of the above.

**4.17** Two interfering light waves have their amplitudes in the ratio 3 : 2. The ratio of the intensity of maxima to that of minima will be

(*a*) 3 : 2 (*b*) 5 : 1 (*c*) 9 : 4 (*d*) 25 : 1

**4.18** One leg of a Michelson's interferometer is lengthened so that the mirror is shifted by 0.020 mm. If the light used has $\lambda = 5000$ Å, then number of dark fringes sweeping through the field of view.

(*a*) 80 (*b*) 100 (*c*) 150 (*d*) 200.

**4.19** A biprism of refracting angle 1° is made up of a material of refractive index 1.55. The biprism is placed at a distance of 13 cm from the slit (source). The separation between the coherent source formed by it is

(*a*) 0.35 cm (*b*) 0.25 cm (*c*) 0.5 cm (*d*) 0.45 cm.

**4.20** A Fresnel's biprism arrangement is set with sodium light ($\lambda = 5893$ Å) and in the field of eyepiece, 62 fringes are seen. Now the source is replaced with a mercury source and a green filter of ($\lambda = 5461$Å) is placed in front of it. The number of fringes now seen will be

(*a*) 54 (*b*) 71 (*c*) 67 (*d*) 81

**4.21** When a thin glass plate ($\mu = 1.5$) is introduced in one of the arms of Michelson interferometer using light of wavelength 5890 Å, there is a shift of 10 fringes. The thickness of the plate will be

(*a*) $5.89 \times 10^{-6}$ m (*b*) $5.48 \times 10^{-4}$ m

(*c*) $5.89 \times 10^{-5}$ m (*d*) none of these.

**4.22** In a biprism experiment when a glass plate of thickness $t$ and refractive index $\mu$ is placed in the path of one of the interfering ray (wave) the entire system shifts through a distance given by

(*a*) $\frac{2d}{D}(\mu - 1)t$ (*b*) $\frac{2d}{D}(\mu + 1)t$ (*c*) $\frac{D}{2d}(\mu - 1)t$ (*d*) $\frac{D}{2d}(\mu + 1)t$

## Answers

| | | | | | |
|---|---|---|---|---|---|
| **4.1** (*a*) | **4.2** (*b*) | **4.3** (*c*) | **4.4** (*a*) | **4.5** (*c*) | **4.6** (*d*) |
| **4.7** (*c*) | **4.8** (*c*) | **4.9** (*b*) | **4.10** (*c*) | **4.11** (*a*) | **4.12** (*b*) |
| **4.13** (*a*) | **4.14** (*b*) | **4.15** (*c*) | **4.16** (*a*) | **4.17** (*d*) | **4.18** (*a*) |
| **4.19** (*b*) | **4.20** (*c*) | **4.21** (*a*) | **4.22** (*c*) | | |