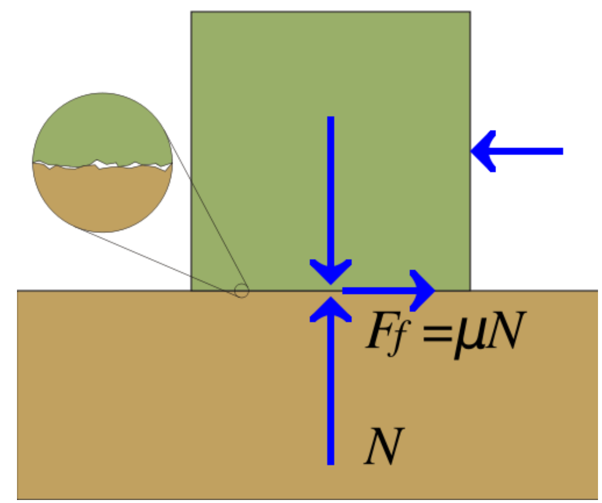
# Engineering Mechanics

Friction in Action

# What is friction?

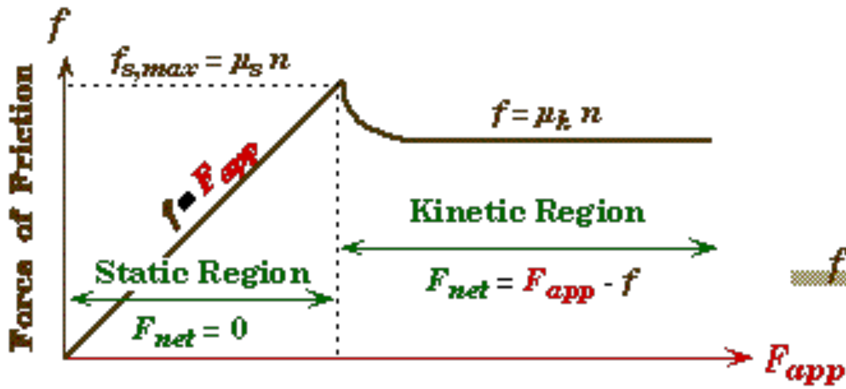
- Friction is a retarding force that opposes motion.
- Friction types:
  - Static friction
  - Kinetic friction
  - Fluid friction

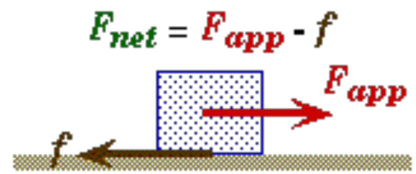
Dry or Coulombic friction

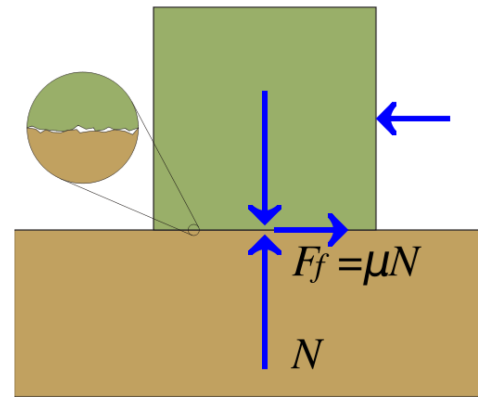



- Sources of dry friction
  - Asperities between contacting surfaces
  - Interactions at the atomic level
- *Tribology* studies sources of friction, lubrication, wear and tear etc.

# Coefficient of Friction




- At impending motion:

$$F = \mu N$$

*This is the maximum force for a given N.*

- $\mu$ is coefficient of friction.
  - for impending *relative* motion $\mu = \mu_s$
  - for actual *relative* motion $\mu = \mu_k$
  - $\mu_k < \mu_s$

# The Laws of Dry Friction. Coefficients of Friction

**Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces**

| | |
|---|---|
| Metal on metal | 0.15–0.60 |
| Metal on wood | 0.20–0.60 |
| Metal on stone | 0.30–0.70 |
| Metal on leather | 0.30–0.60 |
| Wood on wood | 0.25–0.50 |
| Wood on leather | 0.25–0.50 |
| Stone on stone | 0.40–0.70 |
| Earth on earth | 0.20–1.00 |
| Rubber on concrete | 0.60–0.90 |

- Maximum static-friction force:

$$F_m = \mu_s N$$

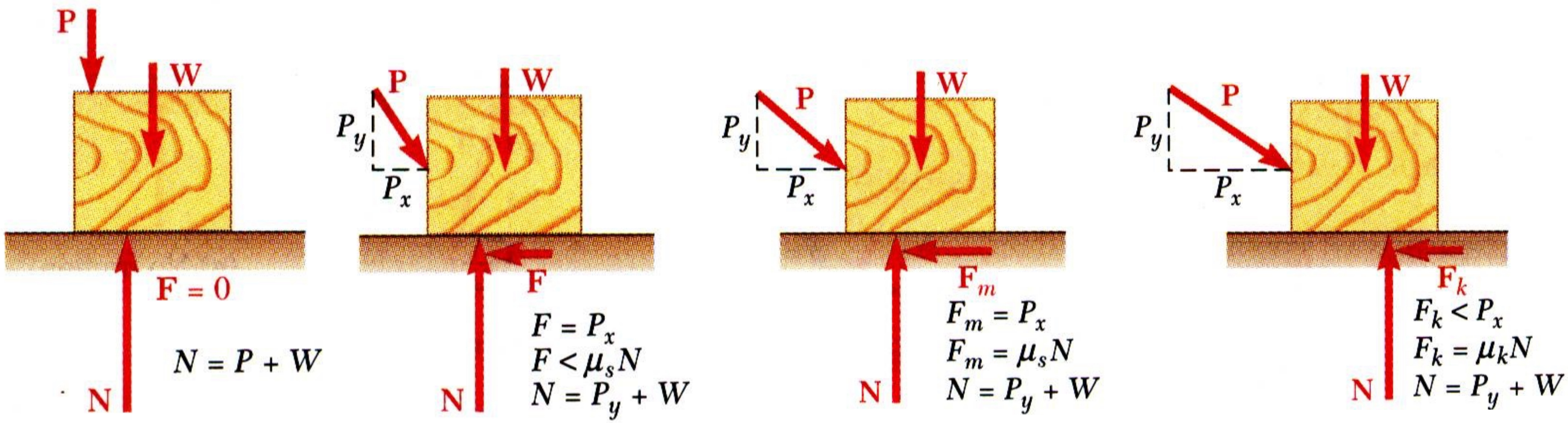
- Kinetic-friction force:

$$F_k = \mu_k N$$

- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

## The Laws of Dry Friction. Coefficients of Friction

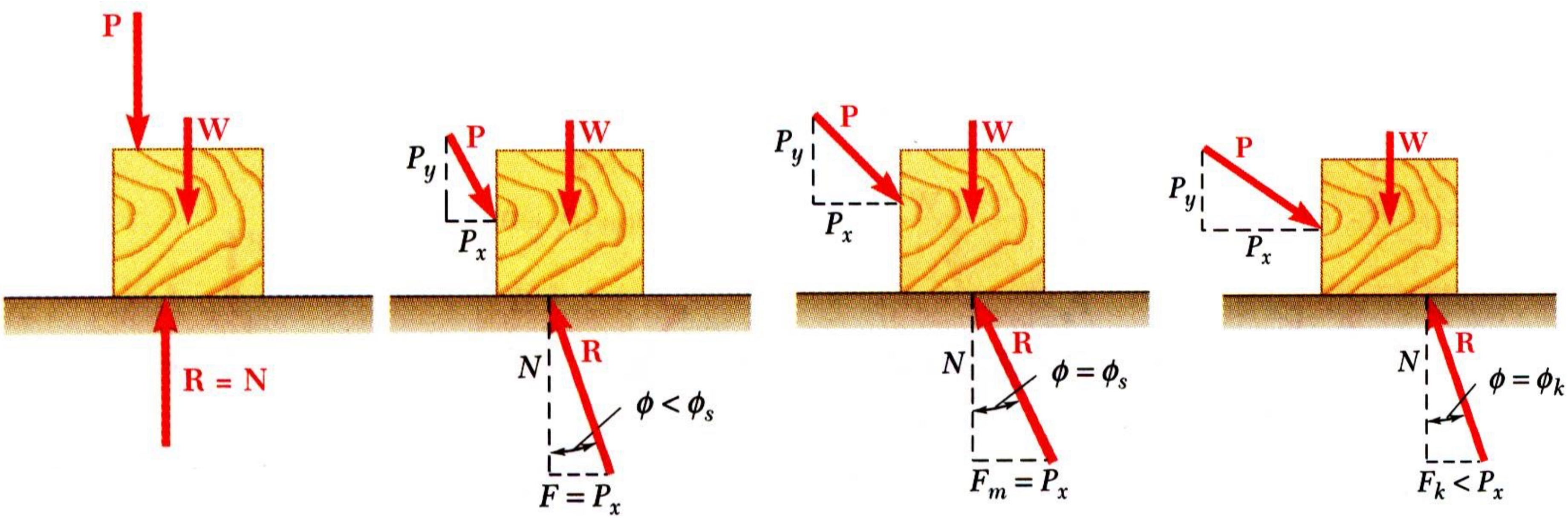
- Four situations can occur when a rigid body is in contact with a horizontal surface:




- No friction, ($P_x = 0$)
- No motion, ($P_x < F$)
- Motion impending, ($P_x = F_m$)
- Motion, ($P_x > F_m$)

## Angles of Friction

- It is sometimes convenient to replace normal force **N** and friction force **F** by their resultant **R**:




- No friction
- No motion
- Motion impending

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

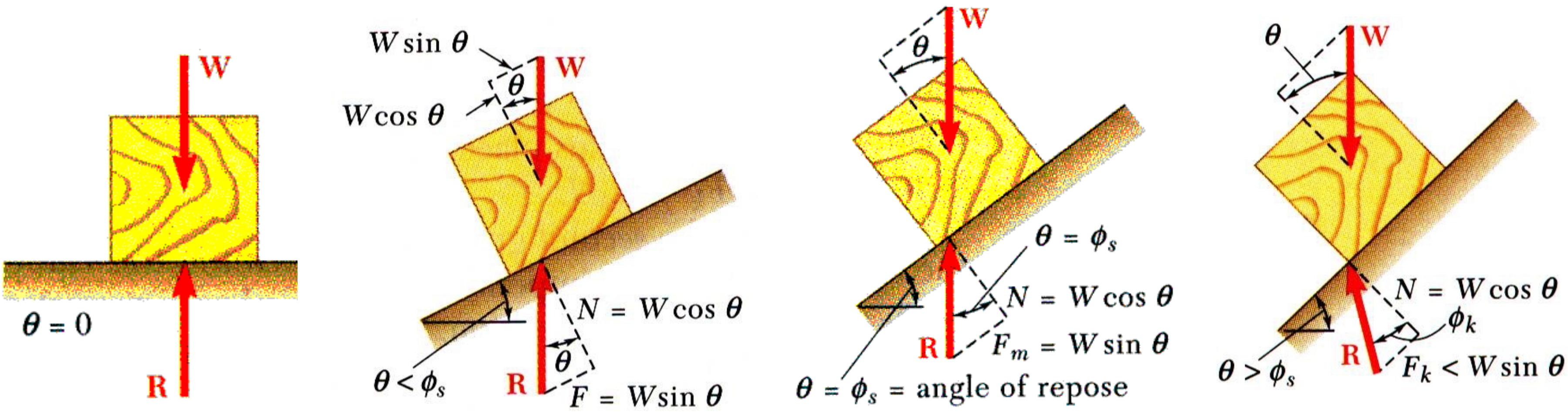
- Motion

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

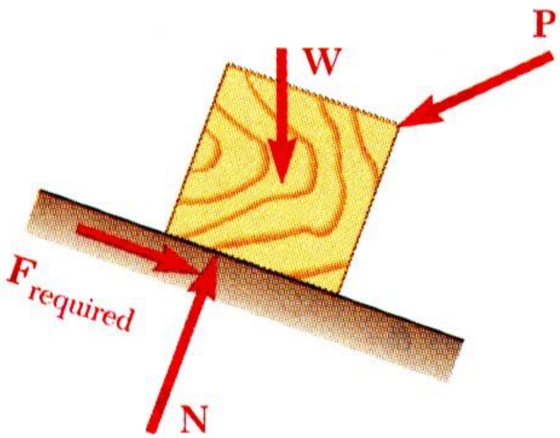
## Angles of Friction

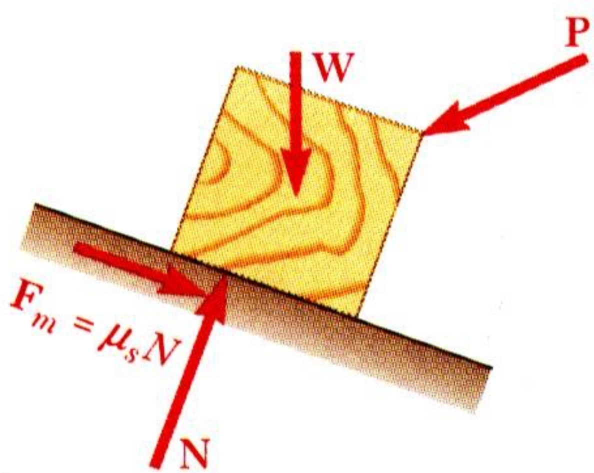
- Consider block of weight **W** resting on board with variable inclination angle θ.

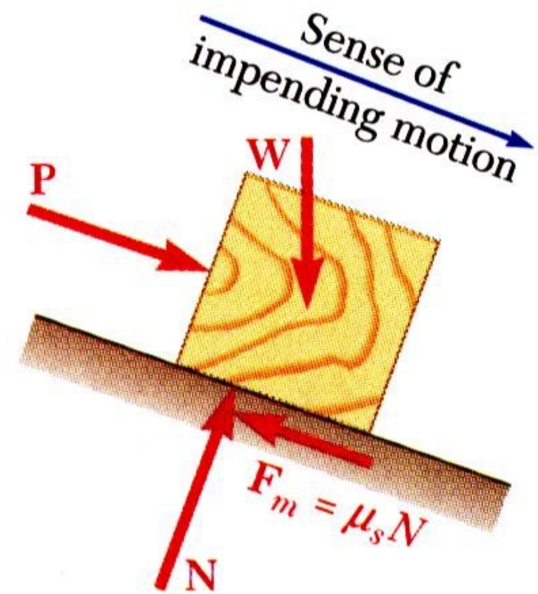



- No friction
- No motion
- Motion impending
- Motion

## Problems involving Dry Friction



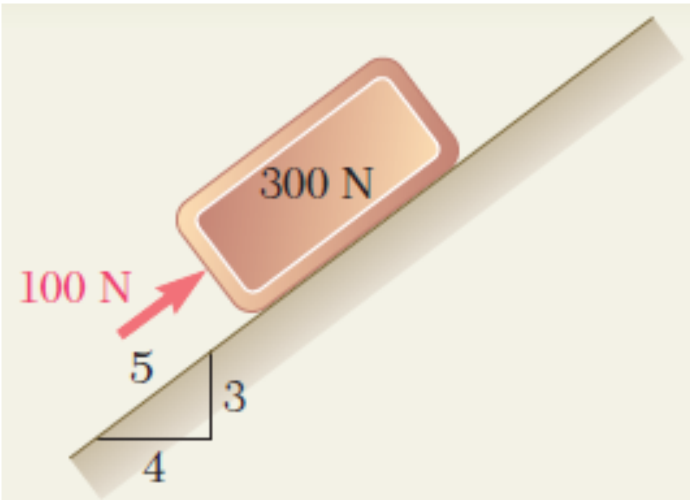




- All applied forces known
- Coefficient of static friction is known
- Determine whether body will remain at rest or slide

- All applied forces known
- Motion is impending
- Determine value of coefficient of static friction.

- Coefficient of static friction is known
- Motion is impending
- Determine magnitude or direction of one of the applied forces

## Sample Problem 8.1

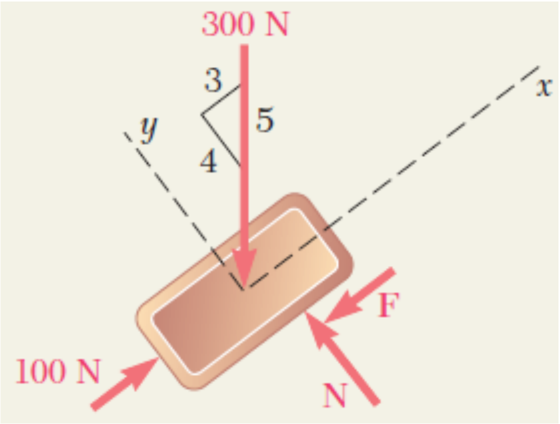



A 100-N force acts as shown on a 300-N block placed on an inclined plane. The coefficients of friction between the block and plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium and find the value of the friction force.

SOLUTION:

- Draw the free body diagram for the block. Remember that the friction force is *opposite the direction of impending motion.*
- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.
- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.
- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

## Sample Problem 8.1




SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.

$$\sum F_x = 0: \qquad 100\ \text{N} - \tfrac{3}{5}(300\ \text{N}) - F = 0$$

$$F = -80\ \text{N}$$

$$\sum F_y = 0: \qquad N - \tfrac{4}{5}(300\ \text{N}) = 0$$

$$N = 240\ \text{N}$$

What does the sign tell you about the assumed direction of impending motion?

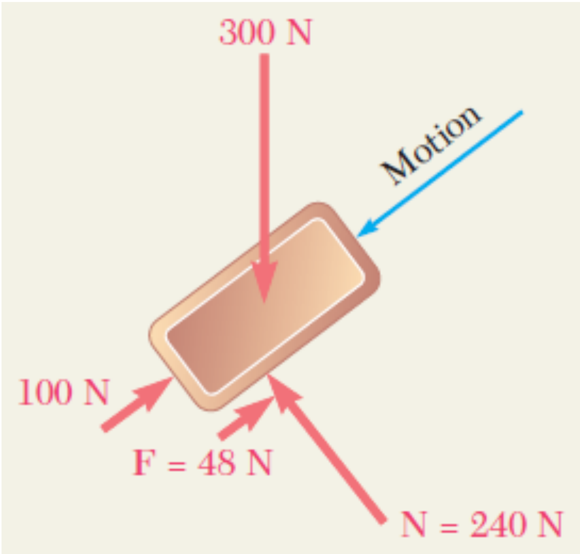
- Calculate maximum friction force and compare with friction force required for equilibrium.

$$F_m = \mu_s N = 0.25(240\ \text{N}) = 60\ \text{N}$$

What does this solution imply about the block?

*The block will slide down the plane.*

## Sample Problem 8.1




- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

$$F_{actual} = F_k = \mu_k N$$
$$= 0.20(240\text{ N})$$

$$\boxed{F_{actual} = 48\text{ N}}$$

# How to invoke laws of friction?

**Key: Always keep track of the number of unknowns and the number of equilibrium equation.**

**Read : Beer and Johnston 8th Ed. pgs 441-442**

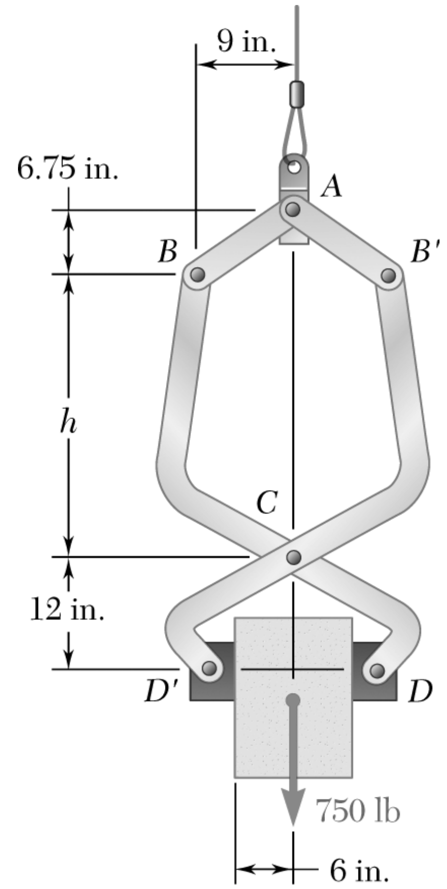
- Problem type 1:
  - Like a usual equilibrium problem
  - Solve using what we learnt earlier
  - Only a couple of changes:
    - Verify if the surface is capable of *handling* the load

      OR
    - Find the *minimum* friction coefficient required.

      OR
    - Given slipping occurs find the friction coefficient.
- Friction law need to be used at the very end.

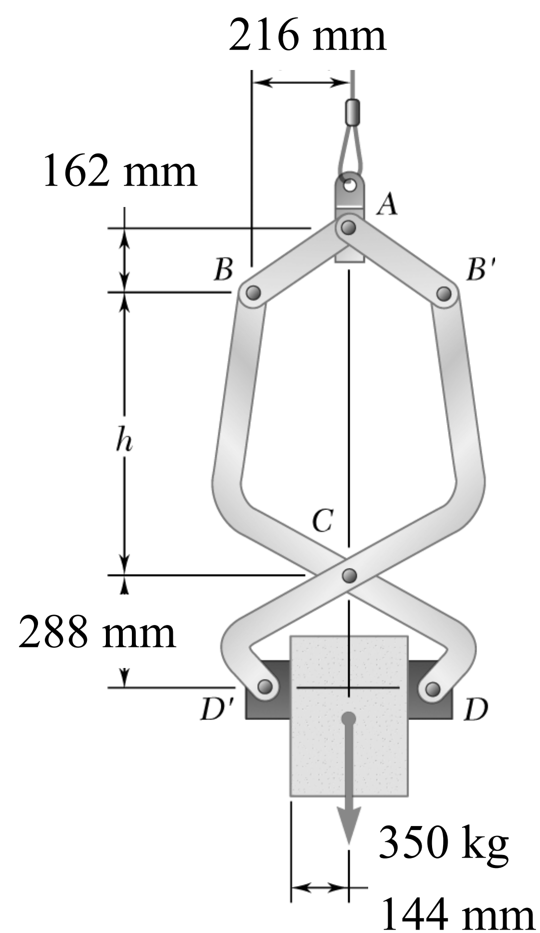
# Problem 1

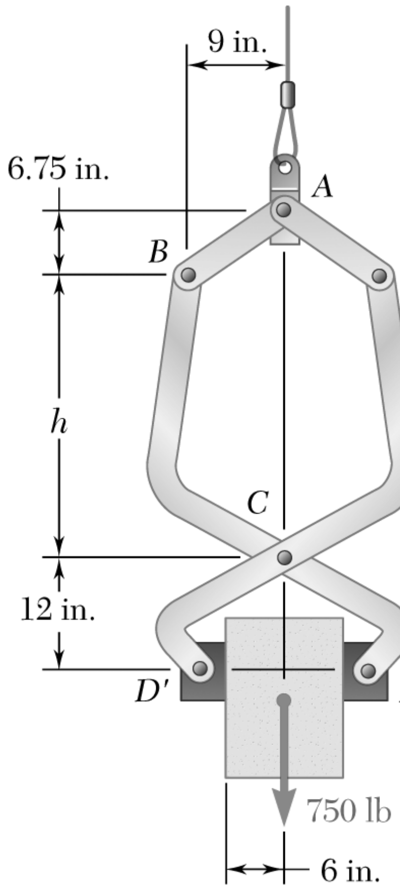
- The friction tongs shown are used to lift a 750-lb casting. Knowing that h = 35in. determine the smallest allowable value of the coefficient of static friction between the casting and blocks *D* and *D*'.

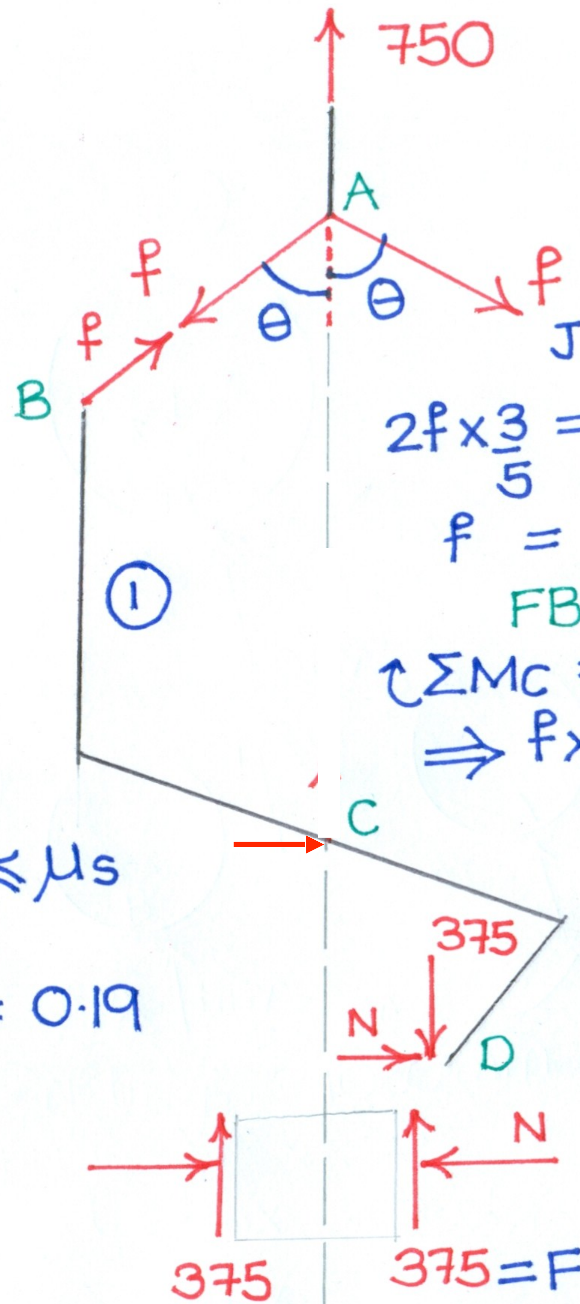


# SI Version

- The friction tongs shown are used to lift a 350 kg casting. Knowing that h = 864 mm, determine the smallest allowable value of the coefficient of static friction between the casting and blocks *D* and *D'*.






Jt. A

$$2f \times \frac{3}{5} = 750\,lb$$

$$f = 625\,lb$$

FBD-1

$$\circlearrowleft \Sigma M_C = 0$$

$$\Rightarrow f \times \frac{4}{5} \times (36+6.75) - N*12 + F*6 = 0$$

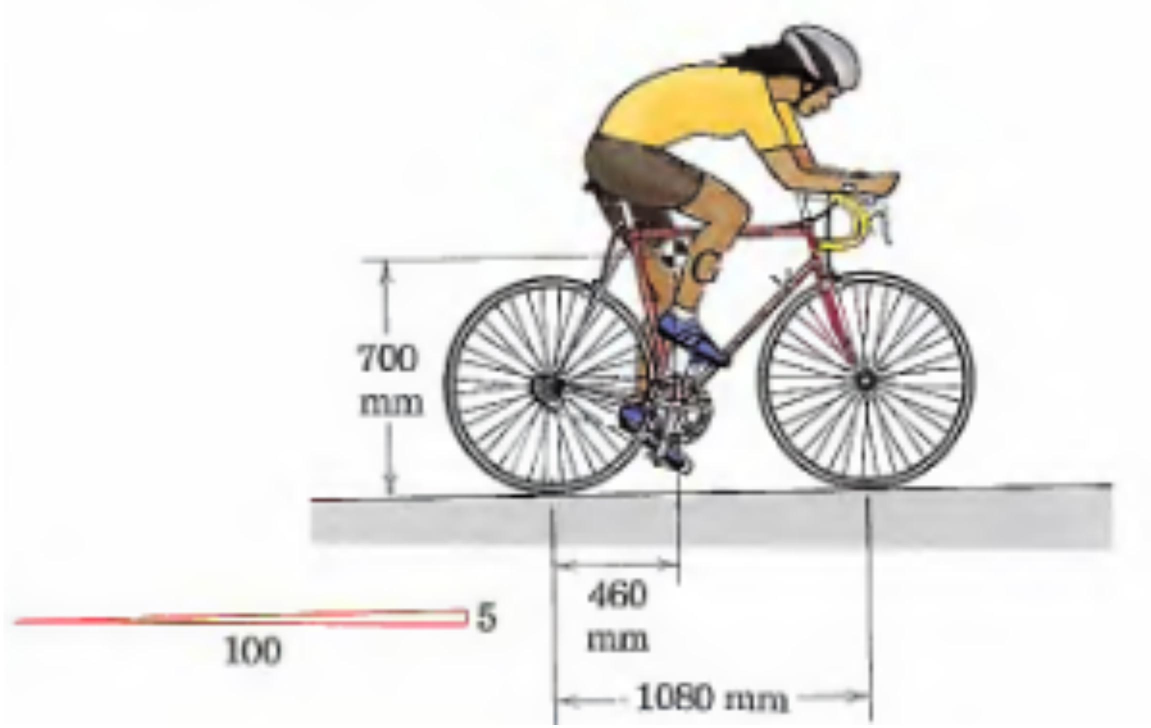
$$N = 1969\,lb$$

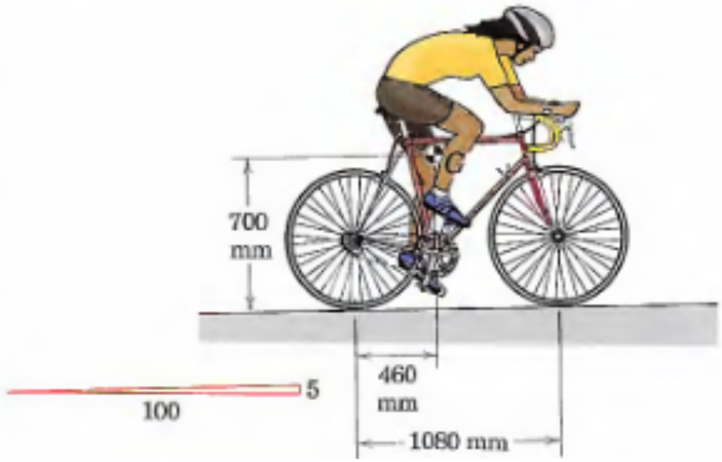
$$\frac{F}{N} \leq \mu_s$$

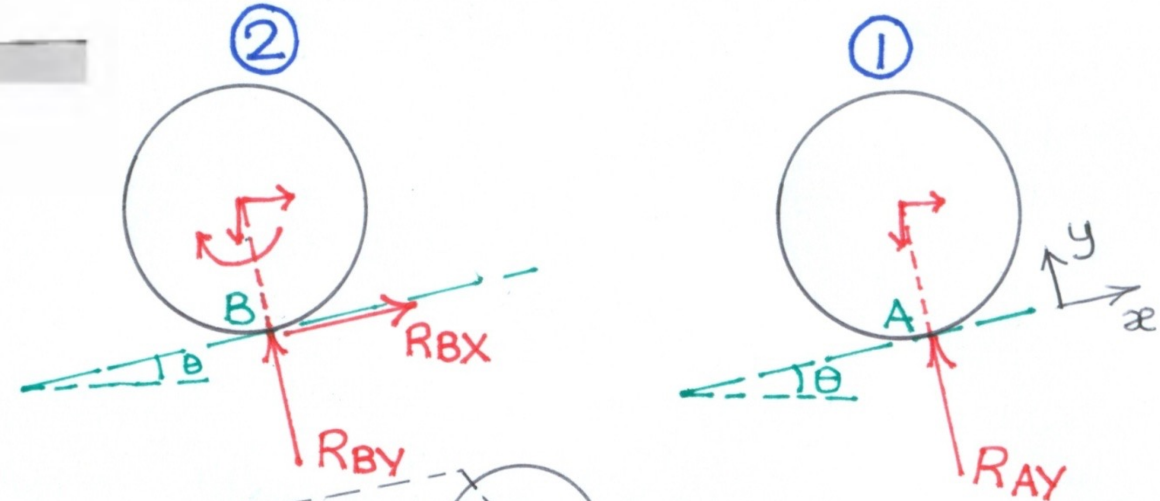
$$\mu_s^{min} = 0.19$$

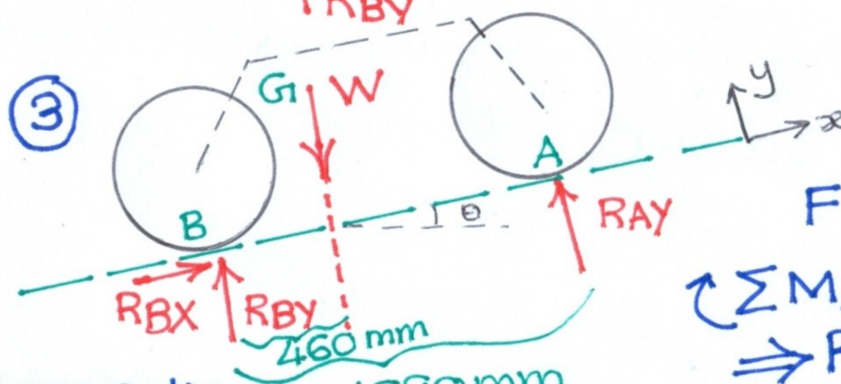
# Problem 2

A woman pedals her bicycle up a 5-percent grade and a slippery road at a steady speed. The woman and bicycle have a combined mass of 82 kg. with mass center at *G*. If the rear wheel is on the verge of slipping, determine the coefficient of friction $\mu_s$ between the rear tire and the road. If the coefficient of friction is doubled, what would be the friction force acting on the rear wheel? (why may we neglect friction under the front wheel)






FBD-3

$\circlearrowleft \sum M_A = 0$

$\Rightarrow R_{BY} = 49.7\,kg$

$\nearrow \sum F_x = 0$

$\Rightarrow W\sin\theta = R_{BX}$

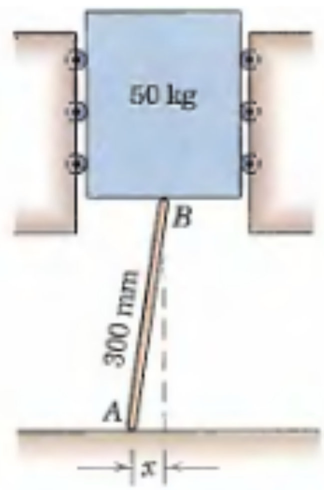
$R_{BX} = 4.09\,kg$

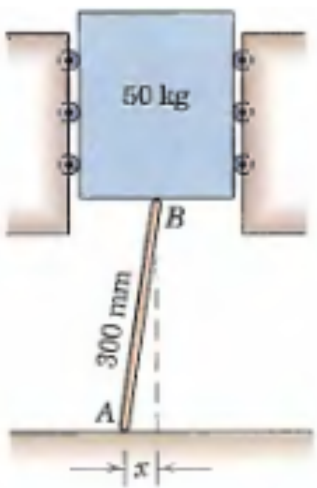
Impending slippage

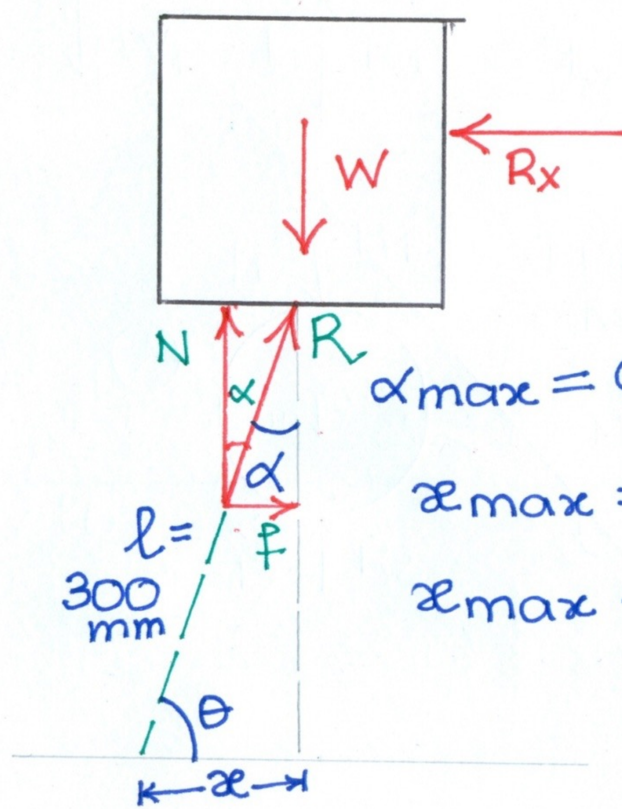
$\mu_s = \dfrac{R_{BX}}{R_{BY}} = 0.082$

# Problem 3

- The light bar is used to support the 50-kg block in its vertical guides. If the coefficient of static friction is 0.3 at the upper end of the bar and 0.4 at the lower end of the bar, find the friction force acting on each end for $x = 75mm$. Also find the maximum value of $x$ for which the bar will not slip.





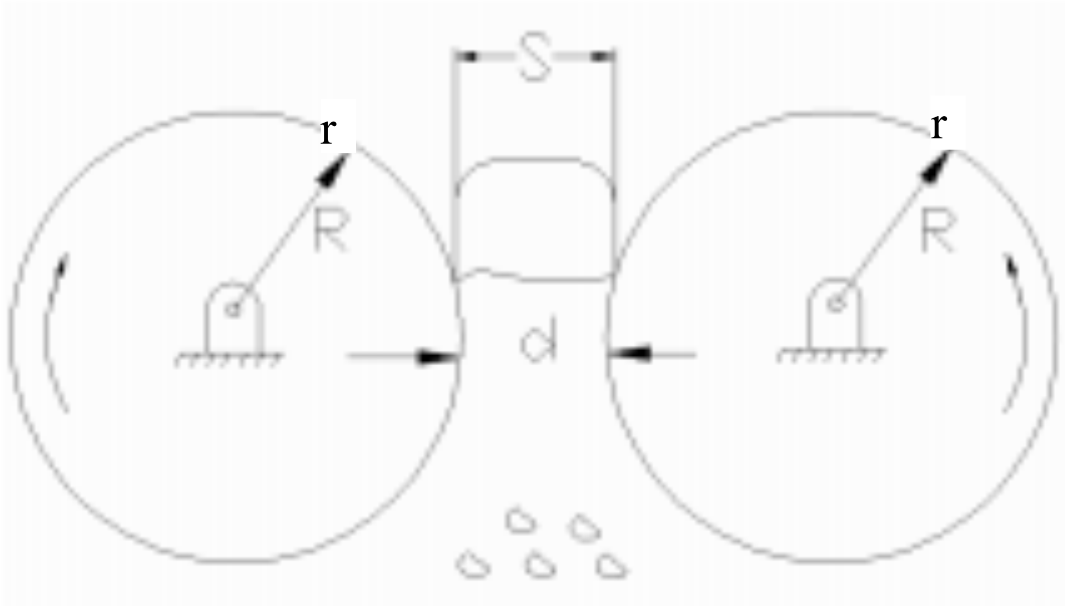

$$\alpha_{max} = \varphi_s = \tan^{-1}\mu_s$$

$$x_{max} = l \sin\alpha_{max}$$

$$x_{max} = 86.2\,mm$$

# Stone Crusher Problem

Two large cylinders, each of radius r = 500 mm rotate in opposite directions and form the main elements of a crusher for stone aggregates. The distance d is set equal to the maximum desired size of the crushed aggregate. If d = 20 mm, $\mu_s = 0.3$, determine the size of the largest stones which will be pulled through the crusher by friction alone. Assume the stone to be symmetrically placed on both wheels, and that the weight of the stone is negligible compared to the contact forces.

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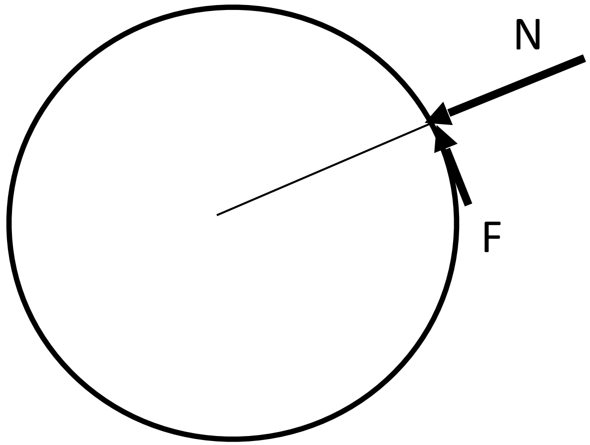
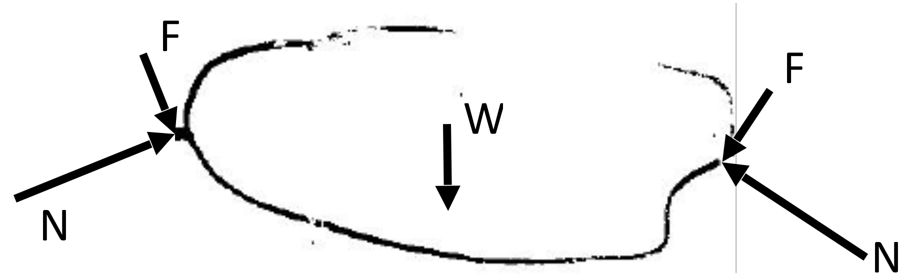
# Stone Crusher Problem - Solution



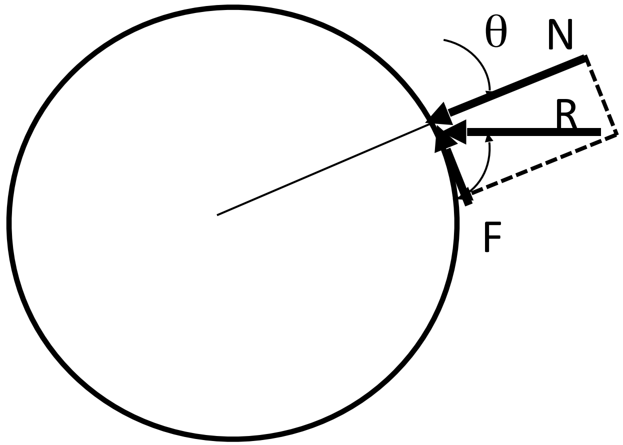

**FBD of Crusher Cylinder**




**FBD of Aggregate**

From FBD of aggregate it can be seen that the aggregate will pass through the crusher cylinders due to its self weight and the friction forces from the two cylinders.
Since the aggregate passes through the crusher due to friction alone, it implies that the self weight of the aggregate is neglected in this case.
Thus the aggregate reduces to a two-force body subjected to reaction forces from the cylinders at two points only and along the same line.

# Stone Crusher Problem - Solution




**FBD of Crusher Cylinder**


**FBD of Aggregate**

$r = 500$ mm
$d = 20$ mm
$\mu_s = 0.3$

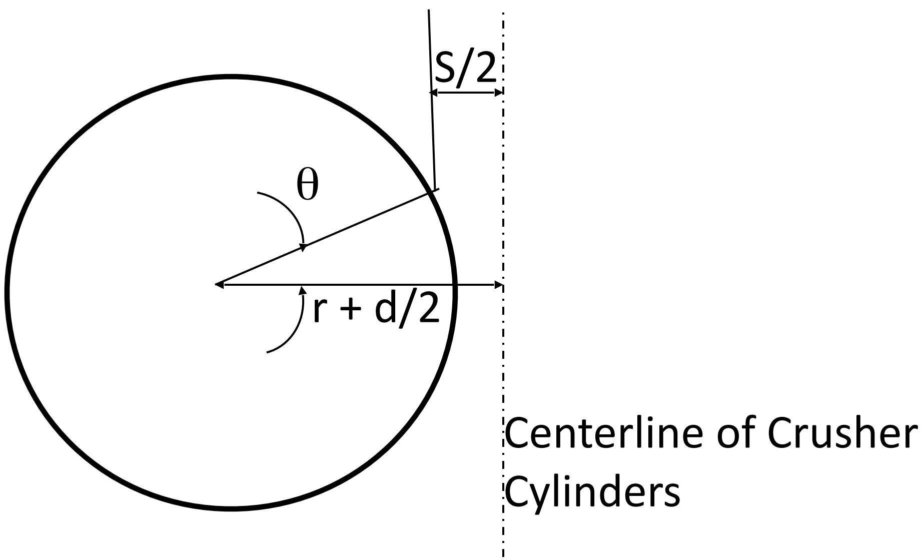
The aggregate and cylinder will be at the point of slipping when the maximum size aggregate passes.

$$\therefore \tan\theta = \frac{F}{N} = \mu_s$$

$$\Rightarrow \theta = \tan^{-1}\mu_s$$

$$\theta = 16.7^\circ$$

# Stone Crusher Problem - Solution




r = 500 mm
d = 20 mm
$\mu_s = 0.3$

$\theta = 16.7^\circ$

From the arrangement of the crusher and the maximum aggregate

$$r + \frac{d}{2} = r\cos\theta + \frac{s}{2}$$

$$s = d + 2r(1 - \cos\theta)$$

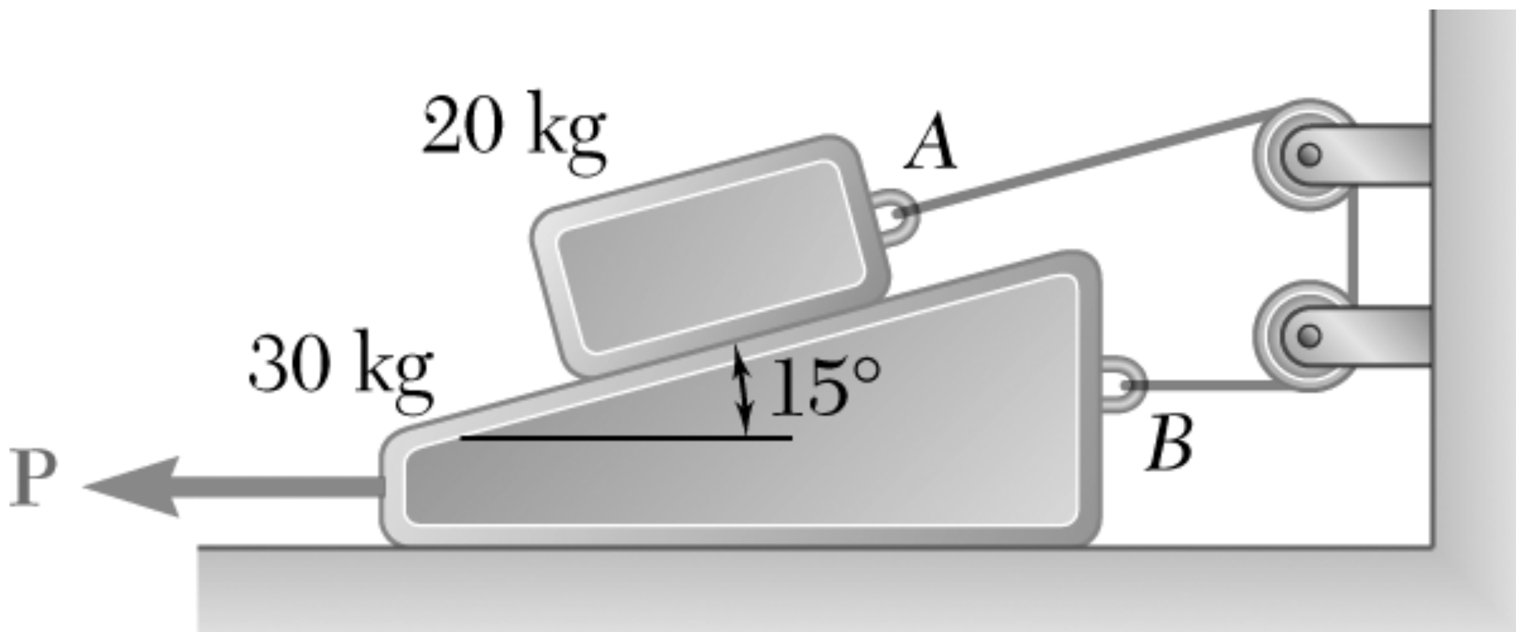
$s = 62.2$ mm

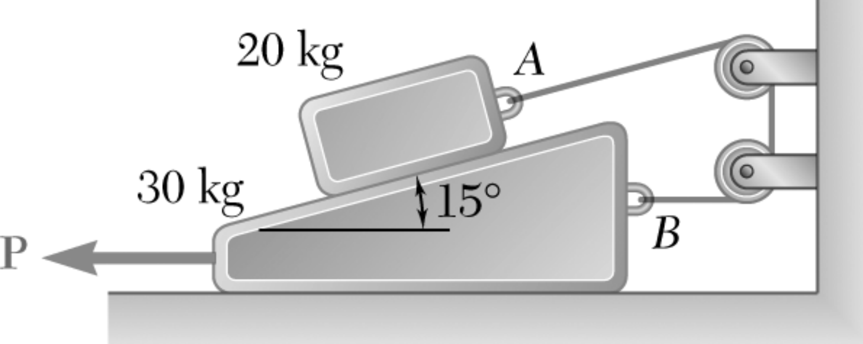
# Problem Type 2

- Number of equations is less than number of unknowns.
- Motion of the body is impending.
- You are asked to obtain:
  - Force/Torque required to start the *impending* motion.
  - Some distance, angle etc. for the impending motion.
- Need to use the law of friction at impending surface.
- *Careful about the sign of forces*.

# Problem 4

- The coefficients of friction are $\mu_s = 0.4$ and $\mu_k = 0.3$ between all surfaces of contact. Determine the force **P** for which motion of the 30-kg block is impending if cable $AB$ ($a$) is attached as shown, ($b$) is removed. Assume that the cord is inextensible and the pulleys are well oiled.



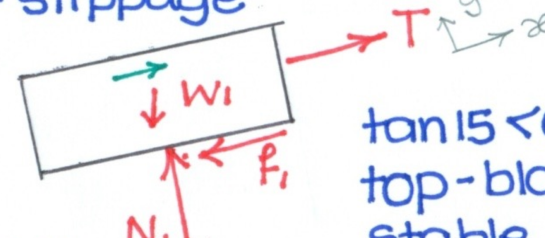



FBD ①

6 unknowns
2×2 = 4 equations
2 - slippage

$\uparrow \Sigma F_y = 0$
$\Rightarrow N_1 = 189.5\,N$

$\nearrow \Sigma F_x = 0$
$\Rightarrow T = 126.56\,N$




①

$f_1 = \mu_s N_1$

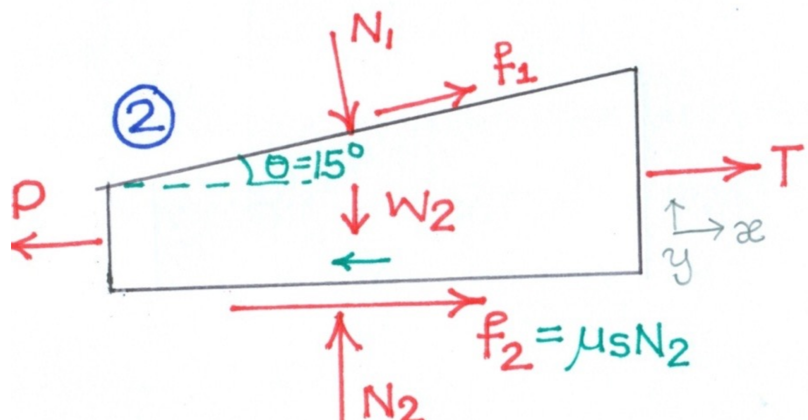
$\tan 15 < 0.4$
top-block
stable

FBD-②

$\uparrow \Sigma F_y = 0$
$\Rightarrow N_2 = 457.74\,N \uparrow$

$\Sigma F_x = 0$
$\Rightarrow P = 361\,N$

②



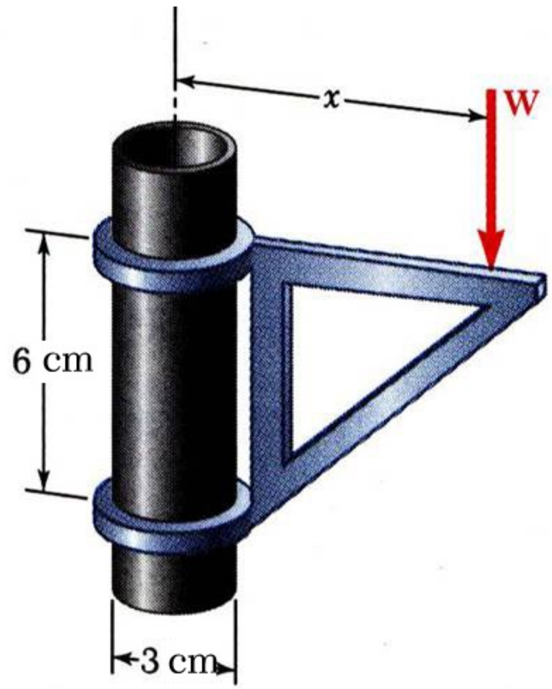

$f_2 = \mu_s N_2$

without rope; $P = \mu_s(\underbrace{\cancel{N_1} + N_2}_{W_1 + W_2})$

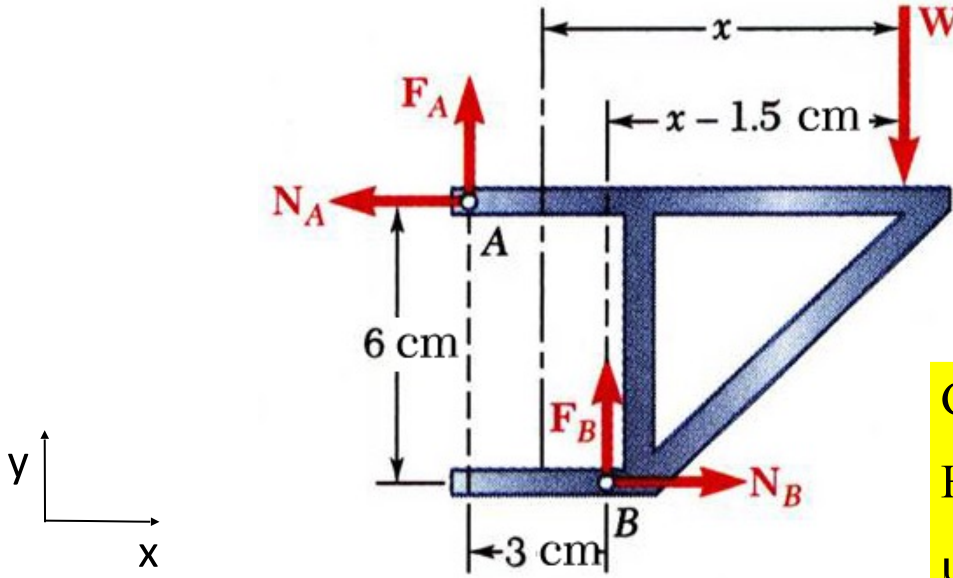
$P = 196.2\,N$

# Problem 40

The moveable bracket shown may be placed at any height on the **30** mm diameter pipe. If the coefficient of friction between the pipe and bracket is 0.25, determine the minimum distance $x$ at which the load can be supported. Neglect the weight of the bracket.



# Problem 40 - Solution




Given- Diameter of the pipe=30mm;

Height of the bracket=60mm.

μ =0.25

**FBD of Bracket**

As the bracket is loaded as shown, the reaction forces on the bracket by the pipe is at only 2 points as shown.

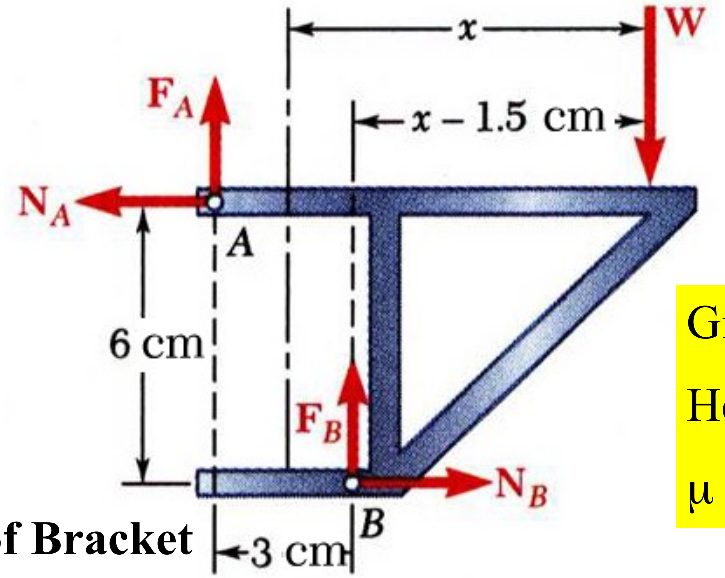
Consider equilibrium of forces acting on the bracket,

$$\sum F_x = 0 \Rightarrow N_A - N_B = 0 \Rightarrow N_A = N_B$$

$$\sum F_y = 0 \Rightarrow F_A + F_B - W = 0 \Rightarrow W = F_A + F_B$$

# Problem 40 - Solution

$$N_A = N_B$$
$$W = F_A + F_B$$




FBD of Bracket

Given- Diameter of the pipe=30mm;

Height of the bracket=60mm.

μ =0.25

$$\sum M_B = 0 \Rightarrow N_A \times 6\text{cm} - F_A \times 3\text{cm} - W \times (x - 1.5\text{cm}) = 0$$

$$\Rightarrow N_A \times 6\text{cm} - F_A \times 3\text{cm} - (F_A + F_B) \times (x - 1.5\text{cm}) = 0$$

$$\Rightarrow x = 1.5\text{cm} + \frac{N_A \times 6\text{cm} - F_A \times 3\text{cm}}{F_A + F_B}$$

Putting the maximum values of $F_A$ and $F_B$ in the above expression will give the minimum value of x at which the load W can be supported.

$$F_{A,\max} = 0.25 N_A ; F_{B,\max} = 0.25 N_B = 0.25 N_A$$

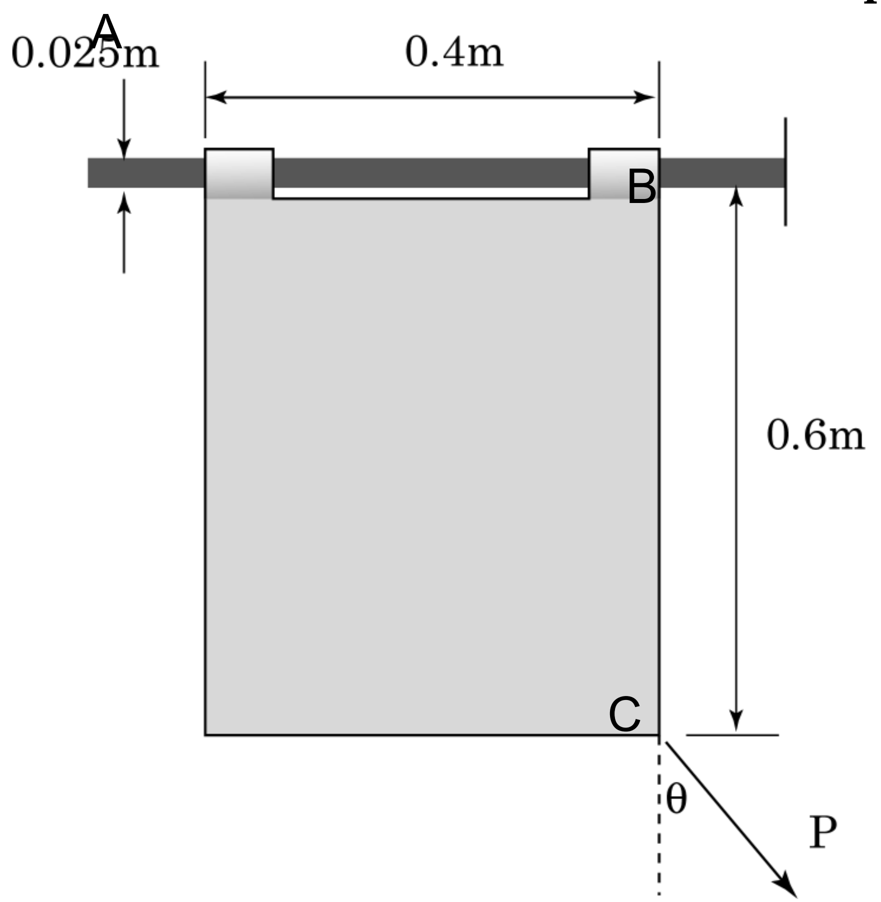
$$\Rightarrow x_{\min} = 12\text{cm}$$

# Problem Type 3

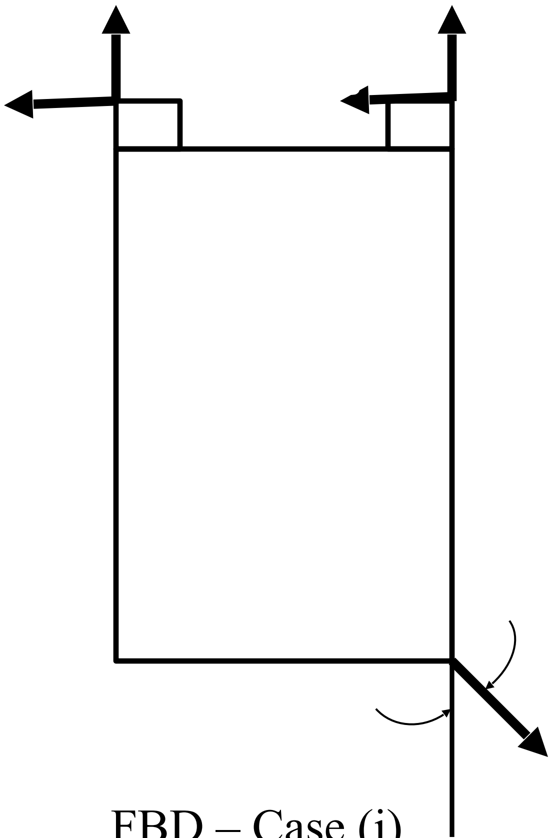
- Similar to Problem Type 2, but with one significant difference
- There can be multiple modes of slipping.
- Which particular contact at which the *impending slippage* occurs have to be decided by
  - Trial and error
  - Inspection
  - Physical intuition.
- Ultimately it must be checked that everything is consistent, i.e., the force on surfaces other than slipping surfaces should be less than $\mu N$.
- *Caution: Be careful about the direction of forces*

# Problem 4a

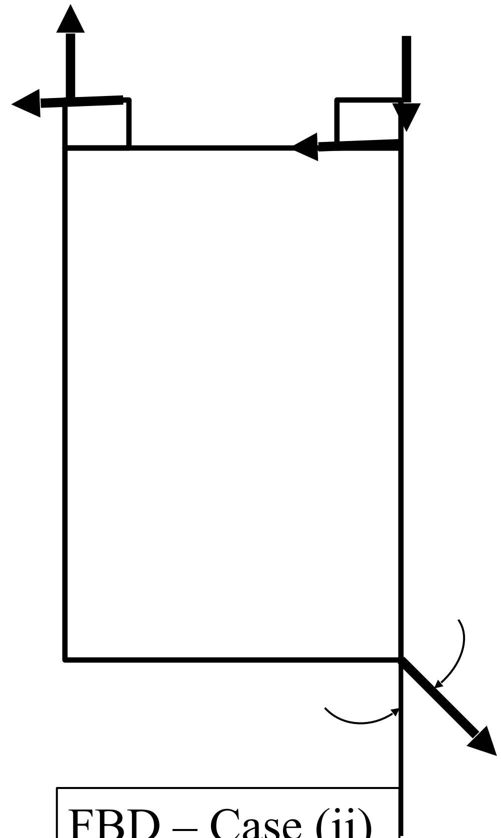
- A light metal panel is welded to two **short sleeves** of 0.025 m inside diameter that can slide on a fixed horizontal rod. The coefficient of friction between the sleeves and the rod are $\mu_s = 0.4$ and $\mu_k = 0.3$. A cord attached to corner $C$ is used to move the panel along the rod. Knowing that the cord lies in the same vertical plane as the panel, determine the range of values of $\theta$ for which the panel will be in impending motion to the right. Assume that **sleeves make contact with the rod at the exterior points A and B.**



# Metal Panel Problem – Two cases



FBD – Case (i)
Small angle



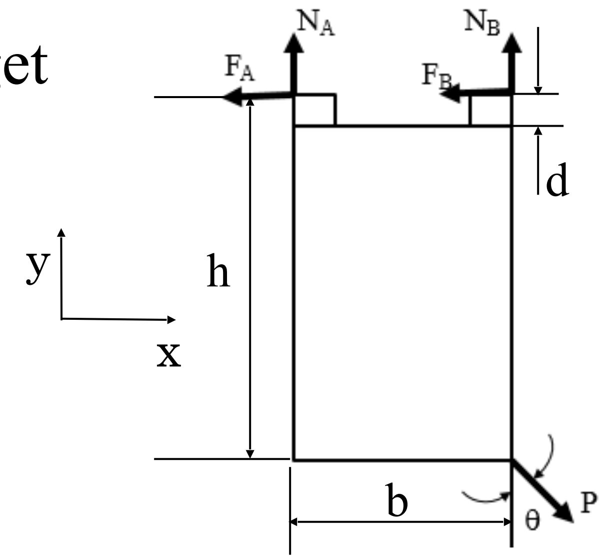
FBD – Case (ii)
Large angle

There will two possible situations of loads acting on the panel as shown in the FBDs above:

# Metal Panel Problem - Solution

Case (i)

From equilibrium conditions of the panel we get




$$\sum M_A = 0 \Rightarrow bN_B = bP\cos\theta - hP\sin\theta$$

$$\therefore N_B \geq 0 \text{ if } \tan\theta \leq \frac{b}{h} \qquad \Rightarrow \theta \leq 33.1^\circ$$

$$\sum F_y = 0 \Rightarrow N_A + N_B = P\cos\theta$$

Panel will start moving to the right if :

$$P\sin\theta = F_A + F_B$$

$$\Rightarrow P\sin\theta = \mu_s N_A + \mu_s N_B$$

$$\Rightarrow P\sin\theta = \mu_s P\cos\theta$$

$$\Rightarrow \tan\theta = \mu_s \qquad \Rightarrow \theta = 21.8^\circ$$

$\mu_s = 0.4$

$\mu_k = 0.3$

b = 400 mm
h = 612.5 mm
d = 25 mm

If $\theta > 21.8^0$ the panel will move to the right under a net force of

$$P(\sin\theta - \mu_k\cos\theta)$$

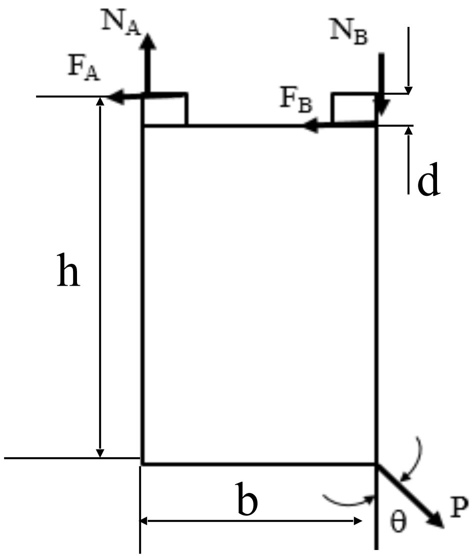
Therefore for case (i) the panel will move to the right if

$$21.8^\circ \leq \theta \leq 33.1^\circ$$

# Metal Panel Problem - Solution

Case (ii)

From equilibrium conditions of the panel we get




$$\sum M_A = 0 \Rightarrow bN_B + dF_B = hP\sin\theta - bP\cos\theta$$

$$F_B = \mu_s N_B$$

$$\Rightarrow N_B = \frac{hP\sin\theta - bP\cos\theta}{b + \mu_s d} \qquad \text{(A)}$$

Can also take Moment w.r.t B

$$\therefore N_B \geq 0 \text{ if } \tan\theta \geq \frac{b}{h} \qquad \Rightarrow \theta \geq 33.1^\circ$$

$$\mu_s = 0.4$$
$$\mu_k = 0.3$$

b = 400 mm
h = 625 mm
d = 25 mm

$$\sum F_y = 0 \Rightarrow N_A - N_B = P\cos\theta$$

Panel will start moving to the right if :

$21.8^\circ \leq \theta \leq 33.1^\circ$ from case (i)

$$P\sin\theta = F_A + F_B$$

$$\Rightarrow P\sin\theta = \mu_s N_A + \mu_s N_B$$

$$\Rightarrow P\sin\theta = \mu_s\left(P\cos\theta + 2N_B\right)$$

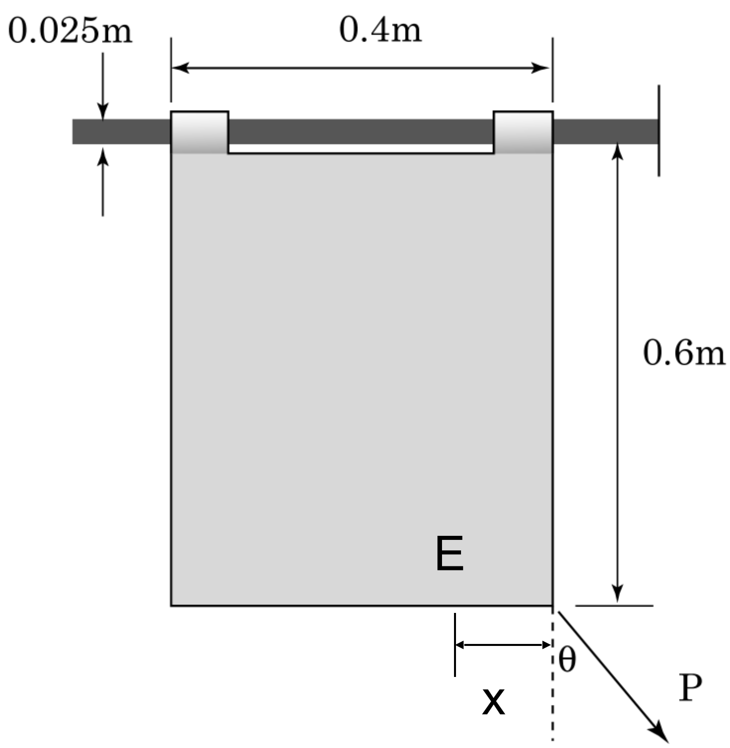
Replacing $N_B$ from (A) in the above we get

$$21.8^\circ \leq \theta \leq 62.9^\circ$$

$$\tan\theta = \frac{2\mu_s b - \mu_s\left(b + \mu_s d\right)}{2\mu_s h - \left(b + \mu_s d\right)} \Rightarrow \theta = 62.9^\circ \qquad \Rightarrow 33.1^\circ \leq \theta \leq 62.9^\circ$$

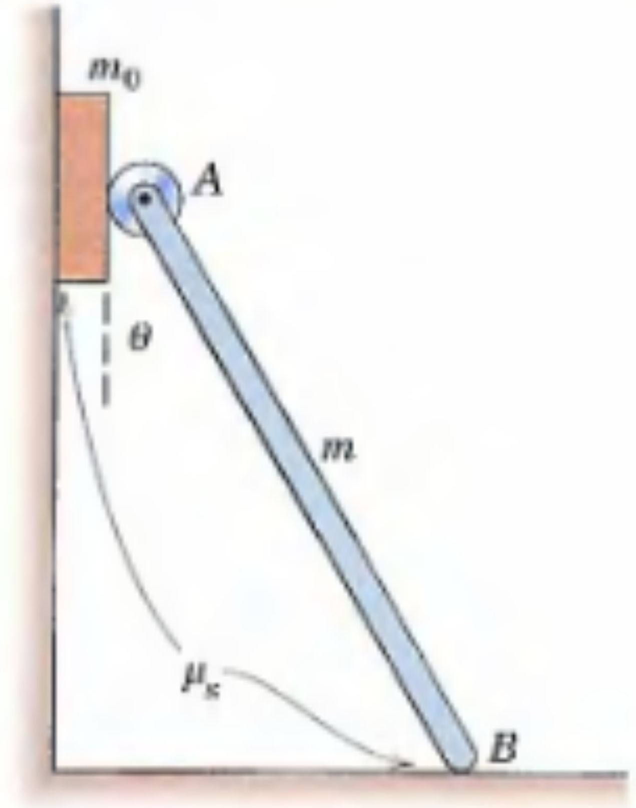
# Problem 4b

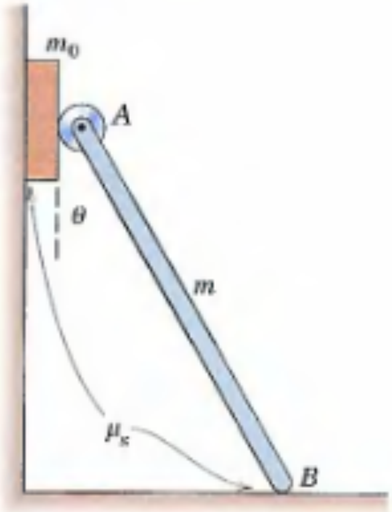
- For the previous problem 4a, assuming that the cord is attached at point $E$ at a distance $x = 0.1$ m from the corner $C$. (b) Determine the largest value of $x$ for which the panel can be moved to the right.

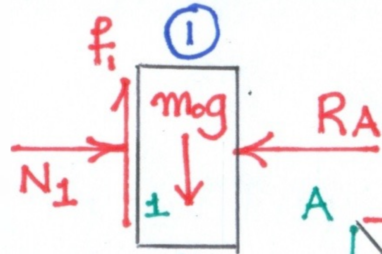


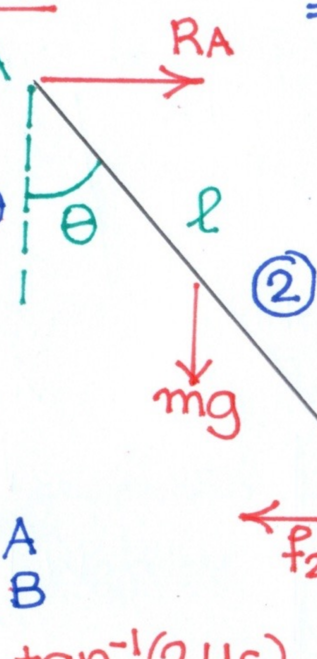
# Problem 5

- A block of mass $m_o$ is placed betwee the vertical wall and the small ideal roller at the upper end $A$ of the uniform slender bar of mass $m$. The lower end $B$ of the bar rests on the horizontal surface. If the coefficient of static friction is $\mu_s$ at $B$ and also between the block and the wall, determine a general expression for the minimum value $\theta_{min}$ of $\theta$ for which the block will remain in equilibrium. Evaluate the expressio for $\mu_s = 0.5$ and $m/m_o = 10$. For these conditions, check for possible slipping at $B$.






Unknowns
$f_1, N_1, f_2, N_2, R_A, \theta$
(6)
Eqns: $2+3=5$
slippage at 1 surface

small '$\theta$' slip at A
large $\theta$ slip at B

$\tan^{-1}\dfrac{2m_0}{m\mu_s} \leq \theta \leq \tan^{-1}(2\mu_s)$

if $\dfrac{2m_0}{m\mu_s} \geq 2\mu_s$; not stable ever!

FBD-2

$\circlearrowleft \Sigma M_B = 0$

$\Rightarrow R_A = \dfrac{mg}{2}\tan\theta$

$f_2 = \dfrac{mg}{2}\tan\theta \quad (\rightarrow \Sigma F_x = 0)$

$N_2 = mg \quad (\uparrow \Sigma F_y = 0)$

$\dfrac{f_2}{N_2} = \dfrac{\tan\theta}{2} \leq \mu_s$

FBD-1

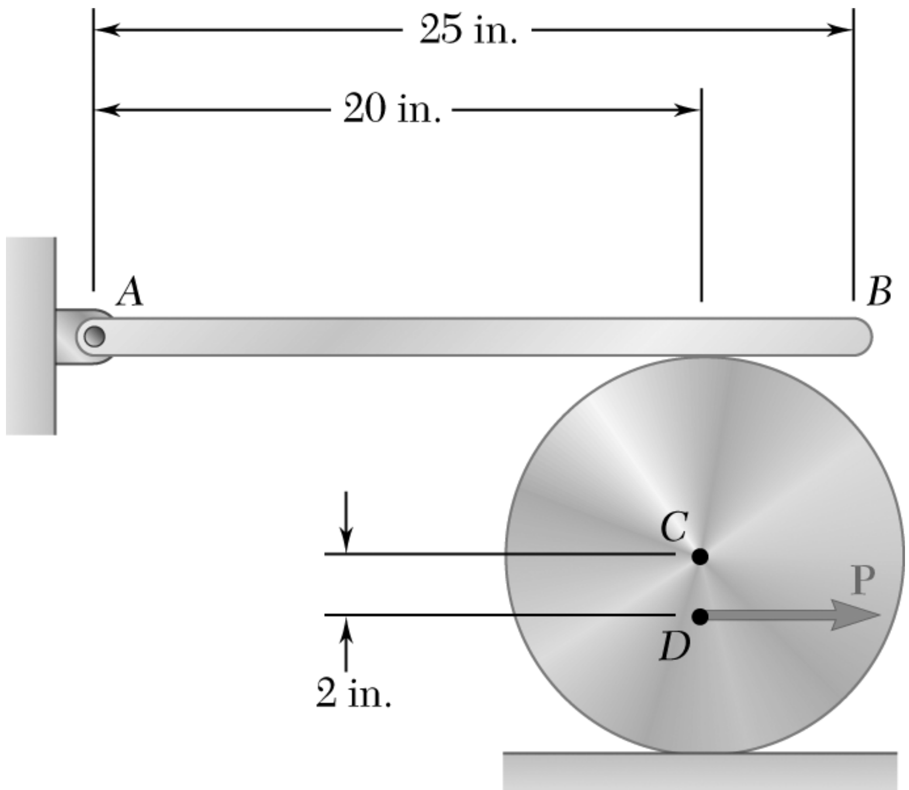
$N_1 = R_A = \dfrac{mg\tan\theta}{2}$

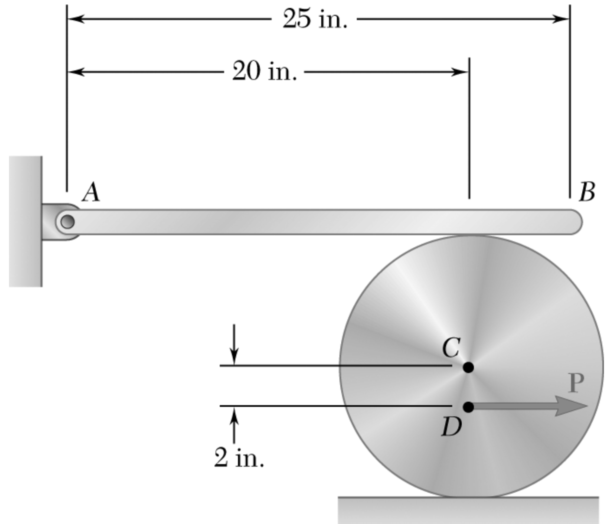
$f_1 = m_0 g$

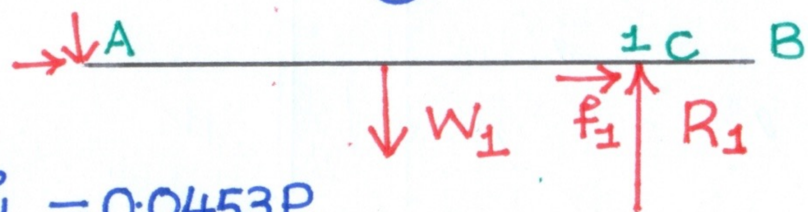
$\dfrac{f_1}{N_1} = \dfrac{2m_0}{m\tan\theta} \leq \mu_s$

# Problem 6

- The 12-lb slender rod $AB$ is pinned at $A$ and rests on the 36-lb cylinder $C$. Knowing that the diameter of the cylinder is 12.5 in. and that the coefficient of static friction is 0.35 between all surfaces of contact, determine the largest magnitude of the force **P** for which equilibrium is maintained






FBD-1

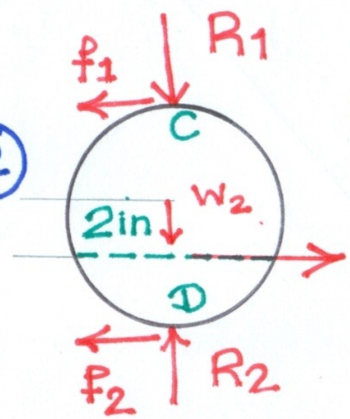
$\circlearrowleft \Sigma M_A = 0 \Rightarrow R_1 = \frac{15}{2}\,lb$

$R_2 = R_1 + W_2 = \frac{87}{2}\,lb$

$\frac{f_1}{N_1} = 0.0453P$

↑ slips

$\frac{f_2}{N_2} = 0.015P$




FBD-2

$\circlearrowleft \Sigma M_C = 0 \Rightarrow f_2 = \frac{16.5}{25} P$

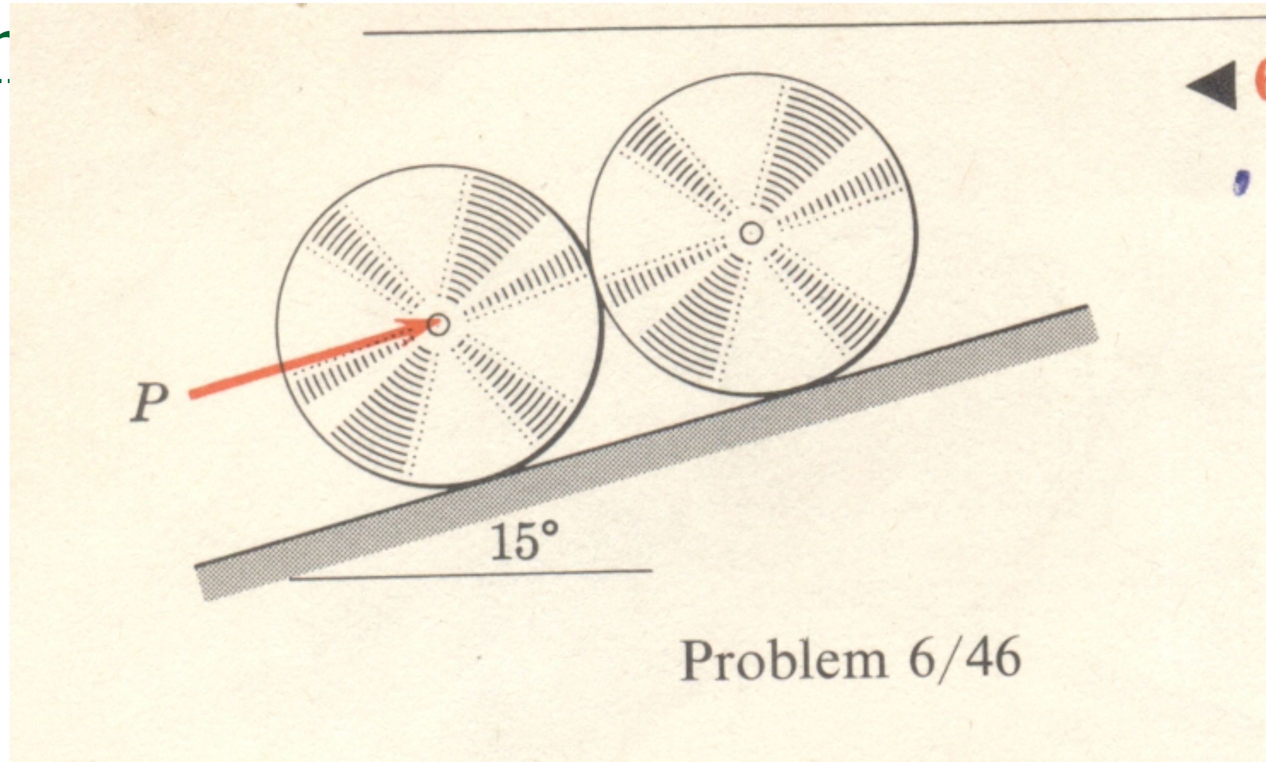
$\circlearrowleft \Sigma M_D = 0 \Rightarrow f_1 = \frac{8.5}{25} P$

$(f_1 + f_2 = P)$

$0.0453P = \mu_s = 0.35$

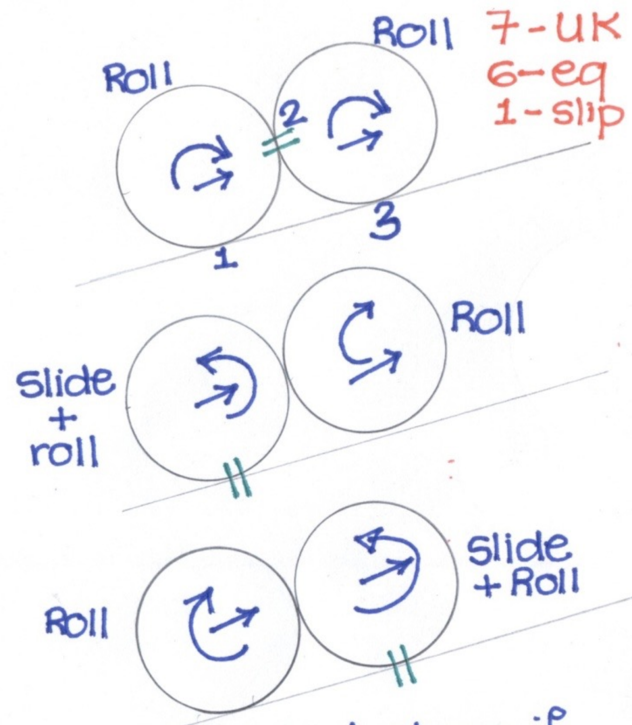
$\Rightarrow P = 7.72\,lb$

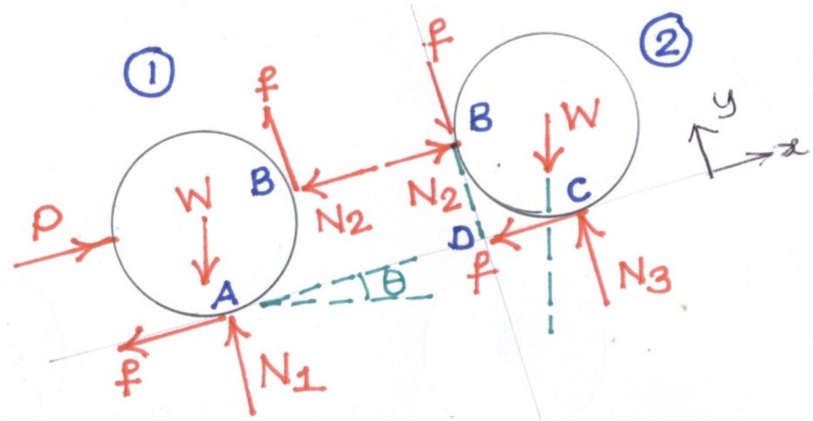
# Probler



Problem 6/46

- Determine the force $P$ required to move the two identical rollers up the incline. Each role weighs 30lb, and the coefficient of friction at all contacting surfaces is 0.2.




FBD-2

$\circlearrowleft \sum M_D = 0 \Rightarrow N_3 r = N_2 r + W d$

$d > 0$

$\theta \lesssim 45°$

or $N_3 > N_2$

no slip at C

FBD-2

$\nearrow \sum F_x = 0 \Rightarrow N_2 = W\sin\theta + f$ —(i)

FBD-1

$\nwarrow \sum F_y = 0 \Rightarrow N_1 + f = W\cos\theta$ —(ii)

Not clear if slip @ A or B

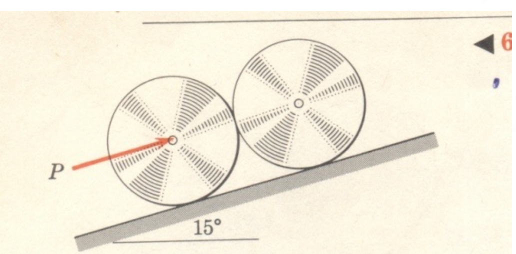
Assume slip @ B

$f = \mu_s N_2$

$N_1 = W\left[\cos\theta - \dfrac{\mu\sin\theta}{-1+\mu}\right] = 0.9W$

$N_2 = W\left[\dfrac{\sin\theta}{1-\mu}\right] = 0.324W$

consistent with slip assumption



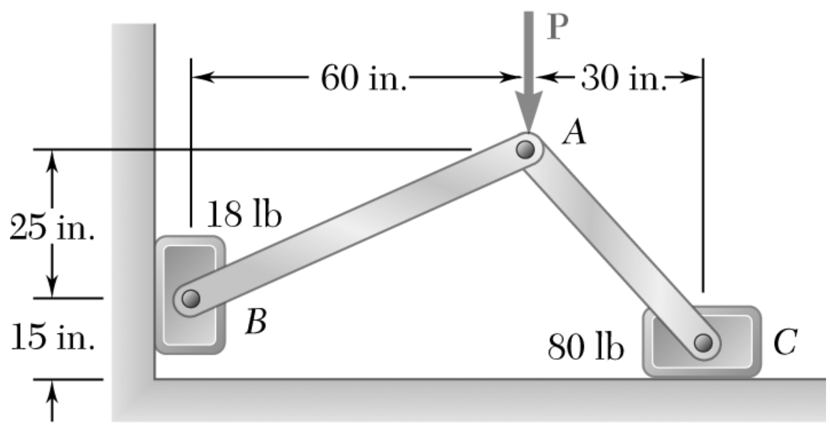

Problem 6/46

For the complete FBD (1) + (2)

$P = 2W\sin\theta + \underbrace{2f}_{2\mu_s N_2} = \underline{19.41\ lb}$

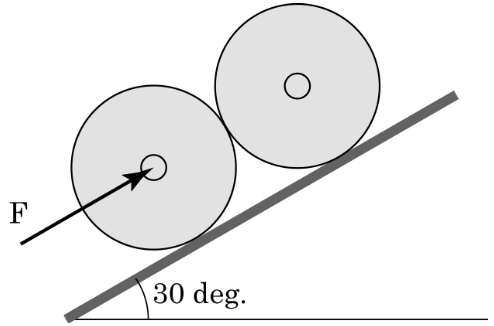
# Problem 5

- Two slender rods of negligible weight are pin-connected at $A$ and attached to the 18-lb block $B$ and the 80-lb block $C$ as shown. The coefficient of static friction is 0.55 between all surfaces of contact. Determine the range of values of $P$ for which equilibrium is maintained



# Problem 3

- What is the force $\boldsymbol{F}$ to hold two cylinders, each having a mass of 50 kg? Take the coefficient of friction equal to 0.2 for all surfaces of contact.

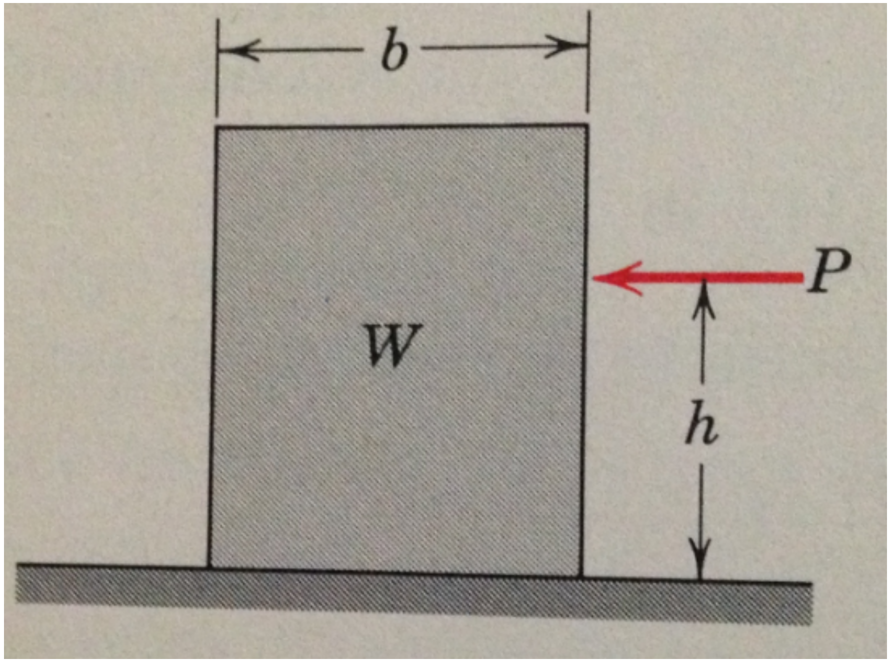


# Problem Type 4

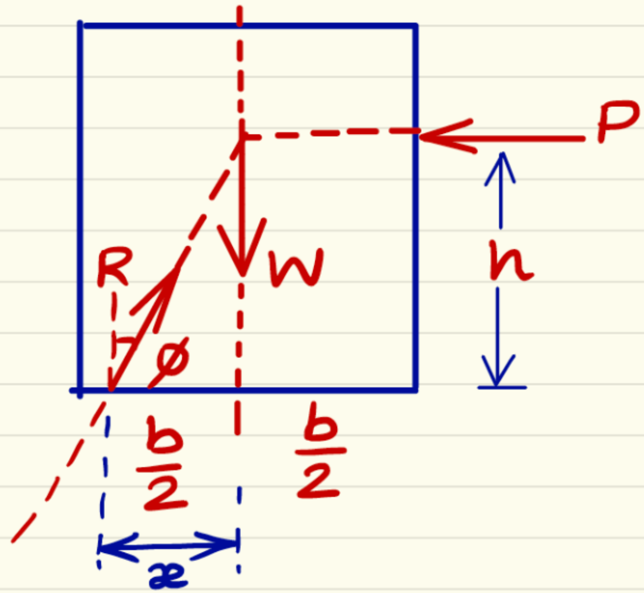
- Whether there is an impending slipping or tipping.
- If slipping occurs:
  - F = $\mu N$

  at the point of slipping
- If tipping occurs
  - Reactions at all points other than the point of tipping is equal to zero.
- Note that this case is just a special case of what we have seen earlier.

# Problem 7




- A homogenous block of weight $W$ rests on a horizontal plane and is subjected to the horizontal force $P$ as shown. If the coefficient of friction is μ, determine the greatest value which $h$ may have so that the block will slide without tipping.




$$R\sin\phi = P, \quad \tan\phi = \frac{P}{W} \leqslant \mu_s$$
$$R\cos\phi = W$$

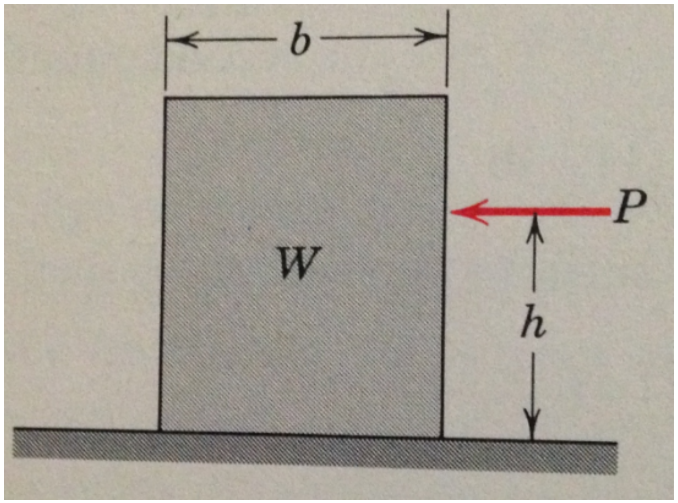
$$\text{also} \quad \tan\phi = \frac{x}{h} \leqslant \frac{b}{2h}$$

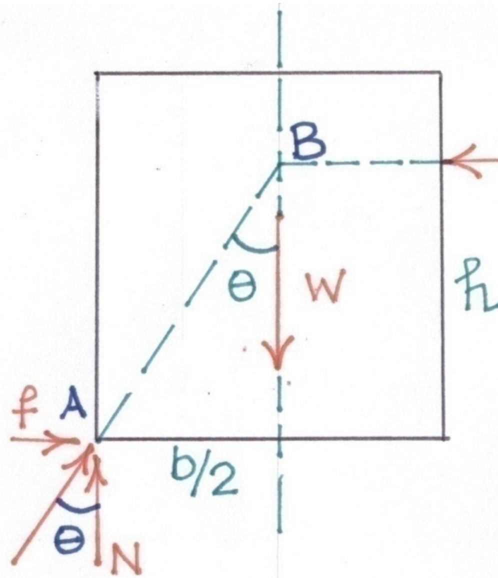
$$\tan\phi \leqslant \min\left(\mu_s, \frac{b}{2h}\right)$$

for system to topple first

$$\mu_s \geqslant \frac{b}{2h}$$

Slide $\mu_s \leqslant \frac{b}{2h}$ ; $h_{max} = \frac{b}{2\mu}$

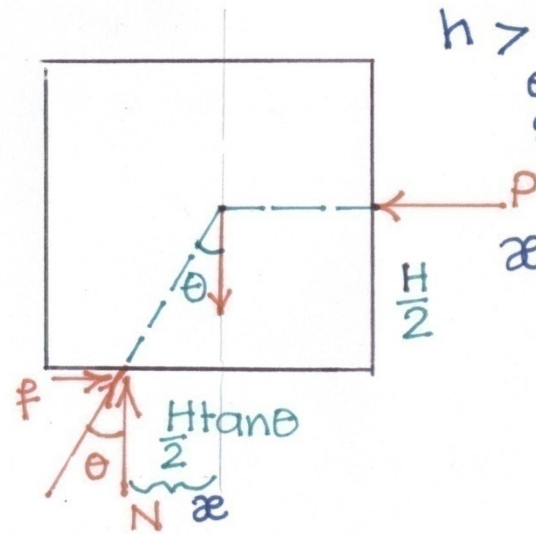



θ is always $\tan^{-1}\mu$ when the body is on the verge of toppling it has contant with the ground only at point A. Body has three forces and all should be concurrent

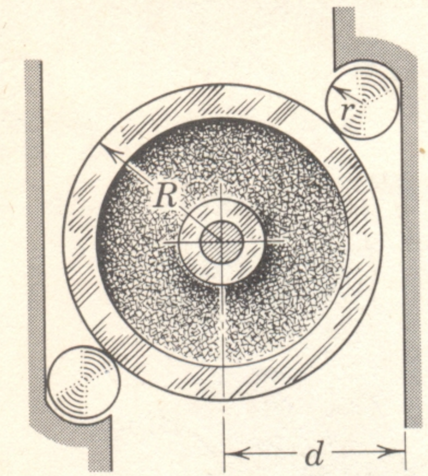
$$\Rightarrow \tan\theta = \frac{b}{2h} \quad \text{or} \quad h = \frac{b}{2\tan\theta} = \frac{b}{2\mu}$$

h > this value moment equilibrium will not be satisfied



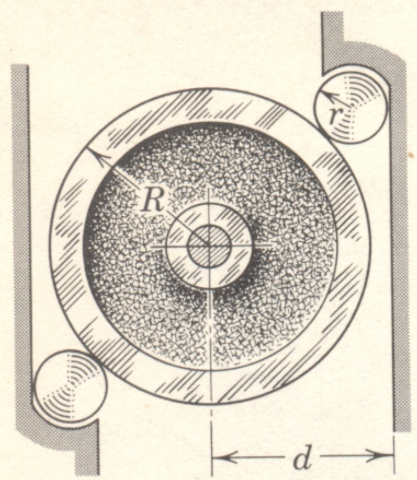

$$x = \frac{\mu H}{2}$$

# Problem 8

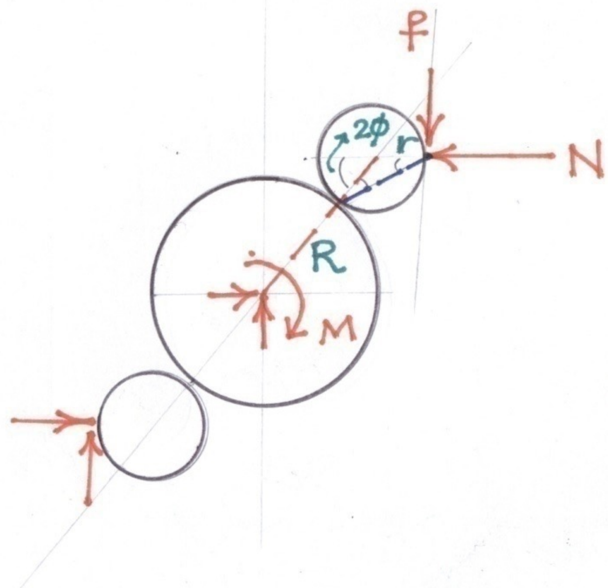


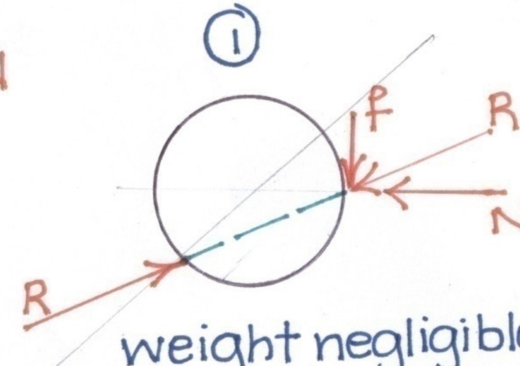
Problem 6/48

- The device shown prevents clockwise rotation in the horizontal plane of the central wheel by means of frictional locking of the two small rollers. For given values of $R$ and $r$ and for a common coefficient of friction μ at all contact surfaces, determine the range of values of $d$ for which the device will operate as described. Neglect weight of the two rollers.

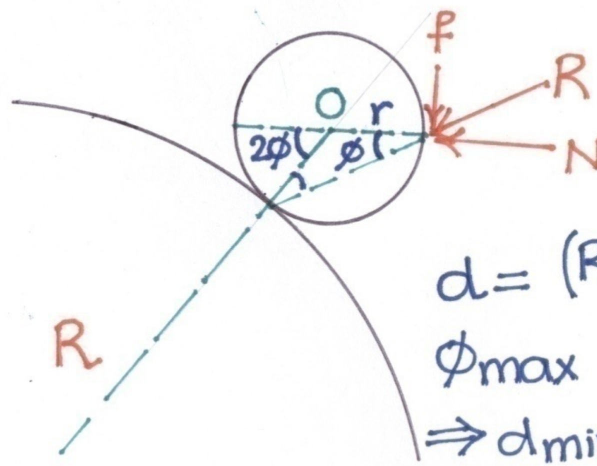



Problem 6/48




weight negligible
2-force member




$$d = (R+r)\cos 2\phi + r$$

$$\phi_{max} = \tan^{-1}\mu_s$$

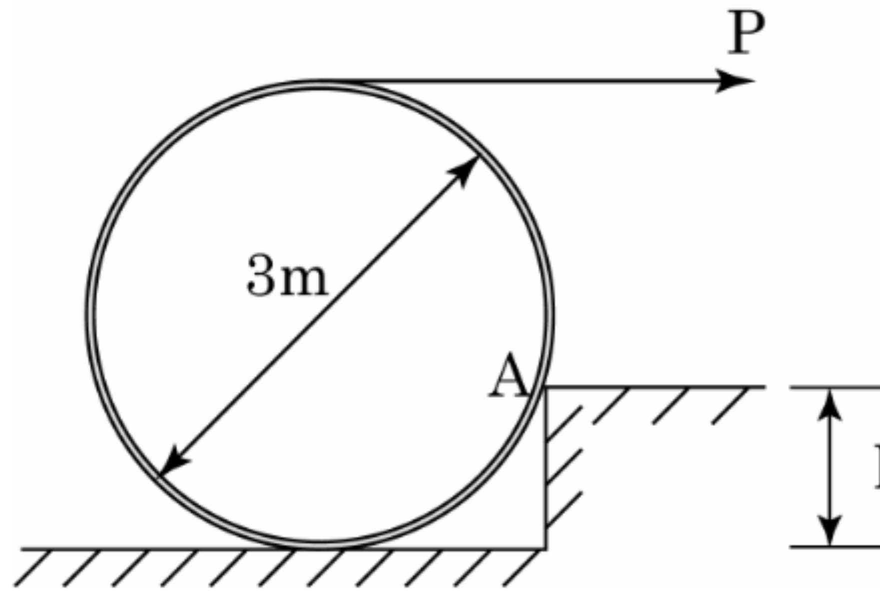
$$\Rightarrow d_{min} \equiv \phi_{max} = \tan^{-1}\mu_s$$

$$= \frac{2r + (1-\mu_s^2)R}{1+\mu_s^2}$$

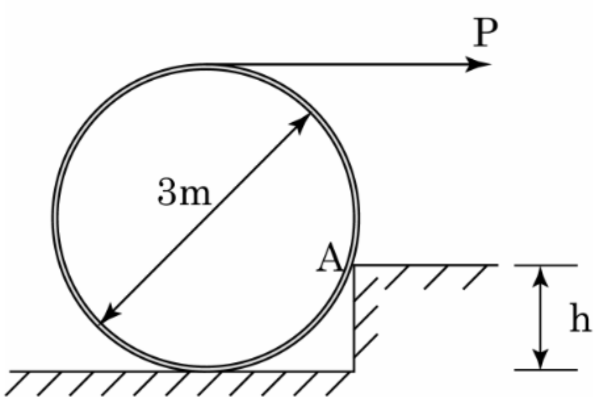
$$d_{max} = R + 2r$$

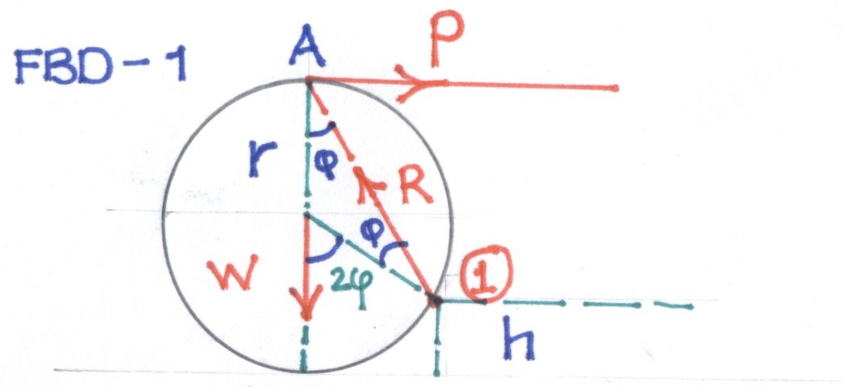
**QUESTION?**
**SUGGESTIONS?**

# Problem 1




- What is the height $h$ of the step so that the force $P$ will role the cylinder of weight 25kg over the step without impending slippage at the point of contact $A$. Take the coefficient of friction to be equal to 0.3.




When the cylinder is pulled over it loses contact with ground
All 3-forces should meet at A

$$r - h = r\cos 2\varphi$$

$$h = r(1 - \cos 2\varphi)$$

$h_{max} \Leftrightarrow \varphi$ as large as possible

$$\varphi_{max} = \tan^{-1}\mu_s$$

$$\Rightarrow h_{max} = r(1 - \cos 2\varphi_s)$$

$$h_{max} \approx 0.248\,m$$

# Problem 1b

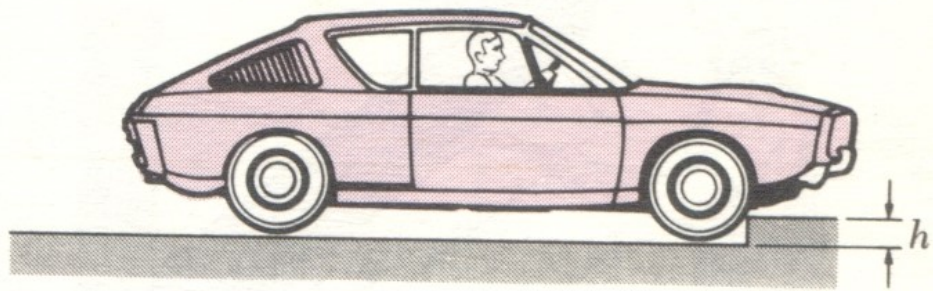
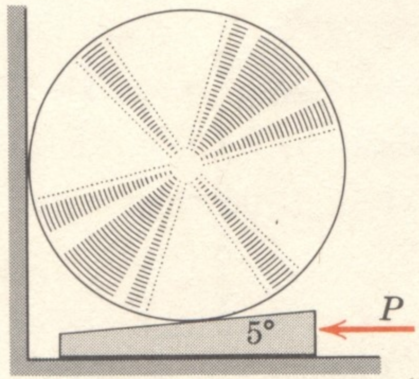



Fig. P8.29

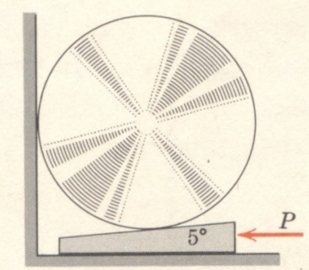
- A car is stopped with its front wheels resting against a curb when its driver starts the engine and tries to drive over the curb. Knowing that the radius of the wheels is 280 mm, that the coefficient of static friction between the tires and the pavement is 0.85, and that the weight of the car is equally distributed over its front and rear wheels, determine the largest curb height $h$ that the car can negotiate, assuming (a) front wheel drive, and (b) rear wheel drive.

# Problem 2

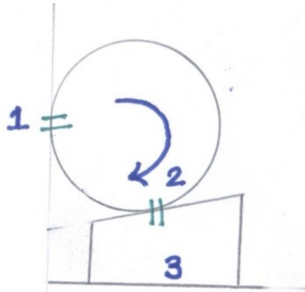



Problem 6/90

- A 5 deg wedge is used to lift the 1000-lb cylinder as shown. If the coefficient of friction is ¼ for all surfaces, determine the force $P$ required to move the wedge.

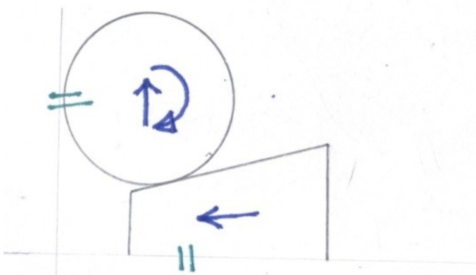


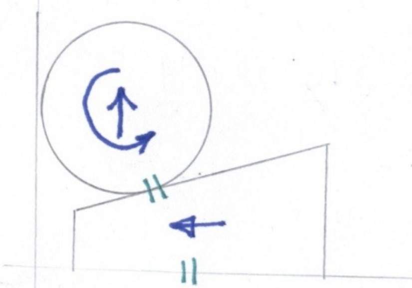

Problem 6/90




1+6 - UK
5 - eqns
2 - slips

wedge doesnot move
so not allowed





For-1

$\circlearrowleft \Sigma M_O = 0$

$\Rightarrow N_1 d + WR = N_2 d$

$\Rightarrow N_2 > N_1$

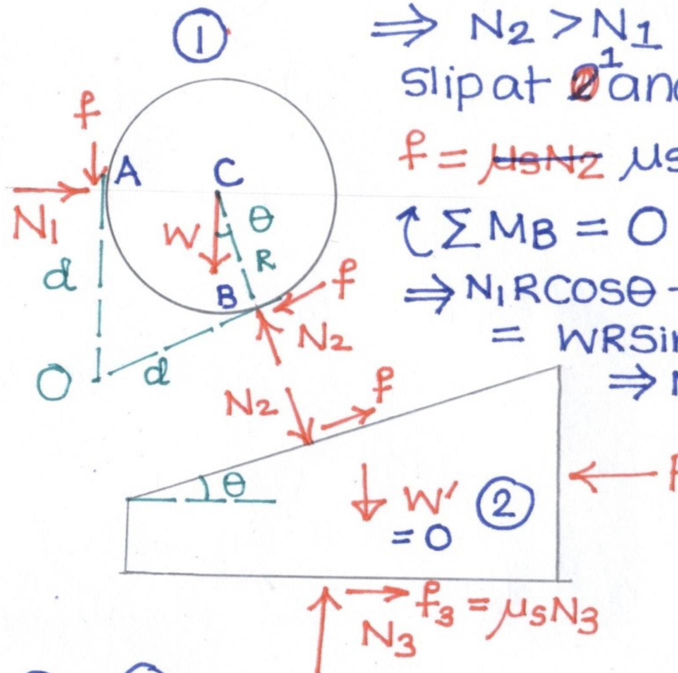
Slip at $\cancel{2}$ 1 and 3

$f = \cancel{\mu_s N_2}\ \mu_s N_1$

$\circlearrowleft \Sigma M_B = 0$

$\Rightarrow N_1 R\cos\theta - \mu N_1 R(1+S) = WR\sin\theta$

$\Rightarrow N_1 = 120.31$




1 + 2

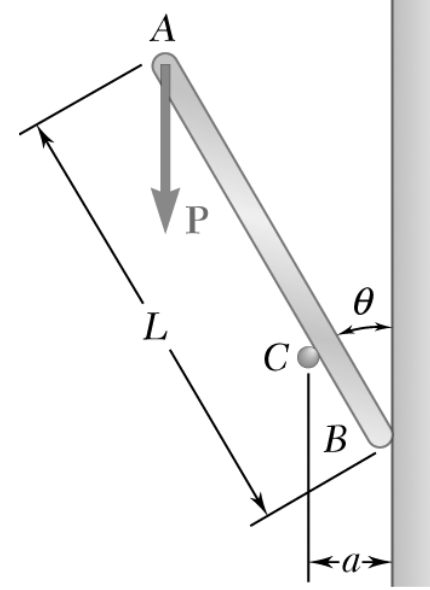
$\uparrow \Sigma F_y = 0 \Rightarrow N_3 = W + \underbrace{f}_{\mu_s N_1} = 1030\,lb$

$\rightarrow \Sigma F_x = 0 \Rightarrow P = N + \underbrace{f_3}_{\mu_s N_3} = 377.83 \approx 378\,lb$

# Problem 3

$\theta = 35$ deg

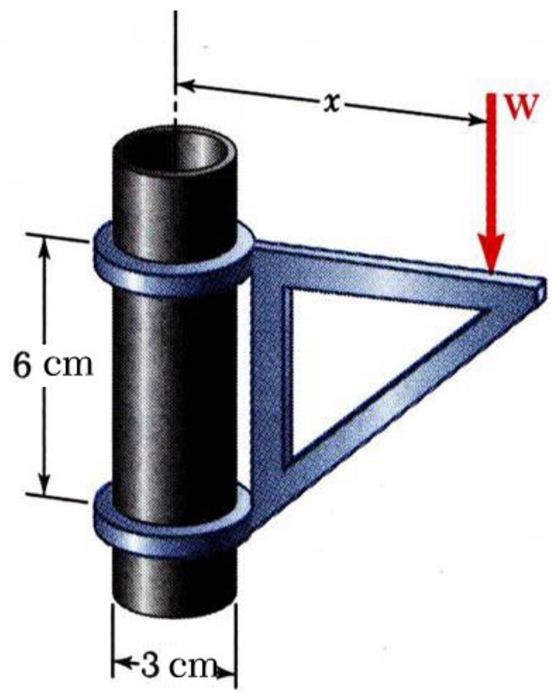
- A slender rod of length $L$ is lodged between peg $C$ and the vertical wall and supports a load **P** at end $A$. Knowing that and that the coefficient of static friction is 0.20 at both $B$ and $C$, determine the range of values of the ratio $L/a$ for which equilibrium is maintained.

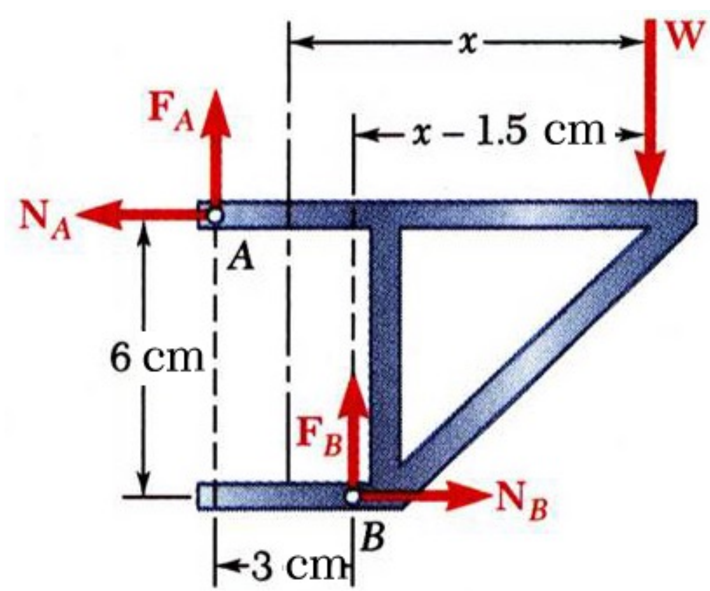



**3.46 < L/a < 13.63**

# Problem 4

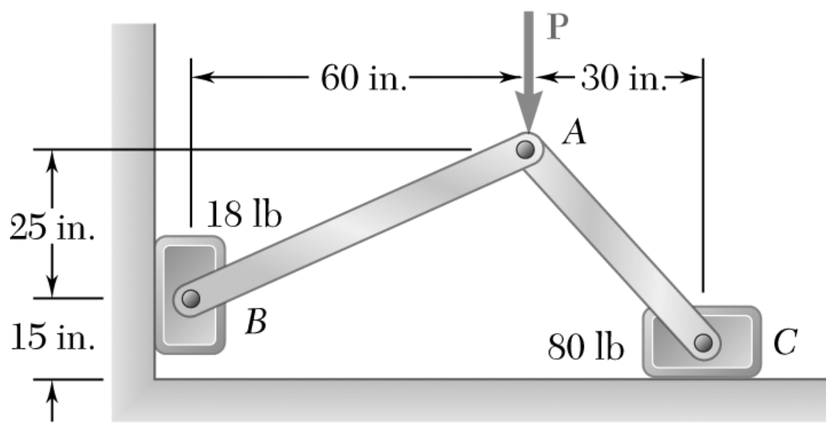
The moveable bracket shown may be placed at any height on the 3-cm diameter pipe. If the coefficient of friction between the pipe and bracket is 0.25, determine the minimum distance *x* at which the load can be supported. Neglect the weight of the bracket.



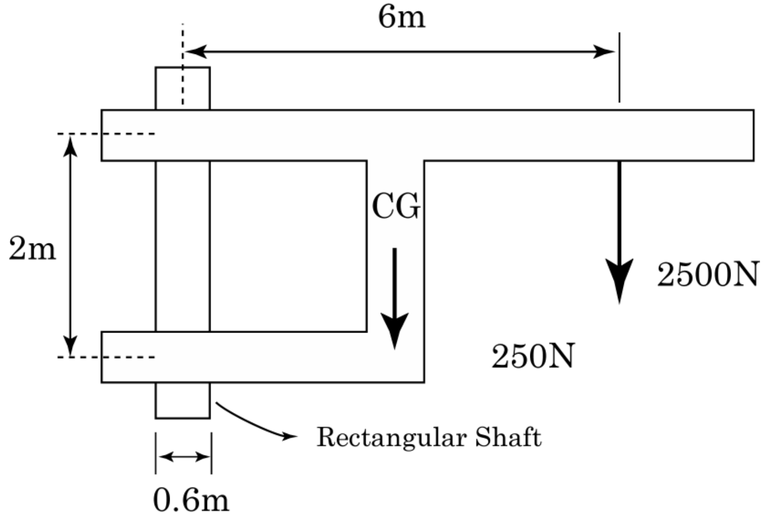
# Problem 5

- Two slender rods of negligible weight are pin-connected at *A* and attached to the 18-lb block *B* and the 80-lb block *C* as shown. The coefficient of static friction is 0.55 between all surfaces of contact. Determine the range of values of *P* for which equilibrium is maintained



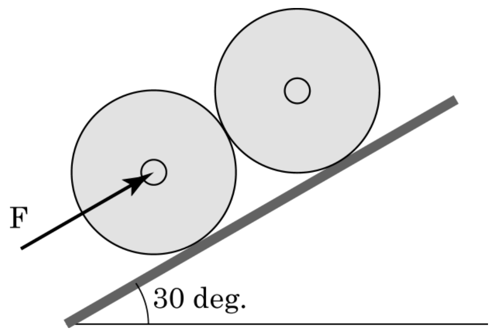
# Problem 2

- What is the minimum coefficient of friction required just to maintain the bracket and its 250 kg load? The center of gravity is 1.8m from the centerline.



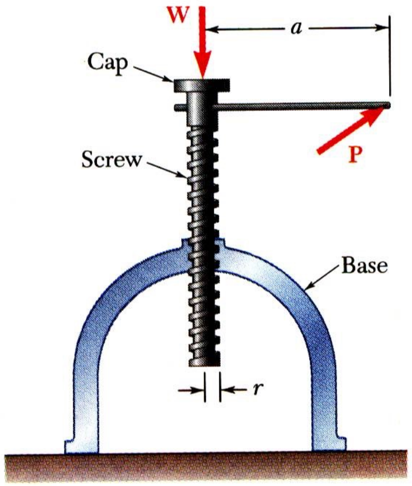
# Problem 3

- What is the force ***F*** to hold two cylinders, each having a mass of 50 kg? Take the coefficient of friction equal to 0.2 for all surfaces of contact.

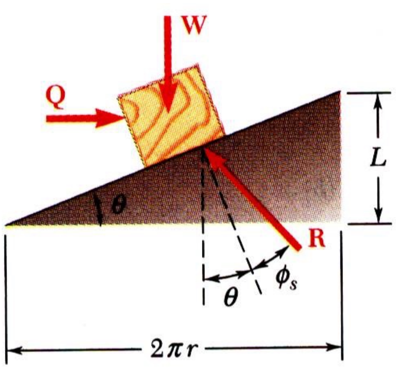


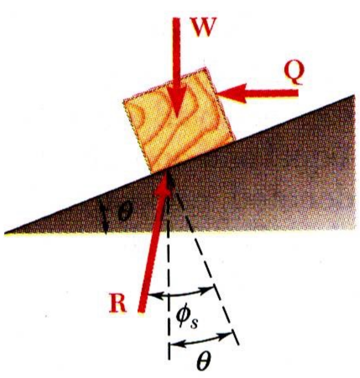
# COMPLEX FRICTION

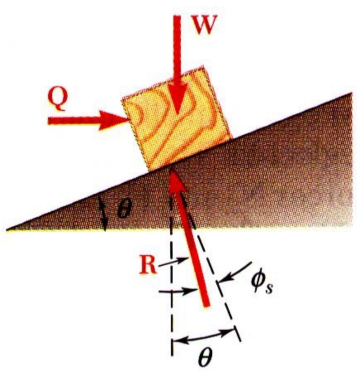
# Square-Threaded Screws




- Square-threaded screws are frequently used in jacks, presses, etc. Analysis is similar to block on inclined plane. Recall that friction force does not depend on area of contact.
- Thread of base has been "unwrapped" and shown as straight line. Slope is $2\pi r$ horizontally by $L$ vertically.
- Moment of force **Q** is equal to moment of force **P**. $Q = Pa/r$

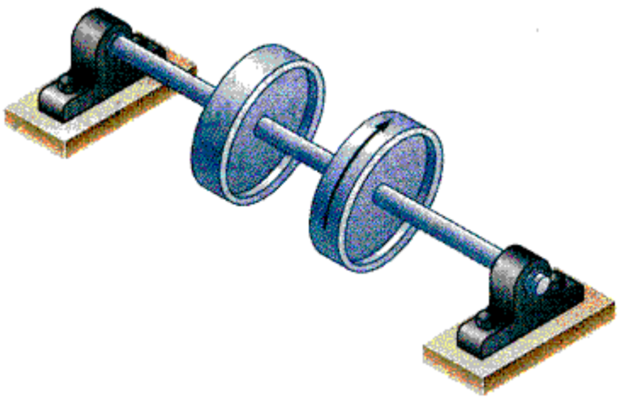


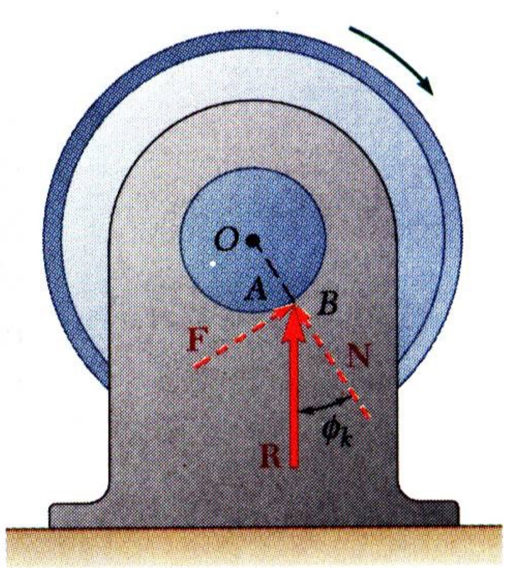




- Impending motion upwards. Solve for **Q**.
- $\phi_s > \theta$, Self-locking, solve for **Q** to lower load.
- $\phi_s < \theta$, Non-locking, solve for **Q** to hold load.

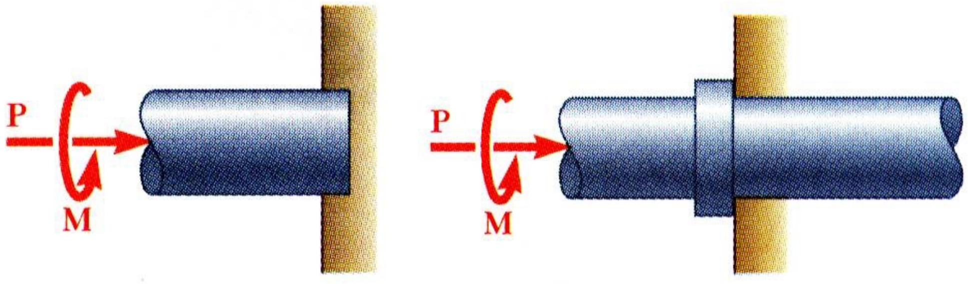
## Journal Bearings. Axle Friction



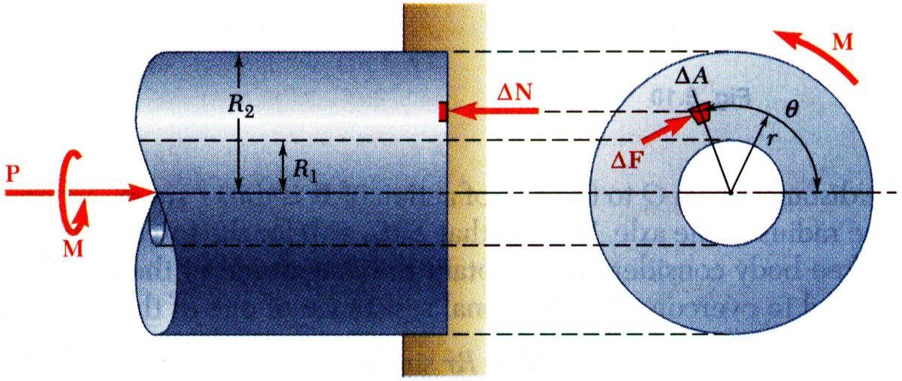



- Journal bearings provide lateral support to rotating shafts. Thrust bearings provide axial support
- Frictional resistance of fully lubricated bearings depends on clearances, speed and lubricant viscosity. Partially lubricated axles and bearings can be assumed to be in direct contact along a straight line.
- Forces acting on bearing are weight **W** of wheels and shaft, couple **M** to maintain motion, and reaction **R** of the bearing.
- Reaction is vertical and equal in magnitude to **W**.
- Reaction line of action does not pass through shaft center *O*; **R** is located to the right of *O*, resulting in a moment that is balanced by **M**.
- Physically, contact point is displaced as axle "climbs" in bearing.

## Thrust Bearings. Disk Friction




(*a*) End bearing (*b*) Collar bearing




Consider rotating hollow shaft:

$$\Delta M = r\Delta F = r\mu_k \Delta N = r\mu_k \frac{P}{A}\Delta A$$

$$= \frac{r\mu_k P \Delta A}{\pi\left(R_2^2 - R_1^2\right)}$$

$$M = \frac{\mu_k P}{\pi\left(R_2^2 - R_1^2\right)} \int_0^{2\pi} \int_{R_1}^{R_2} r^2 \, dr \, d\theta$$

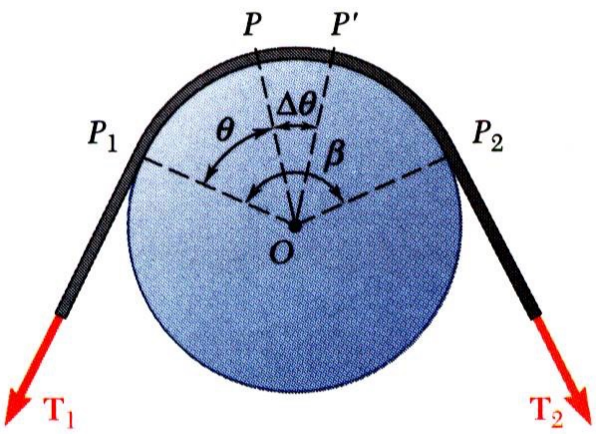
$$= \tfrac{2}{3}\mu_k P \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

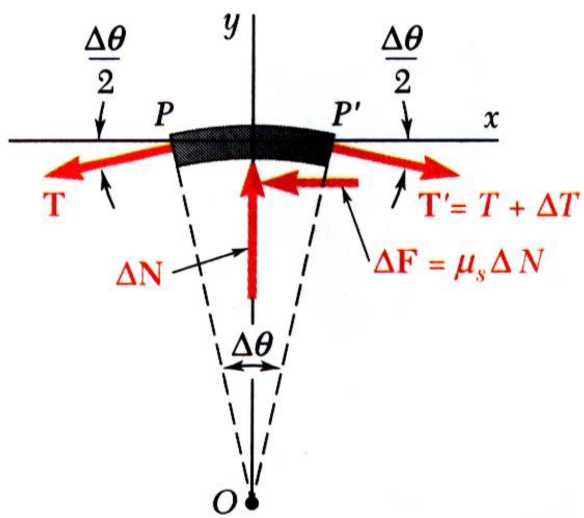
For full circle of radius *R*,

$$M = \tfrac{2}{3}\mu_k PR$$

# Belt Friction

## Belt Friction




- Relate $T_1$ and $T_2$ when belt is about to slide to right.
- Draw free-body diagram for element of belt

$$\sum F_x = 0: \quad (T + \Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu_s \Delta N = 0$$

$$\sum F_y = 0: \quad \Delta N - (T + \Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0$$

- Combine to eliminate $\Delta N$, divide through by $\Delta\theta$,

$$\frac{\Delta T}{\Delta\theta}\cos\frac{\Delta\theta}{2} - \mu_s\left(T + \frac{\Delta T}{2}\right)\frac{\sin(\Delta\theta/2)}{\Delta\theta/2} = 0$$

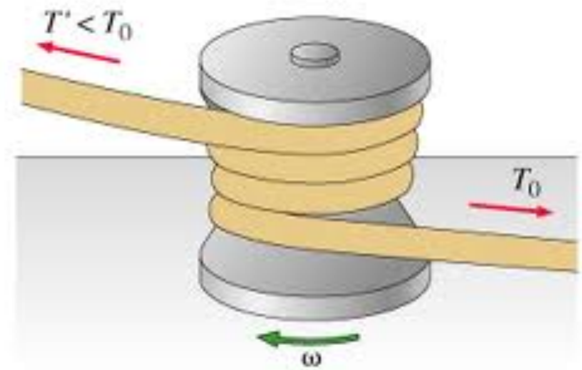
- In the limit as $\Delta\theta$ goes to zero,

$$\frac{dT}{d\theta} - \mu_s T = 0 \qquad \frac{dT}{T} = \mu_s d\theta$$

- Separate variables and integrate from $\theta = 0$ to $\theta = \beta$

$$\ln\frac{T_2}{T_1} = \mu_s\beta \quad \text{or} \quad \frac{T_2}{T_1} = e^{\mu_s\beta}$$

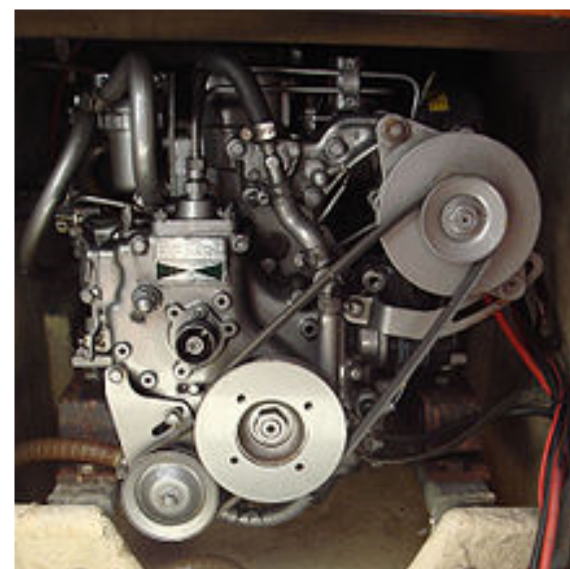
# Belt Friction Application




capstan

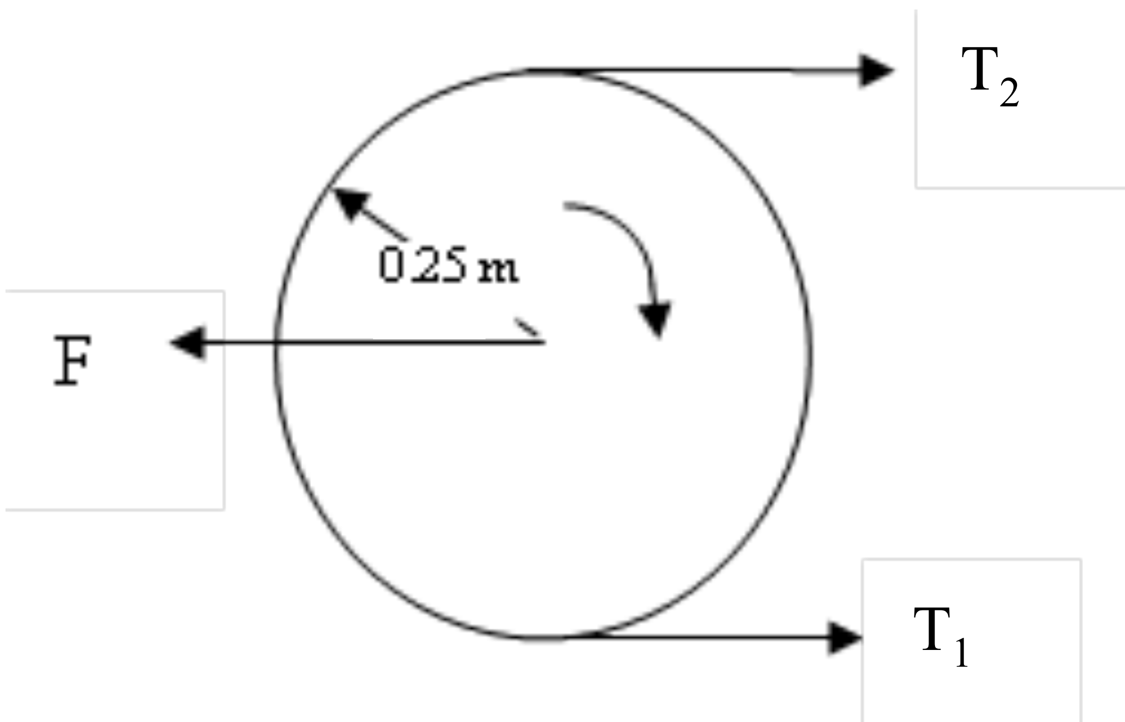
Diesel engine

Band Brake

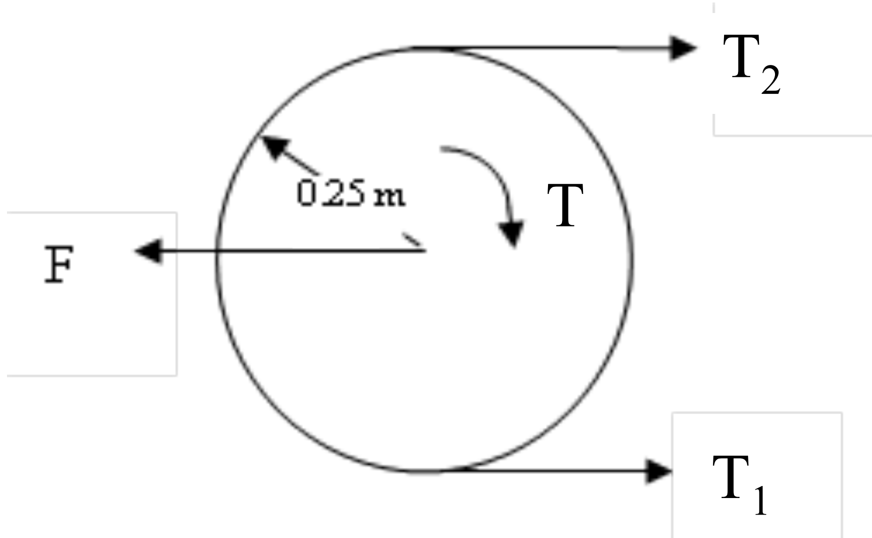


# Problem 46

- A pulley requires 200 Nm torque to get it rotating in the direction as shown. The angle of wrap is $\pi$ radians, and $\mu_s = 0.25$. What is the minimum horizontal force F required to create enough tension in the belt so that it can rotate the pulley?



# Problem 46 - Solution




For torque T = 200 Nm in given direction, $T_2 > T_1$

Equating moments acting on the pulley we get:

$(T_2\text{-}T_1)*0.25m = 200Nm$ (i)

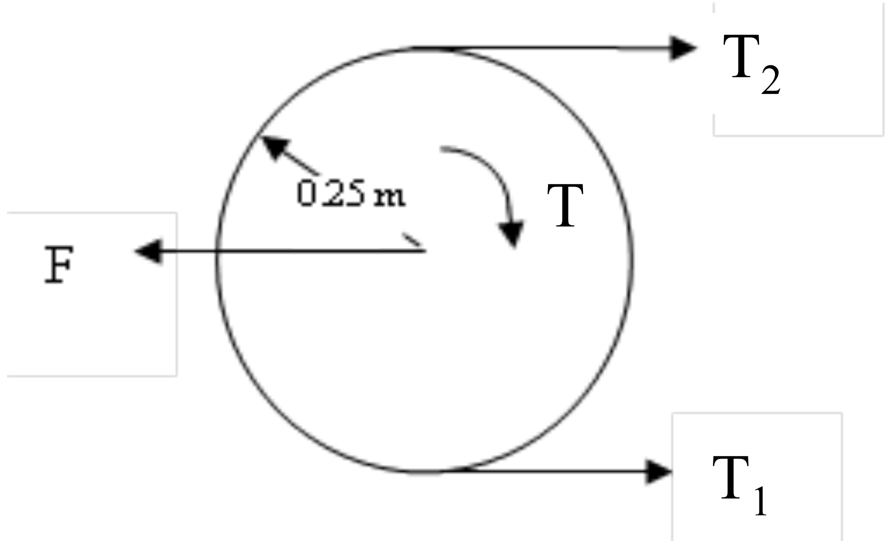
Considering friction in the pulley the forces T1 and T2 can be correlated as:

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad \text{(ii)}$$

Combining $T_2$ from eqn. (ii) in eqn. (i) we get:

$$\left(e^{\mu_s \beta} - 1\right) T_1 = 800N$$

# Problem 46 - Solution




$$\left(e^{\mu_s \beta} - 1\right) T_1 = 800N$$

Putting $\mu_s = 0.25$ and $\beta = \pi$ radians in the above equation we get

$1.193\ T_1 = 800N$

$\therefore T_1 = 670.6N$

$$T_2 = e^{\mu_s \beta} T_1 = 2.193 \times 670.6N = 1470.6N$$

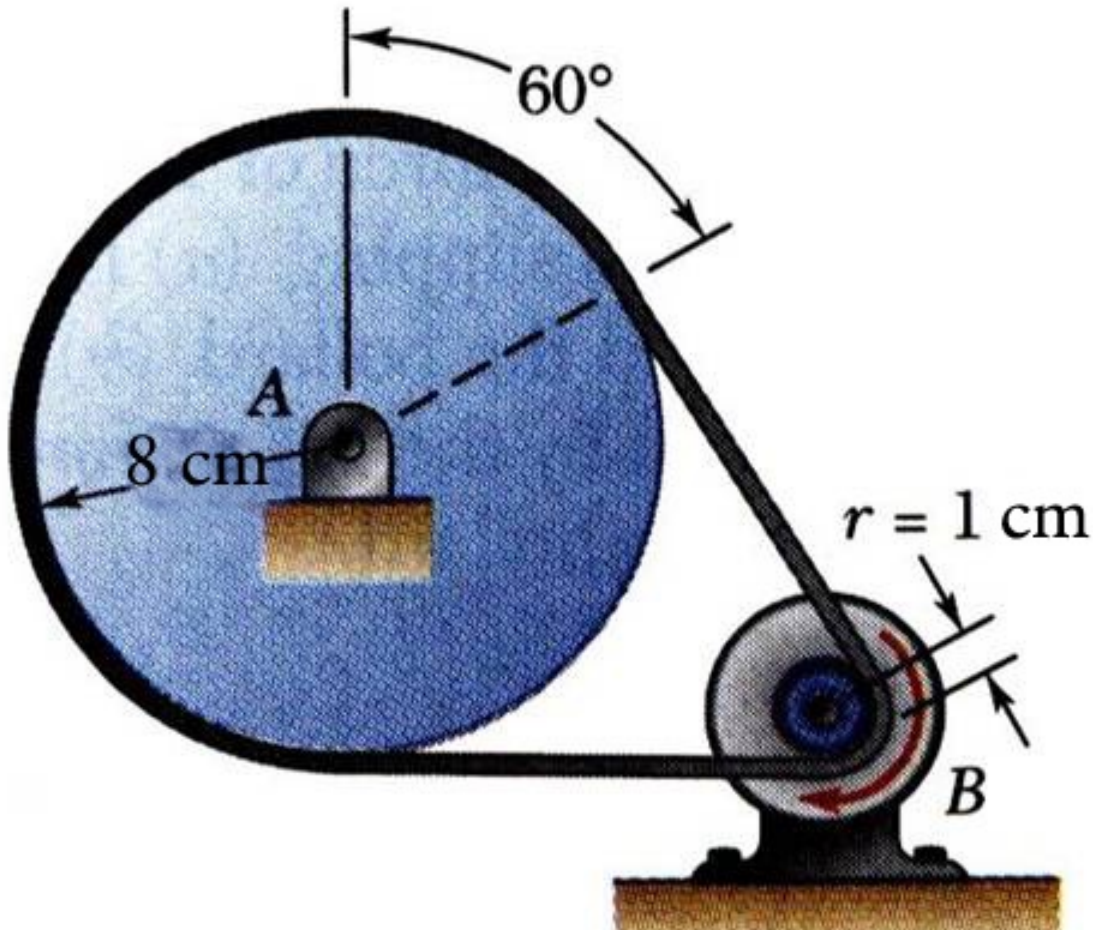
Equating forces acting on the pulley we get:

$F = T_1 + T_2$

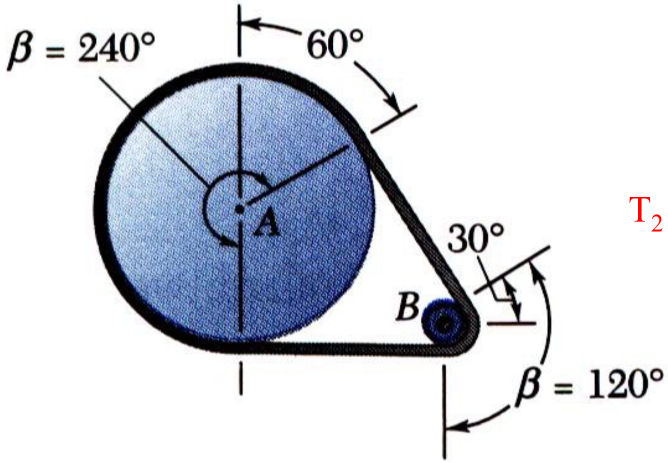
$\boxed{F = 2141N}$

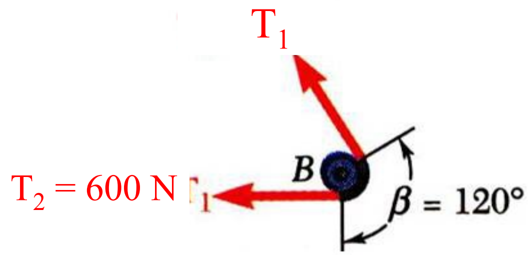
# Problem 47

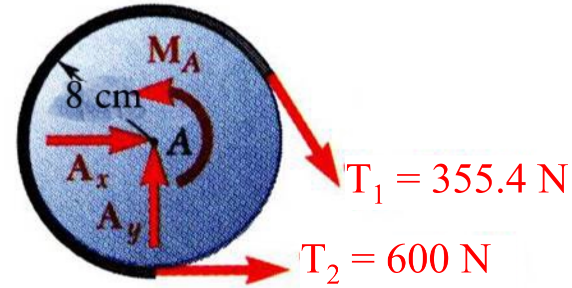
A flat belt connects pulley *A* to pulley *B*. The coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt. Knowing that the maximum allowable tension in the belt is 600 N, determine the largest torque which can be exerted by the belt on pulley *A*.



# Problem 47 - Solution




$T_1 \geq \dfrac{T_2}{e^{\mu_s \beta}}$; for a given $T_2$

$T_2 \leq T_1 e^{\mu_s \beta}$; for a given $T_1$

- Since angle of contact is smaller, slippage will occur on pulley *B* first. Determine belt tensions based on pulley *B*.

$$T_1 = \frac{T_2}{e^{\mu_s \beta}}$$

$$\Rightarrow T_1 = \frac{600\,\mathrm{N}}{e^{0.25(2\pi/3)}} = \frac{600\,\mathrm{N}}{1.688} = 355.4\,\mathrm{N}$$

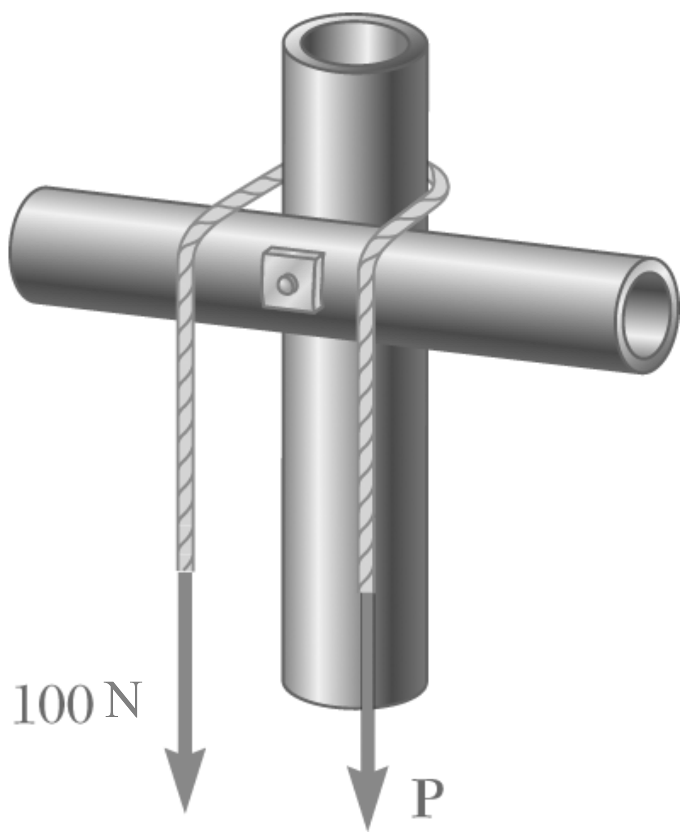
- Equating moments acting on pulley A we get

$$M_A = (8\,cm)(600\,\mathrm{N} - 355.4\,\mathrm{N})$$

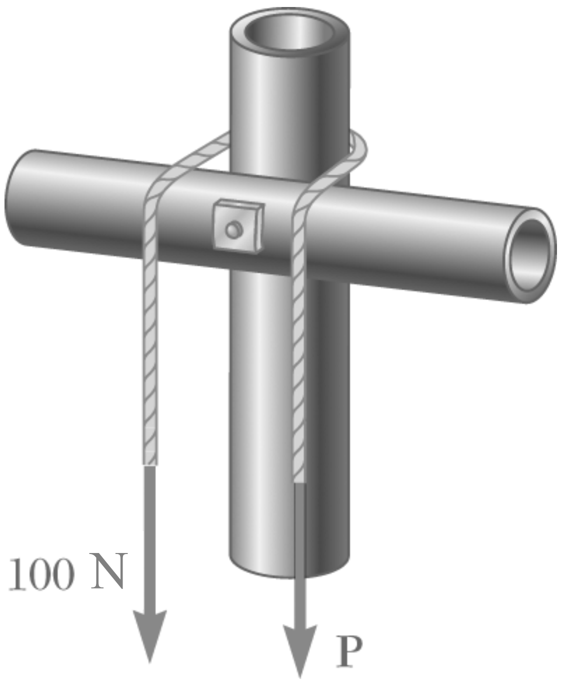
$$M_A = 1956.8\,\mathrm{N \cdot cm}$$

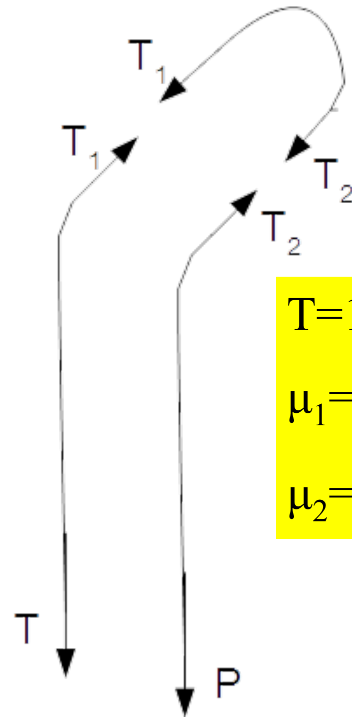
# Problem 48

Knowing that the coefficient of static friction is 0.25 between the rope and the horizontal pipe and 0.20 between the rope and the vertical pipe, determine the range of values of $P$ for which equilibrium is maintained.



# Problem 48 - Solution




T=100N

$\mu_1$=0.25 (between rope & horizontal pipe)

$\mu_2$=0.20 (between rope &vertical pipe)

Consider the sections of the rope as shown. Due to the friction, the tension in each part is different.

P will be max if there is impending slip in the direction of P (i.e. $P>T_2>T_1>T$).

P will be min if there is impending slip in the direction of T (i.e. $P<T_2<T_1<T$).

# Problem 48 - Solution

T=100N

$\mu_1$=0.25 (between rope & horizontal pipe)

$\mu_2$=0.20 (between rope &vertical pipe)

$P = P_{max}$ : $P>T_2>T_1>T$

$P = P_{min}$ : $P<T_2<T_1<T$

$$When\ P = P_{min}\left(P < T_2 < T_1 < T\right)$$

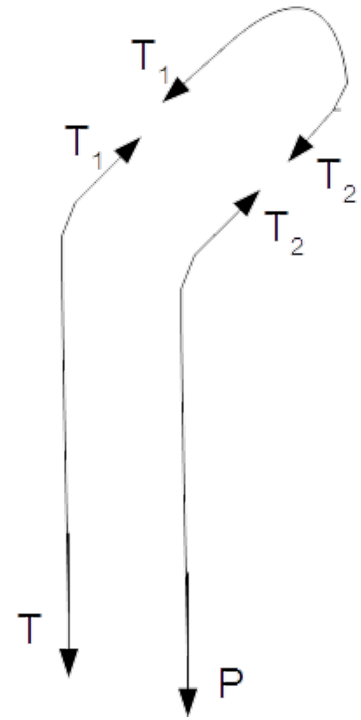
$$T_1 = \frac{T}{e^{\mu_1\beta_1}} \quad \left(\beta_1 = \frac{\pi}{2}\right)$$

$$T_2 = \frac{T_1}{e^{\mu_2\beta_2}} = \frac{T}{e^{\mu_1\beta_1+\mu_2\beta_2}} \quad \left(\beta_2 = \pi\right)$$

$$P = \frac{T_2}{e^{\mu_1\beta_3}} = \frac{T}{e^{\mu_1\beta_1+\mu_2\beta_2+\mu_1\beta_3}} \quad \left(\beta_3 = \frac{\pi}{2}\right)$$

*Putting the values of* $T$, $\mu$*'s and* $\beta$*'s, we get*

$$P_{min} = 24.3N$$



# Problem 48 - Solution

T=100N

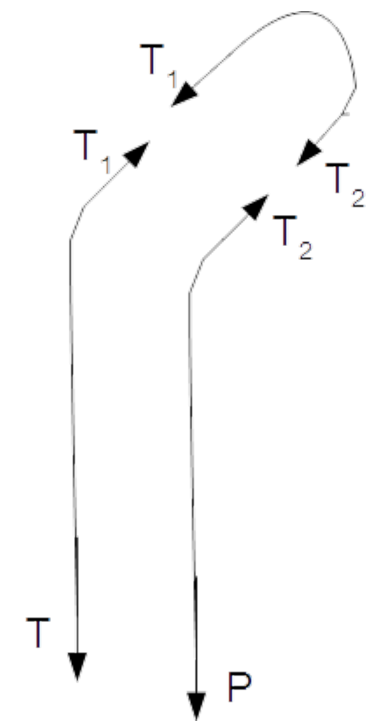
$\mu_1$=0.25 (between rope & horizontal pipe)

$\mu_2$=0.20 (between rope &vertical pipe)

$P = P_{max}$ : $P>T_2>T_1>T$

$P = P_{min}$ : $P<T_2<T_1<T$

$P_{min} = 24.32N$




$$When\ P = P_{max}\left(P > T_2 > T_1 > T\right)$$

$$T_1 = Te^{\mu_1\beta_1} \qquad \left(\beta_1 = \frac{\pi}{2}\right)$$

$$T_2 = T_1 e^{\mu_2\beta_2} \Rightarrow T_2 = Te^{\mu_1\beta_1+\mu_2\beta_2} \qquad \left(\beta_2 = \pi\right)$$

$$P = T_2 e^{\mu_1\beta_3} \Rightarrow P = Te^{\mu_1\beta_1+\mu_2\beta_2+\mu_1\beta_3} \qquad \left(\beta_3 = \frac{\pi}{2}\right)$$

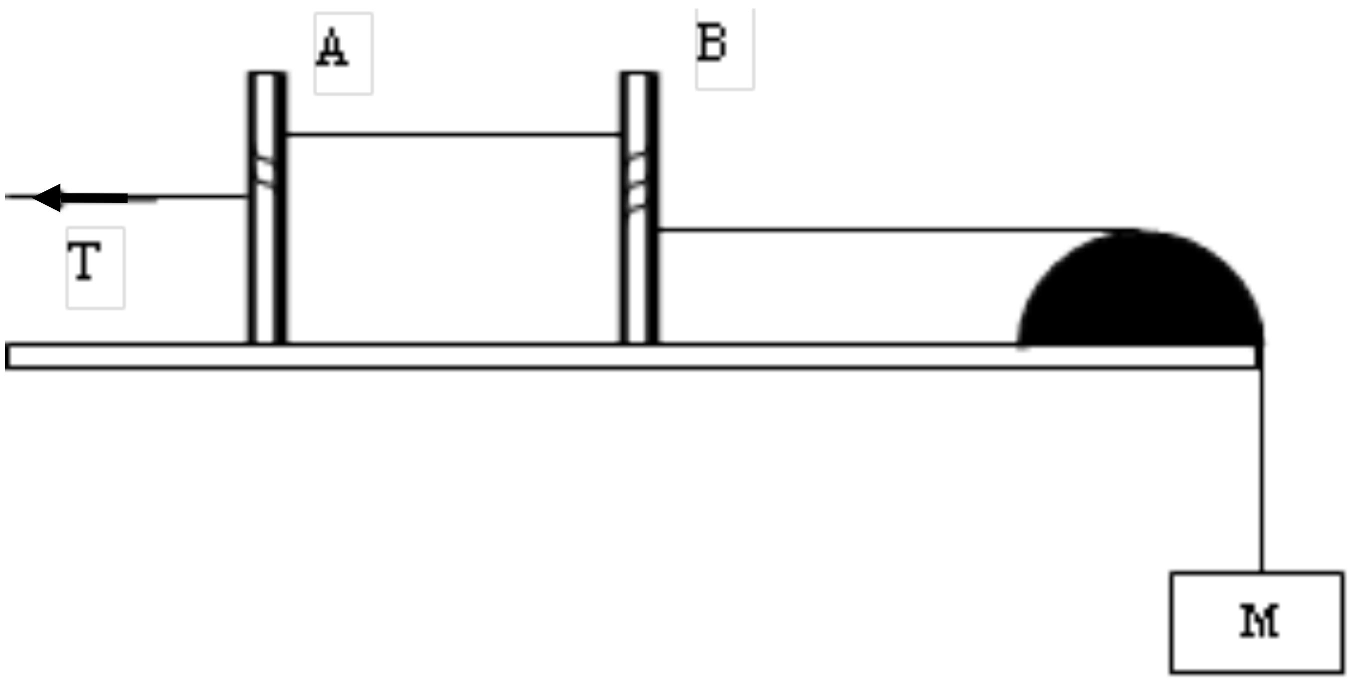
*Putting the values of* $T$, $\mu$'s *and* $\beta$'s, *we get*

$$P_{max} = 411.1N$$

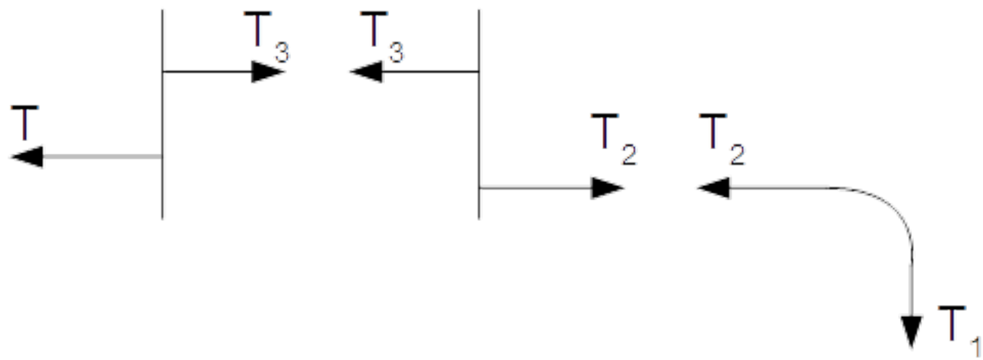
$$24.3N \le P \le 411.1N$$

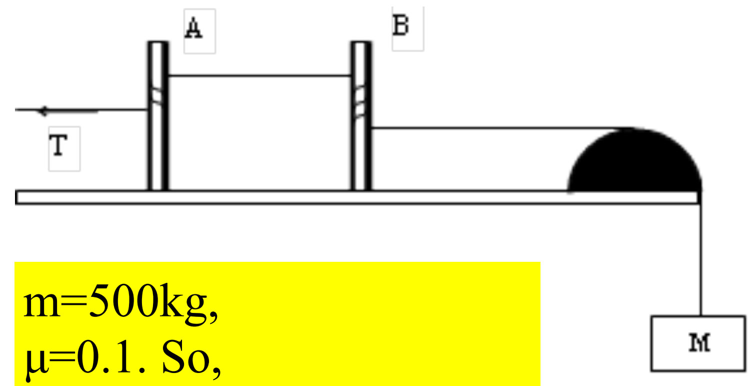
# Problem 49

- A cord is wiped twice around a pole A and three times around second pole B. Finally cord goes over a half barrel section and supports a mass M of 500 kg. What is tension T required to maintain this load? Take coefficient of friction 0.1 for all surfaces of contact.



# Problem 49 - Solution




m=500kg,
μ=0.1. So,
$T_1$=Mg=500*9.81=4905N

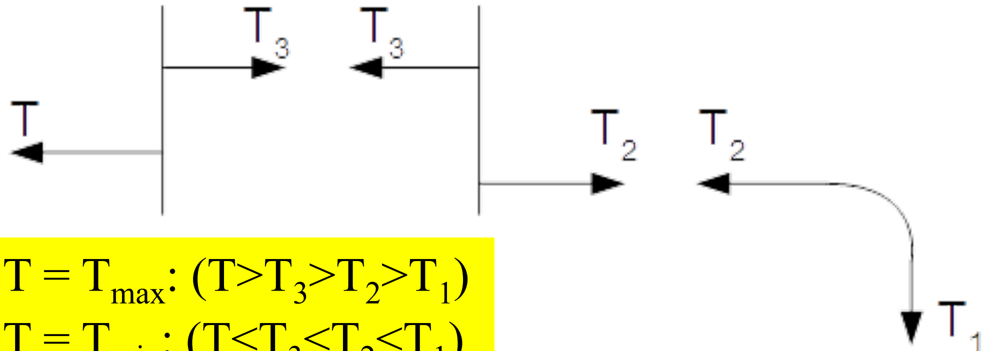
Consider the sections of the rope as shown. Due to the friction, the tension in each part is different.

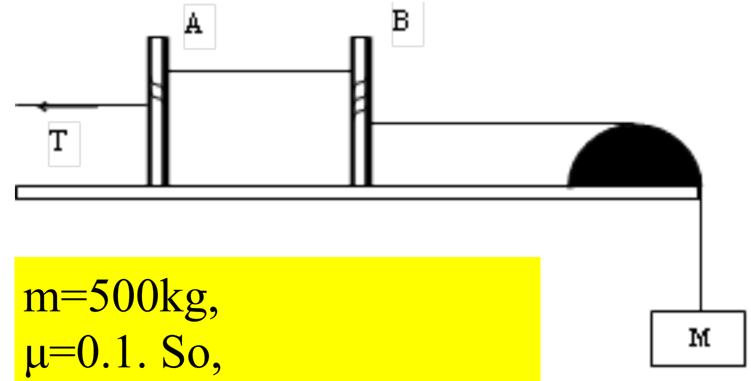
T will be max if there is impending slip in the direction of T (i.e. $T>T_3>T_2>T_1$).

T will be min if there is impending slip in the direction opposite to T (i.e. $T<T_3<T_2<T_1$).

For the values of T greater than min and lesser than max, it can maintain the load.

# Problem 49 - Solution




$T = T_{max}$: $(T>T_3>T_2>T_1)$
$T = T_{min}$: $(T<T_3<T_2<T_1)$

m=500kg,
μ=0.1. So,
$T_1$=Mg=500*9.81=4905N

$$When\, T = T_{\min}\left(T < T_3 < T_2 < T_1\right)$$

$$T_2 = \frac{T_1}{e^{\mu\beta_1}} \quad \left(\beta_1 = \frac{\pi}{2}\right)$$

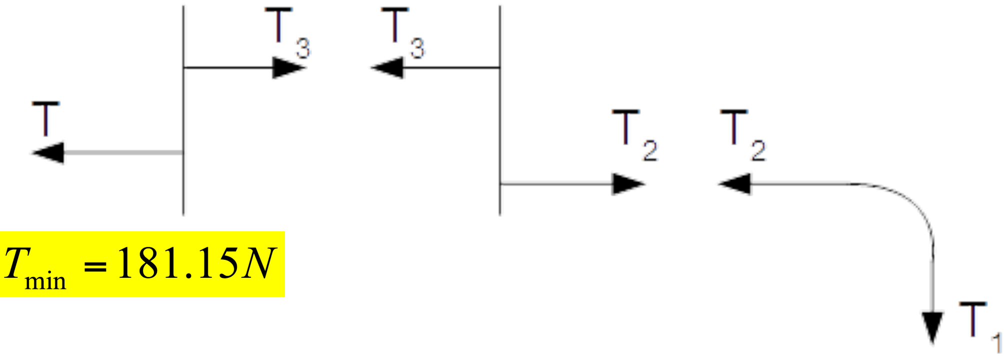
$$T_3 = \frac{T_2}{e^{\mu\beta_2}} = \frac{T_1}{e^{\mu\beta_1+\mu\beta_2}} \quad \left(\beta_2 = 3\times 2\pi\right)$$

$$T = \frac{T_3}{e^{\mu\beta_3}} = \frac{T_1}{e^{\mu\beta_1+\mu\beta_2+\mu\beta_3}} \quad \left(\beta_3 = 2\times 2\pi\right)$$

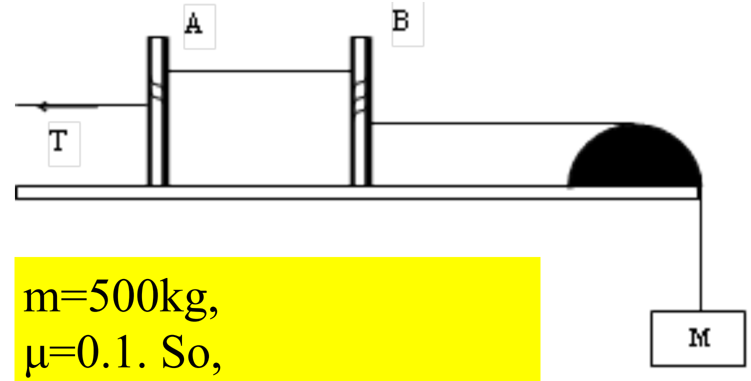
*Putting the values of* $T_1, \mu$ *and* $\beta$'s, *we get*

$$T_{\min} = 181.2N$$

# Problem 49 - Solution




$T_{\min} = 181.15N$




m=500kg,
μ=0.1. So,
$T_1$=Mg=500*9.81=4905N

$T = T_{max}$: $(T>T_3>T_2>T_1)$
$T = T_{min}$: $(T<T_3<T_2<T_1)$

$$When\, T = T_{\max}\left(T > T_3 > T_2 > T_1\right)$$

$$T_2 = T_1 e^{\mu\beta_1} \qquad \left(\beta_1 = \frac{\pi}{2}\right)$$

$$T_3 = T_2 e^{\mu\beta_2} \Rightarrow T_3 = T_1 e^{\mu\beta_1 + \mu\beta_2} \qquad \left(\beta_2 = 3\times 2\pi\right)$$

$$T = T_3 e^{\mu\beta_3} \Rightarrow T = T_1 e^{\mu\beta_1 + \mu\beta_2 + \mu\beta_3} \qquad \left(\beta_3 = 2\times 2\pi\right)$$

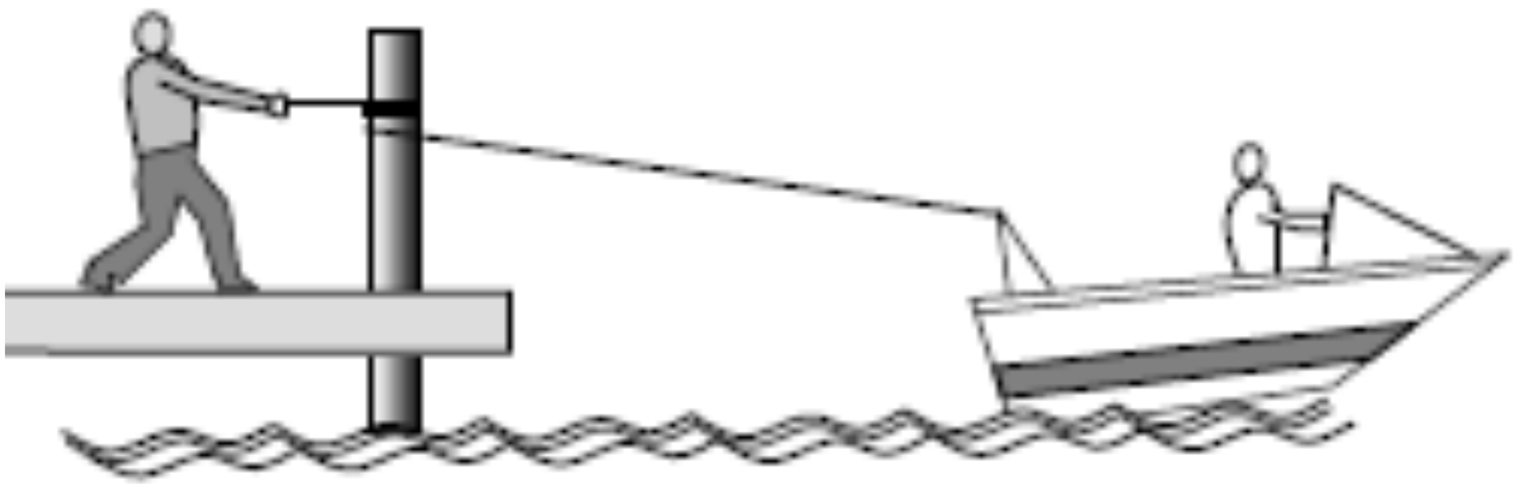
*Putting the values of* $T_1$, $\mu$*'s and* $\beta$*'s, we get*

$$T_{\max} = 132.8kN$$

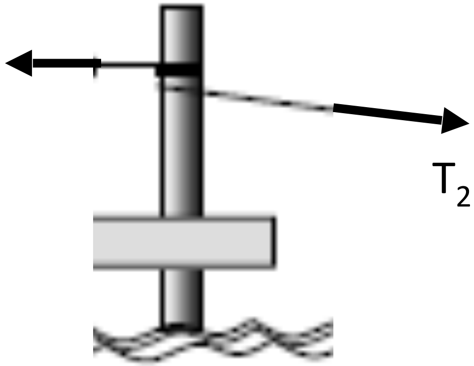
$\therefore$ *Mass M is balanced when* $181.2N \le T \le 132.8kN$

# Extra Problem

The seaman pulls with 100 N force and wants to stop the motor boat from moving away from the dock. How many wraps he must make around the post if the motor boat develops a thrust of 3500 N. ($\mu_s$= 0.2 between rope and the post)



# Extra Problem - Solution




Considering friction in the pulley the forces T1 and T2 can be correlated as:

$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

$$\beta = \frac{1}{\mu_s} \ln\left(\frac{T_2}{T_1}\right) = 17.78$$

But $\beta = 2n\pi$

where n = number of wraps of the rope around the post

From above n = 2.8

Therefore 3 wraps of the rope is necessary to restrain the motor boat

1. As shown in Figure 1, blocks **A** and **B** have a mass of **7 kg** and **10 kg**, respectively. Using the coefficients of static friction indicated, determine the **largest** vertical force **P** which can be applied to the cord **without causing** any **impending** motion. Take acceleration due to gravity ***g* = *10* m/s²**.

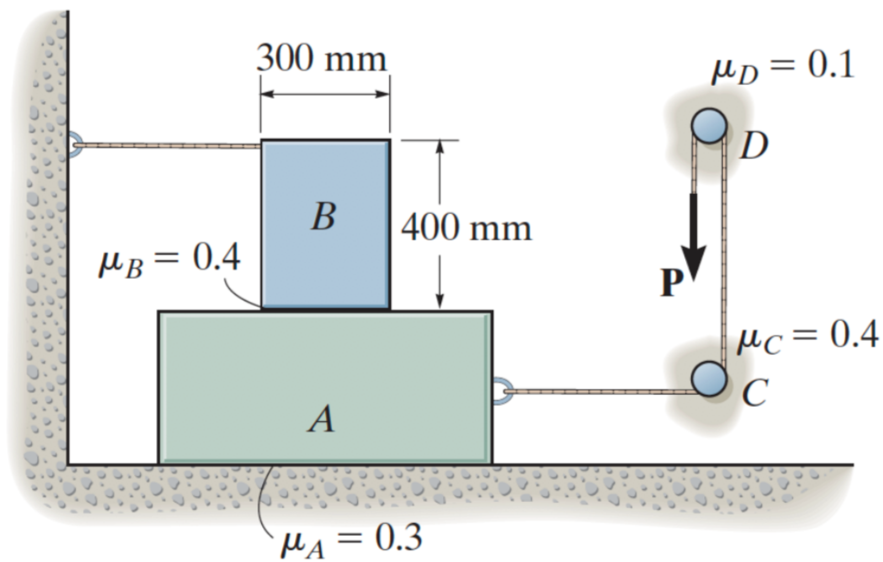
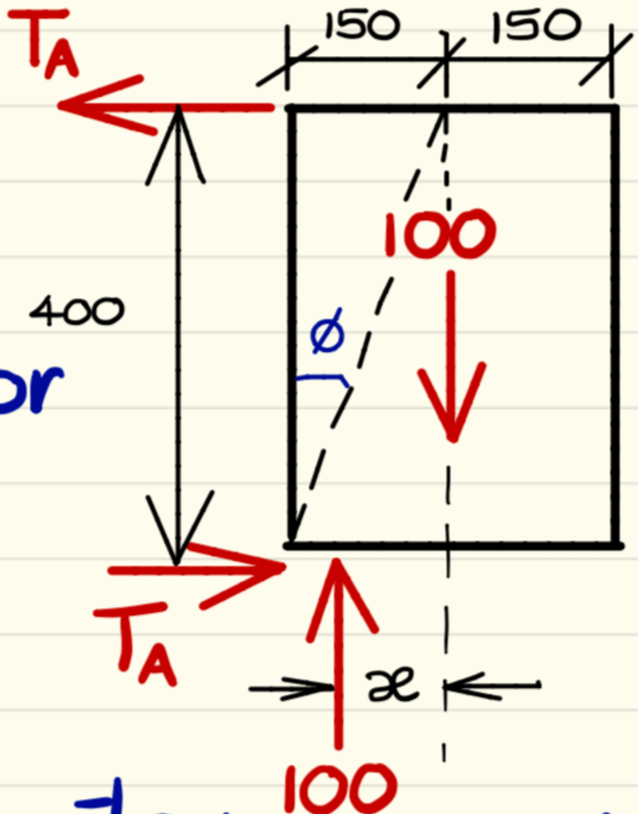


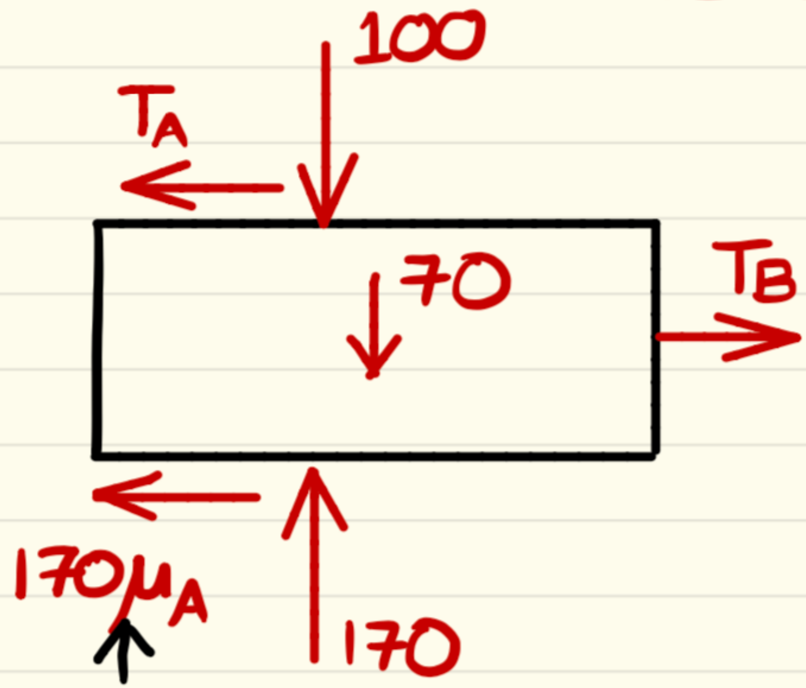

**Figure 1**

FBD-A

A - tips or slide




FBD-B




impending

$\phi_{max} = \tan^{-1} \underbrace{0.4}_{\mu_B}$ impending

$\phi = \tan^{-1} \frac{150}{400} = \tan^{-1} 0.375$

$\phi < \phi_{max}$, when $x = 150mm$

Block B tips before slide

$x = 150mm$

$T * 400 = 100N \times 150$

$T = \frac{100 \times 150N}{400}$

$T = 37.5N$

For FBD-B

$$T_B = T_A + 170\mu_A$$

$$= 37.5 + 170 \times 0.3 = 88.5$$

For the rope:

$$\frac{T_{DC}}{T_B} = e^{\mu_C \pi/2} \quad , \quad \frac{P}{T_{DC}} = e^{\mu_D \pi}$$

$$\Rightarrow \quad \frac{P}{T_B} = e^{\mu_C \frac{\pi}{2} + \mu_D \pi}$$

$$\Rightarrow \quad \boxed{P = 227.12\,\text{N}}$$

1. A cable is attached to a **50 kg** plate **B**, passes over a fixed disk at **C**, and is attached to the block at **A**. Using the coefficients of static friction shown in **Fig. 1**, determine the smallest weight of block **A** that will prevent sliding motion of **B** down the plane. (6)

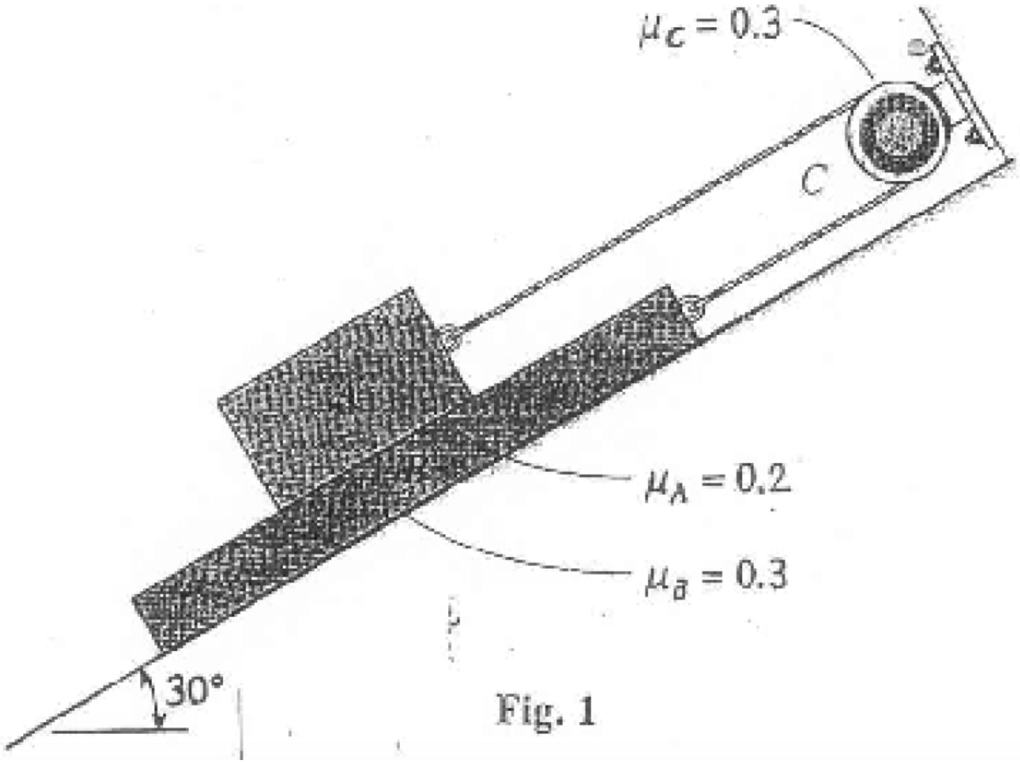



Fig. 1

3. The homogenous blocks ***A*** and ***B*** in **Fig. 3** weigh **200 N** and **300 N**, respectively. The pulley at ***C*** is smooth. The coefficient of friction between ***A*** and ***B***, between ***B*** and the plane, and between the cable and the drum ***D*** are all equal to **0.2**. Determine the maximum weight ***W***, which does not disturb the equilibrium of the system.

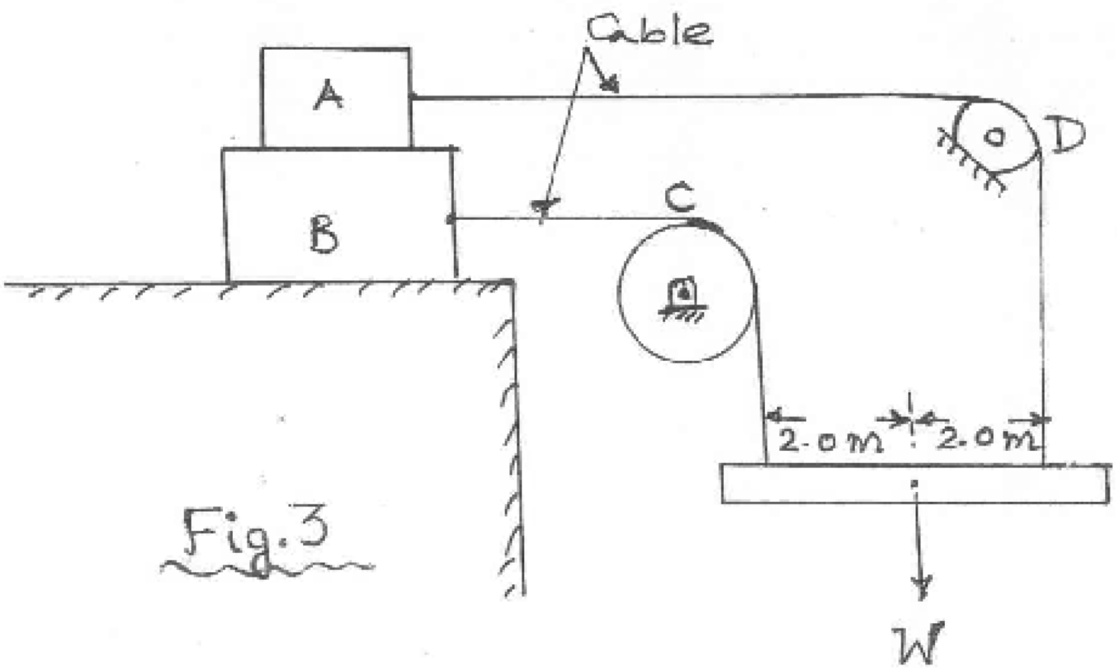



Fig.3

1. A light flexible cord ***DABC*** is passed around the circular disk of mass ***m*** and ends in a small massless, frictionless pulley at *C* that is free to find its equilibrium position on the cord (Figure 1). The coefficient of friction between the cord and the disk is **$\mu$ = 0.5**. Show that, for the position where the disk is on the *verge of turning* under the action of a couple ***M*** applied to the disk, the angle (*BOA*) between the **normals** (*OB* and *OA*) to the cord at the tangency points (*B* and *A*) is equal to $\alpha = 87.3^{o}$. (9 marks)

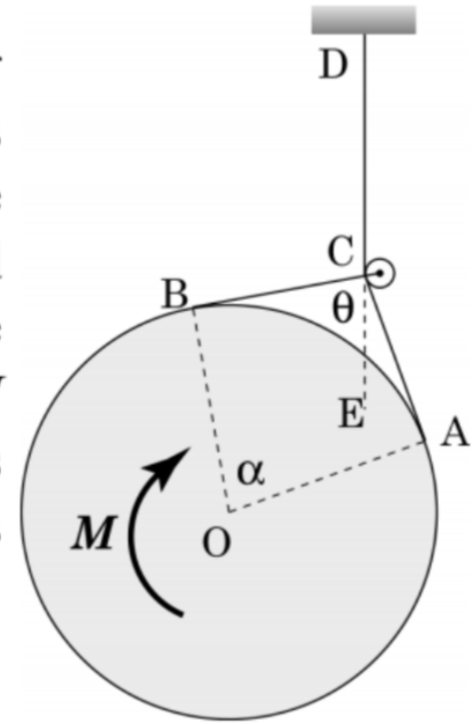



Figure 1

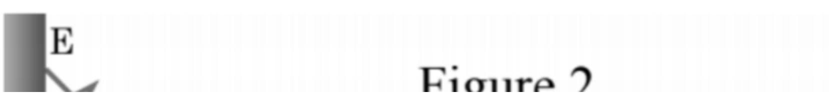
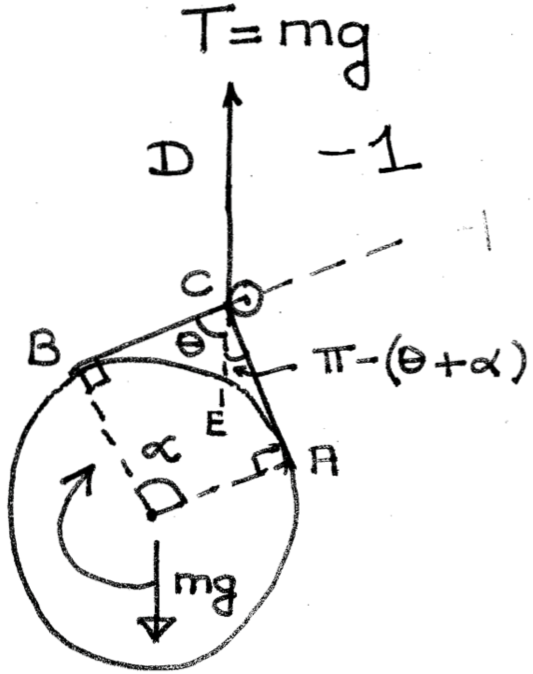



Figure 2

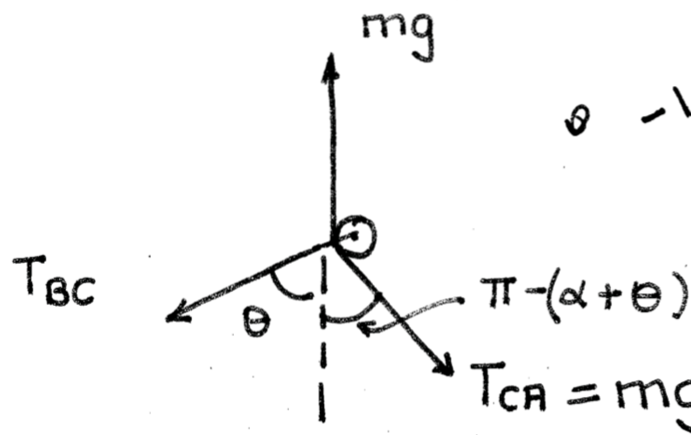
4




−1

$T = mg$ by equilibrium for complete FBD in the ↓ direction

Consider FBD − (2)




−1

−1

Since pulley is frictionless $T_{CA} = mg$ (moment about the center of pulley)

For this FBD.

$$\rightarrow \sum F = mg\sin[\pi - (\alpha + \theta)] - T_{BC}\sin\theta = 0$$

$$\Rightarrow \quad \frac{T_{BC}}{mg} = \frac{\sin(\theta + \alpha)}{\sin\theta} \quad \text{— (1)}$$

−1

$\uparrow \Sigma F = mg - T_{BC}\cos\theta - mg\cos[\pi - (\theta + \alpha)] = 0$ - 1

$\Rightarrow \quad mg = mg\left[\frac{T_{BC}}{mg}\cos\theta - \cos(\theta+\alpha)\right]$ - (2)

$1 = \frac{\sin(\theta+\alpha)}{\sin\theta}\cos\theta - \cos(\theta+\alpha) = \frac{\sin\alpha}{\sin\theta}$ - (3) - 1

$\Rightarrow \sin\alpha = \sin\theta \Longrightarrow \cos\alpha = \cos\theta$

$\frac{T_{BC}}{mg} = \frac{2\sin\alpha\cos\alpha}{\sin\alpha} = 2\cos\alpha$ - (4) - 1

" on the verge of slipping with $\circlearrowleft M$" $\Rightarrow$

$\frac{T_{BC}}{mg} = \exp(-\mu_s\beta) = e^{-0.5(2\pi-\alpha)} = 2\cos\alpha$ - (5)

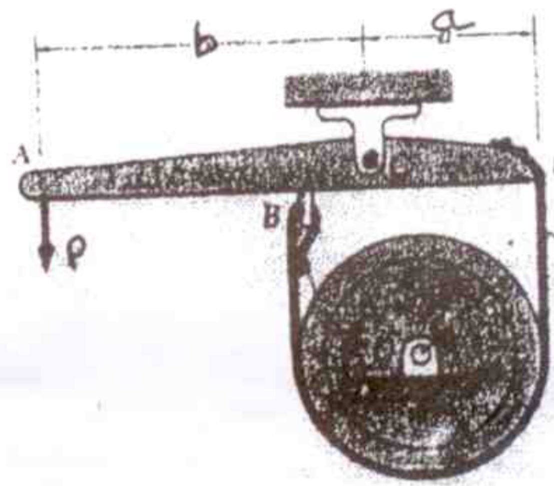
-1

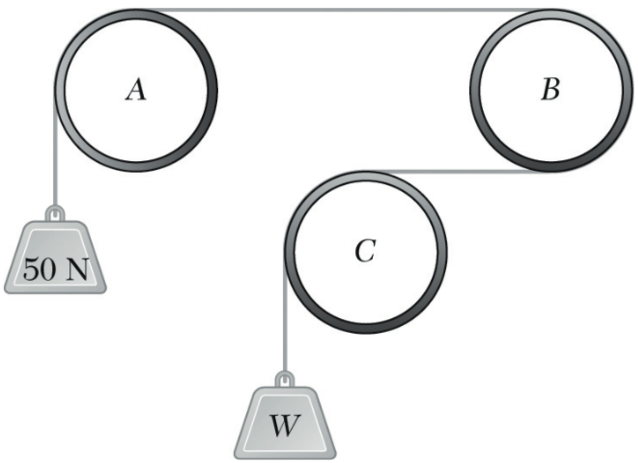
$\alpha = 87.3°$ satisfies this equation.

A brake drum of radius $r = 150$ mm is rotating counterclockwise when a force **P** of magnitude 60 N is applied at A. Knowing that the coefficient of kinetic friction is 0.40, determine

(a) the moment about $O$ of the friction forces applied to the drum when $a = 250$ mm, and $b = 300$ mm, and

(b) the maximum value of the coefficient of kinetic friction for which the brake is not self-locking.

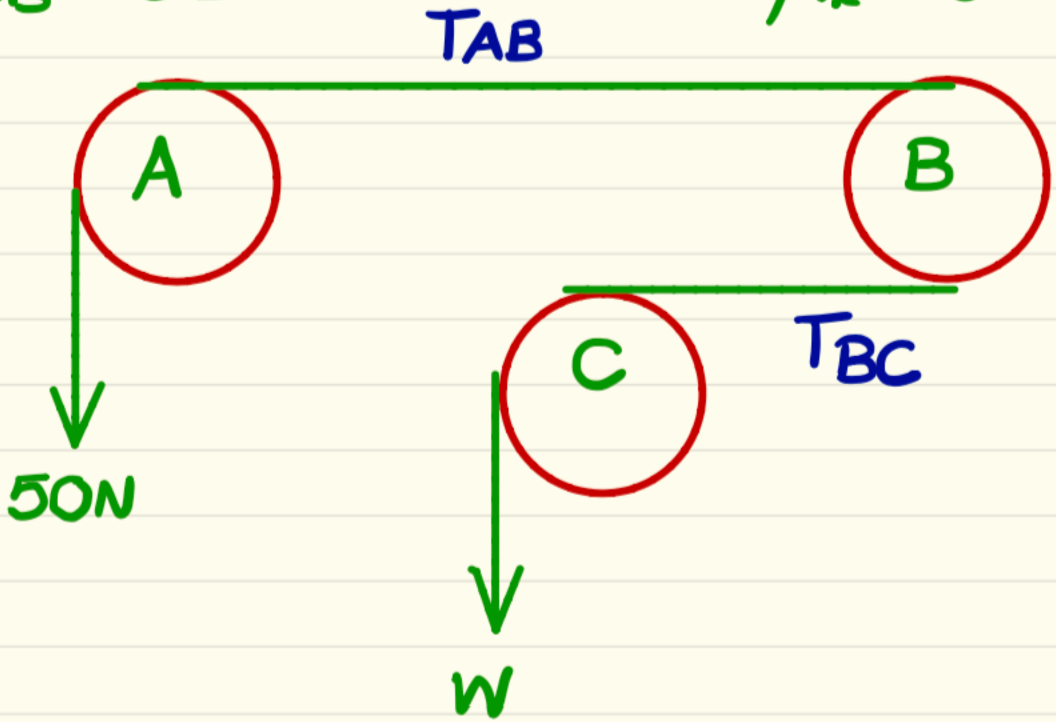





## PROBLEM 8.120

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the smallest weight $W$ for which equilibrium is maintained, (*b*) the largest weight $W$ that can be raised if pipe $B$ is slowly rotated counterclockwise while pipes $A$ and $C$ remain fixed.

$\mu_s = 0.25$ $\qquad$ $\mu_k = 0.2$

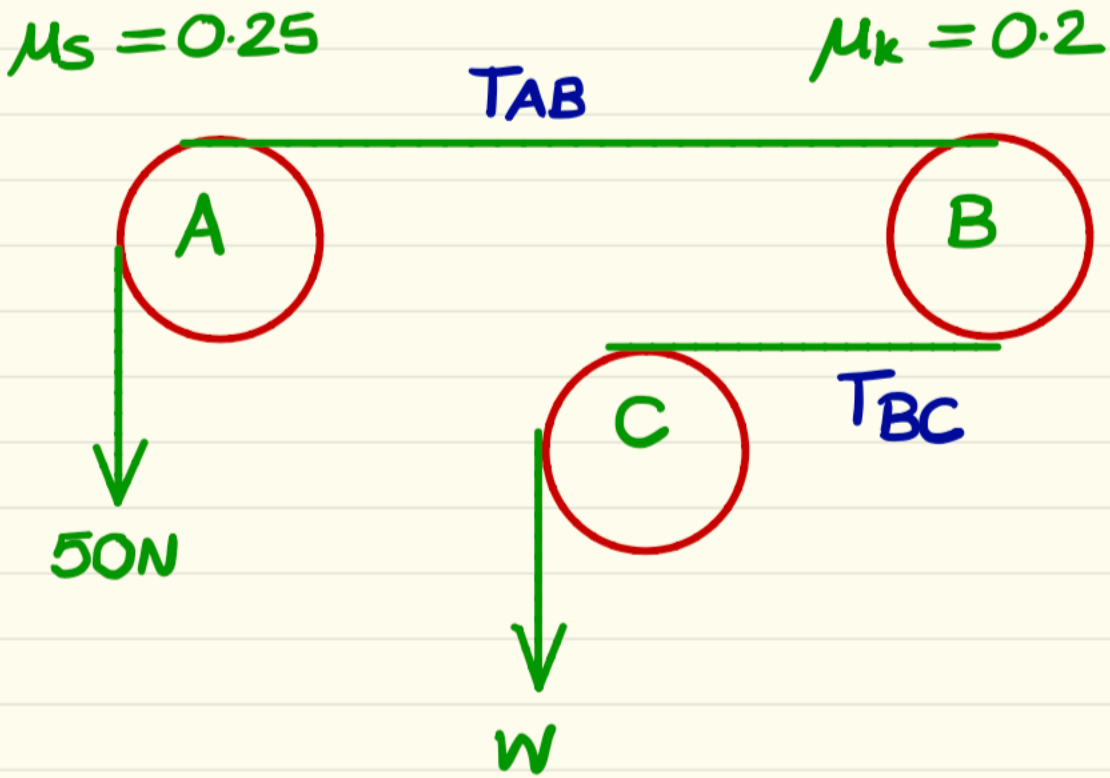



$$\frac{50}{T_{AB}} = e^{\mu_s \times \pi/2}$$

$$\frac{T_{AB}}{T_{BC}} = e^{\mu_s \times \pi} \quad \Rightarrow \quad \frac{50}{W} = e^{\mu_s \times 2\pi}$$

$$\frac{T_{BC}}{W} = e^{\mu_s \times \pi/2} \qquad W = 50 \times e^{-2\pi \mu_s}$$

$$\boxed{W = 10.4\,N}$$




ii) $$\frac{50}{T_{AB}} = e^{\mu_k \times \pi/2} \qquad \frac{T_{BC}}{T_{AB}} = e^{\mu_s \times \pi}$$

$$\frac{T_{BC}}{W} = e^{\mu_k \times \pi/2}$$

$$\frac{50}{W} = e^{\pi \mu_k} \times e^{-\mu_s \pi} \Rightarrow \boxed{W = 58.50N}$$