University School of Automation and Robotics
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Delhi - 110092

Engineering Mechanics

By: Dr. Divya Agarwal

- **UNIT- I**

- **Force system:** Introduction, force, principle of transmissibility of force, resultant of a force system, resolution of a force, moment of force about a line, Varigon's theorem, couple, resolution of a force into force and a couple, properties of couple and their application to engineering problems.
- **Equilibrium:** Force body diagram, equations of equilibrium, and their applications to engineering problems, equilibrium of two force and three force members.
- **Distributed forces:** Determination of centre of gravity, centre of mass and centroid by direct integration and by the method of composite bodies., mass moment of inertia and area moment of inertia by direct integration and composite bodies method, radius of gyration, parallel axis theorem, polar moment of inertia.

- **UNIT- II**

- **Structure:** Plane truss, perfect and imperfect truss, assumption in the truss analysis, analysis of perfect plane trusses by the method of joints, method of section, graphical method.
- **Friction:** Static and Kinetic friction, laws of dry friction, co-efficient of friction, angle of friction, angle of repose, cone of friction, frictional lock, friction in pivot and collar bearing, friction in flat belts.

- **UNIT-III**

- **Kinematics of Particles:** Rectilinear motion, plane curvilinear motion, rectangular coordinates, normal and tangential coordinates
- **Kinetics of Particles:** Equation of motion, rectilinear motion and curvilinear motion, work energy equation, conservation of energy, concept of impulse and momentum, conservation of momentum, impact of bodies, co-efficient of restitution, loss of energy during impact.

- **UNIT-IV**

- **Kinematics of Rigid Bodies:** Concept of rigid body, type of rigid body motion, absolute motion, introduction to relative velocity, relative acceleration (Corioli's component excluded) and instantaneous center of zero velocity, velocity and acceleration.
- **Kinetics of Rigid Bodies:** Equation of motion, translatory motion and fixed axis rotation, application of work energy principles to rigid bodies conservation of energy.
- **Beam:** Introduction, types of loading, methods for the reactions of a beam, space diagram, types of end supports, beams subjected to couple

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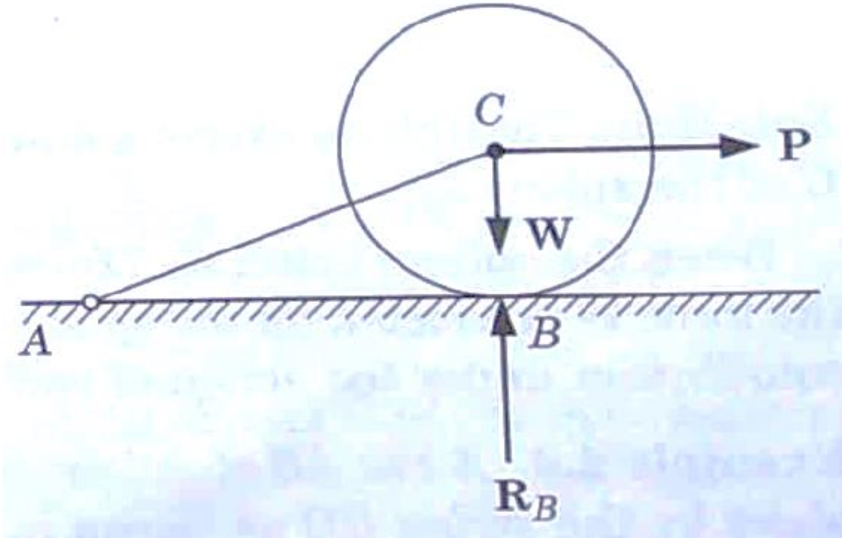
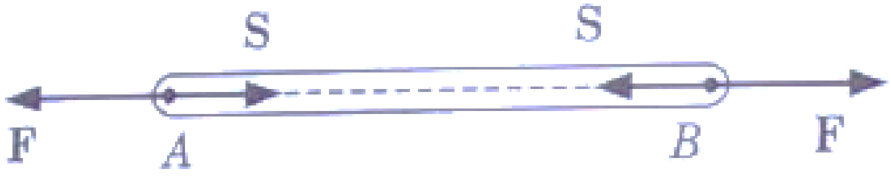
WHAT IS A FREE BODY DIAGRAM?

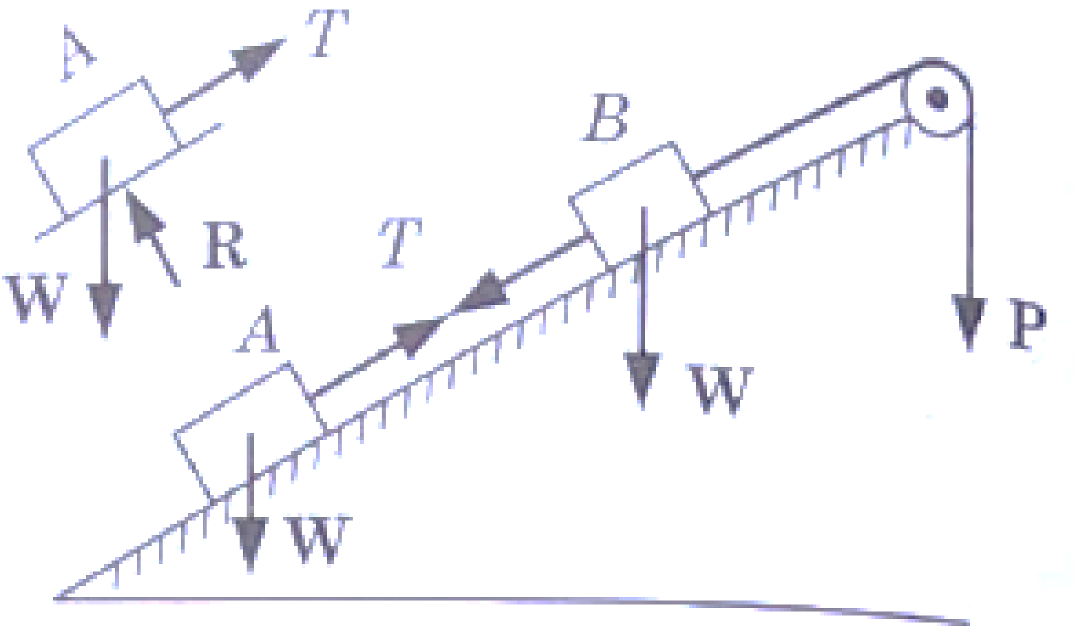
- To clearly identify the various forces acting on a body in equilibrium we have to draw its FBD. Only then we can write the equations of the equilibrium of the body.
- To draw the FBD of a body we remove all the supports (like wall, floor, hinge or any other body) and replace them by the reactions which these supports exert on the body.
- **External forces.** These are forces which act on a body or a system of bodies from outside. For example, in the case of the roller shown in the figure, (i). Weight of roller **W**, (ii). Applied forces **P**, and (iii) the reaction $\mathbf{R_B}$ at the point of contact, are the external forces acting on the roller. (Fig a)
- **Internal forces.** Are those forces which hold together the particles of a body. And if more than one body is involved, it may be the force that holds the two bodies together. If we try to pull a bar by applying two equal and opposite forces **F** then an internal force **S** comes into play to hold the body together. (Fig b)





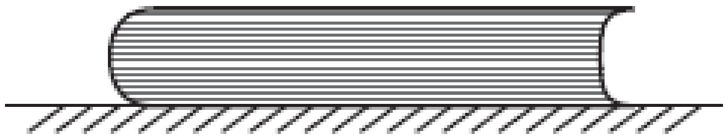
WHAT IS A FREE BODY DIAGRAM?

- If two bodies **A** and **B** connected by a string are held on an inclined plane by a force **P**, then the force of tension **T** in the string is the internal force.
- But if we consider the equilibrium of a single body **A**, this force of tension **T** becomes an external force acting on the body **A**.



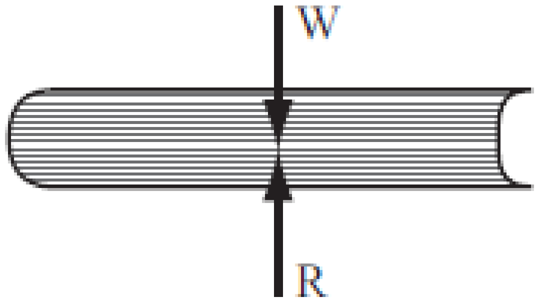
FREE BODY DIAGRAMS

- No system, natural or man-made, consists of a single body alone or is complete by itself.
- A single body or a part of the system can, however, be isolated from the rest by appropriately accounting for its effect.
- FBD consists of a diagrammatic representation of a single body or a subsystem of bodies isolated from its surroundings but shown under the action of forces and moments due to external actions.
- Consider, for example, a book lying flat on a table Fig. (a). the book exerts its weight on the table and the table exerts its own weight as well as transmits the weight of the book on the ground.
- A FBD for the book alone would consist of its weight W acting through the centre of gravity and the reaction exerted on the book by the table top as shown in Fig. (b).



(a)

(a) Book on a table top



(b)

(b) Free-body diagram

WHAT IS A FREE BODY DIAGRAM?

- FBD are useful aids for representing the relative magnitude and direction of all forces acting upon an object in a given situation.
- The first step in analysing and describing most physical phenomena involves the careful drawing of a FBD.
- In a FBD, the size of the arrow denotes the magnitude of the force while the direction of the arrow denotes the direction in which the force acts.
- A FBD is defined as: **A FBD is a graphic, dematerialized, symbolic representation of the body (structure, element or segment of an element) in which all connecting "pieces" have been removed.**
- **Purpose of FBD:** FBDs are tools that are used to visualize the force and moments applied to a body and to calculate the resulting reactions in many types of mechanics problems.

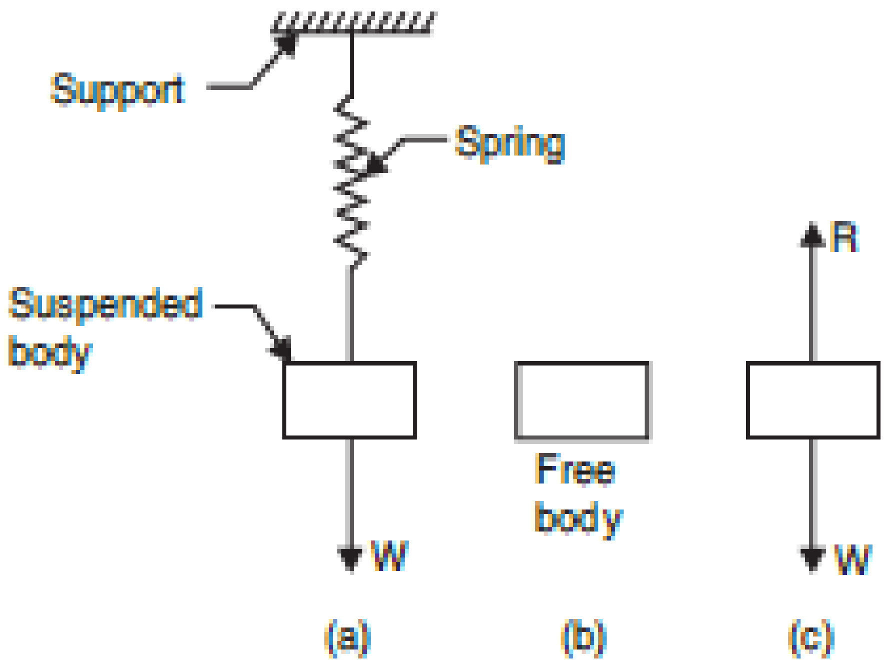
FEATURES OF FREE BODY DIAGRAM

Normally, a FBD consists of the following components:

- A simplified version of the body (most commonly a box)
- A coordinate system
- Forces are represented as arrows pointing in the direction they act on the body. The number of forces acting on a body depends on the specific problem and the assumptions made.
- Moments showed as curved arrows pointing in the direction they act on the body
- Exclusions in FBD
  1. Bodies other than the FBD
  2. Constraints, Internal Forces
  3. Velocity and Acceleration Vectors

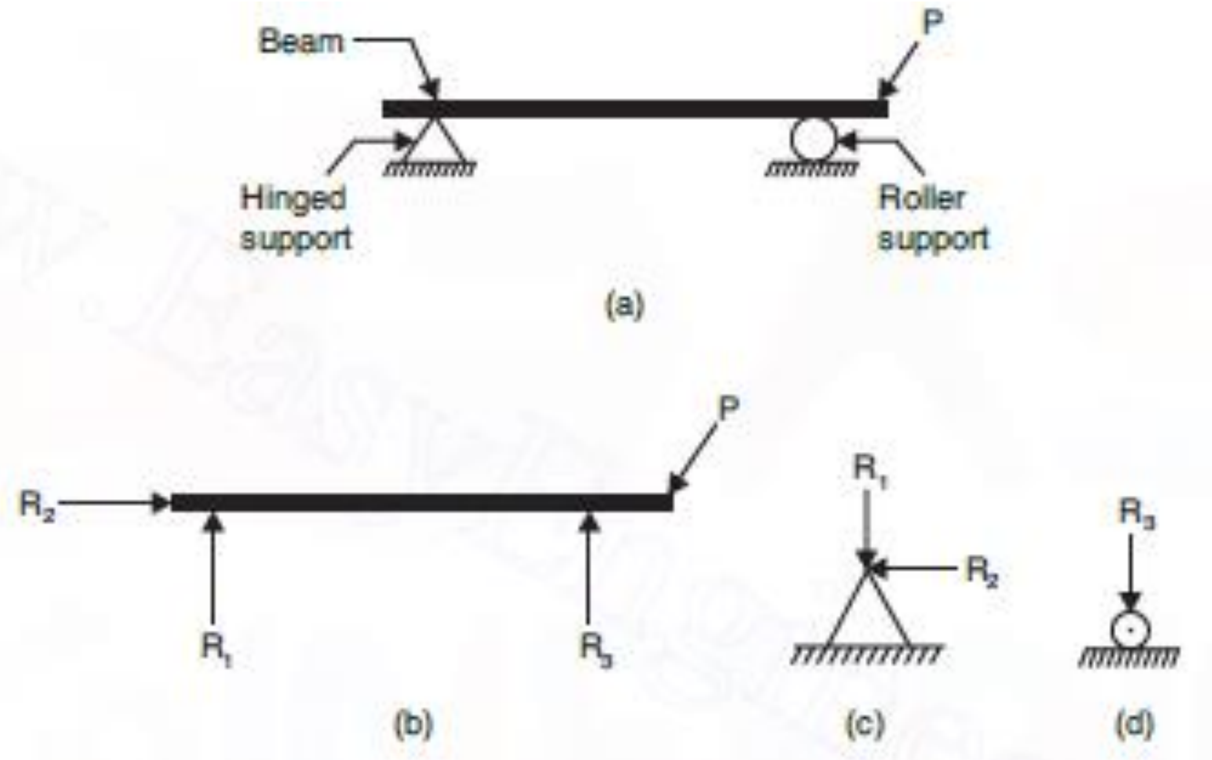
FREE BODY DIAGRAMS

- A body may consist of more than one element and supports. Each element or support can be isolated from the rest of the system by incorporating the net effect of the remaining system through a set of forces.
- This *diagram of the isolated element or a portion of the body along with the net effects of the system on it is called a 'FBD'.*
- Free-body diagrams are useful in solving the forces and deformations of the system.
- In case of a body shown here, we remove the supporting springs and replace it by the reactive force *R* equal to *W* in magnitude. The Fig. (*c*) in which the body is completely isolated from its support and in which all forces acting on it are shown by vectors is called a *FBD*.



FREE BODY DIAGRAMS

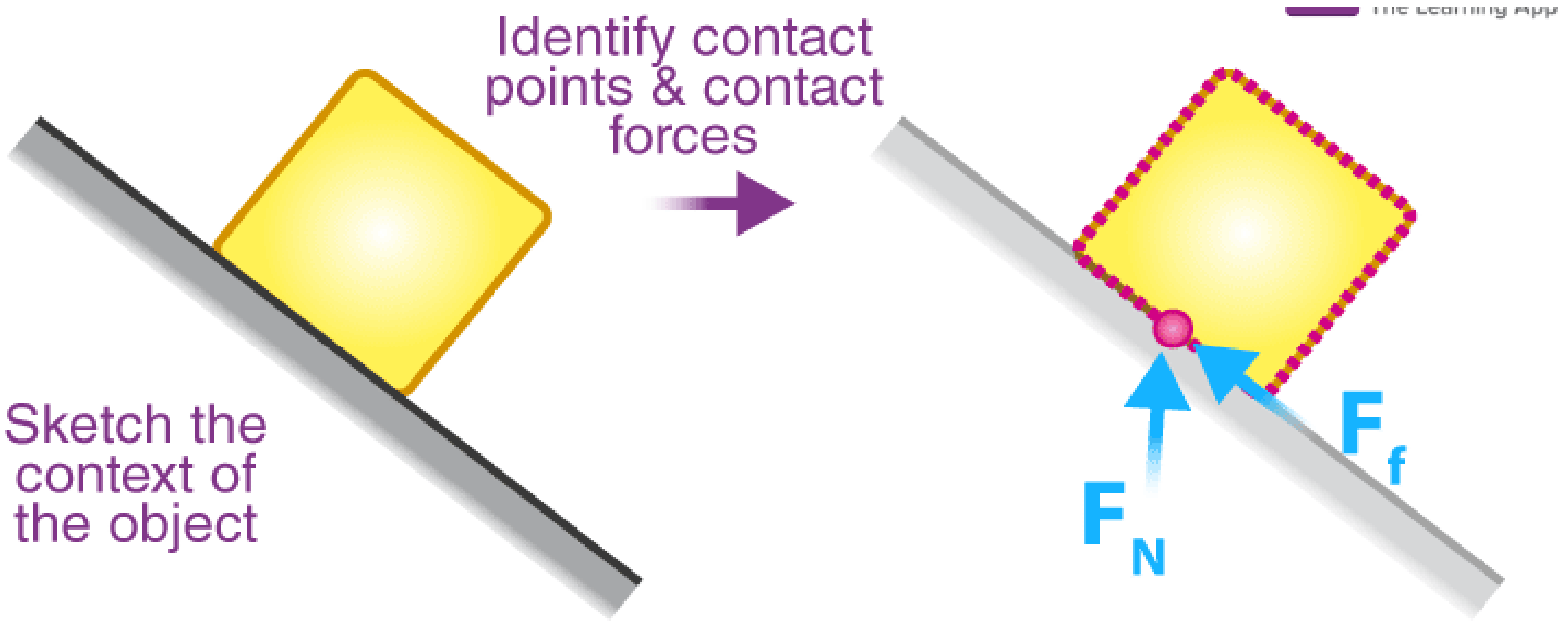
- Let us consider another case of a beam shown in Fig. (*a*).
- The beam is supported on a hinge at the left end and on a roller at the right end. The hinge offers vertical and horizontal reaction whereas the roller offers vertical reaction.
- The beam can be isolated from the supports by setting equivalent forces of the supports.
- Fig. (*b*) illustrates the FBD of the beam in which *R*1 and *R*2 are reactions of the hinge support and *R*3 the reaction of the roller support.
- Similarly, the FBDs of hinge and roller supports are shown in Figs. (*c*) and (*d*) respectively.



HOW TO MAKE A FREE BODY DIAGRAM?

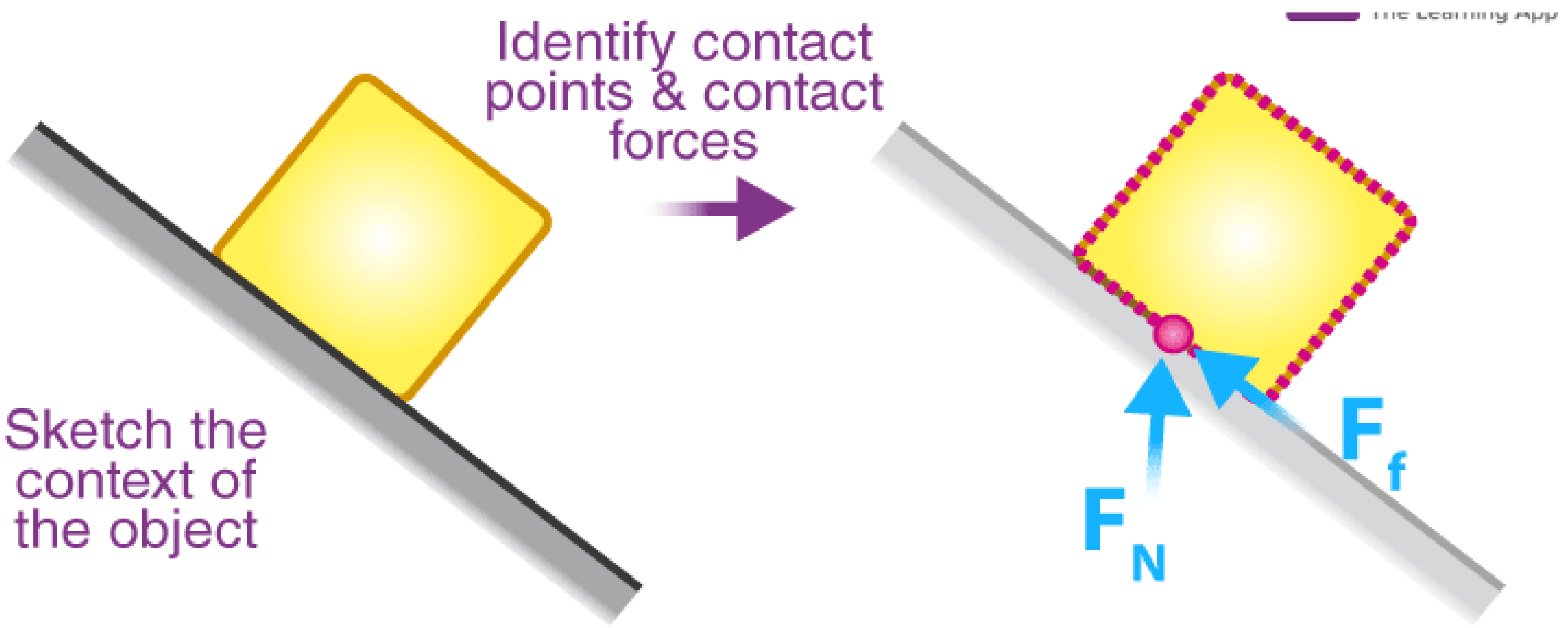
Step 1: Identify the Contact Forces:

To identify the forces acting on the body, draw an outline of the object with dotted lines as shown in the figure. Make sure to draw a dot when something touches the object. When there is a dot, it indicates that there is at least one contact force acting on the body. Draw the force vectors at the contact points to represent how they push or pull on the object.



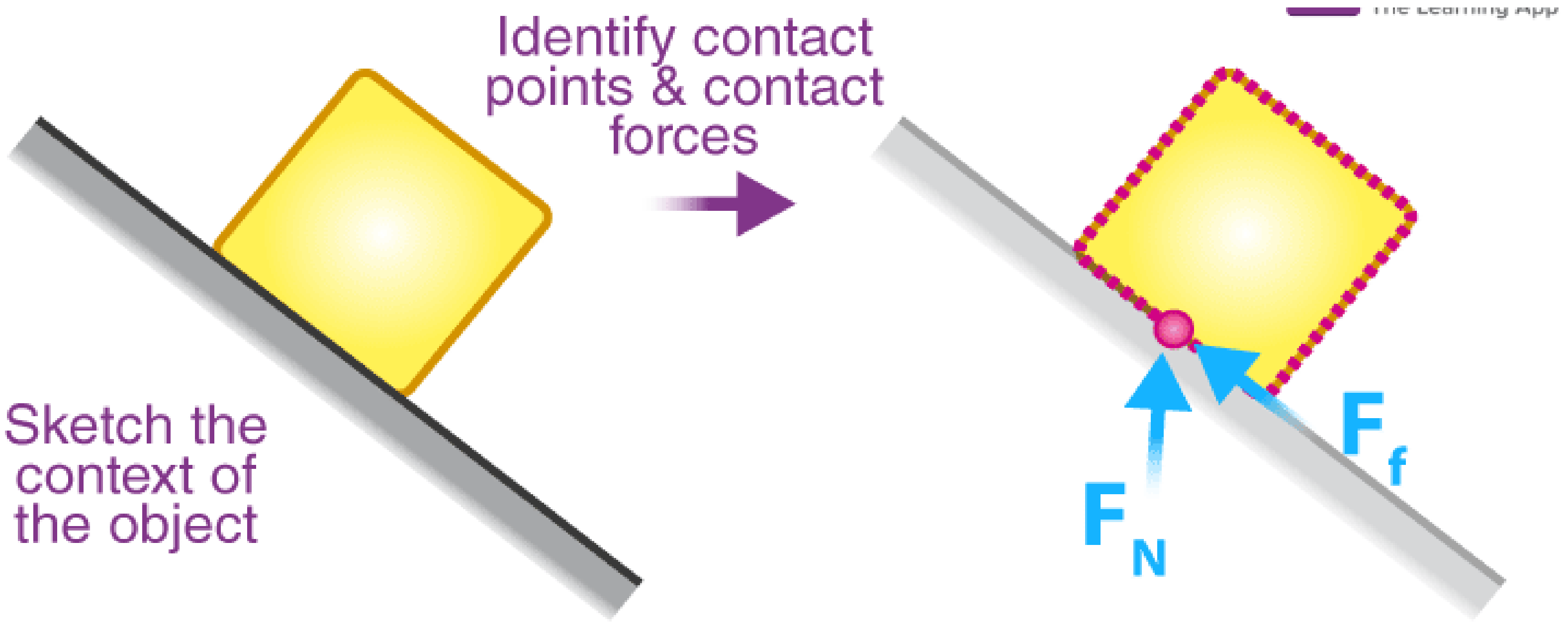
HOW TO MAKE A FREE BODY DIAGRAM?

Step 2. After identifying the contact forces, draw a dot to represent the object that we are interested in. Here, we are only interested in determining the forces acting on our object.



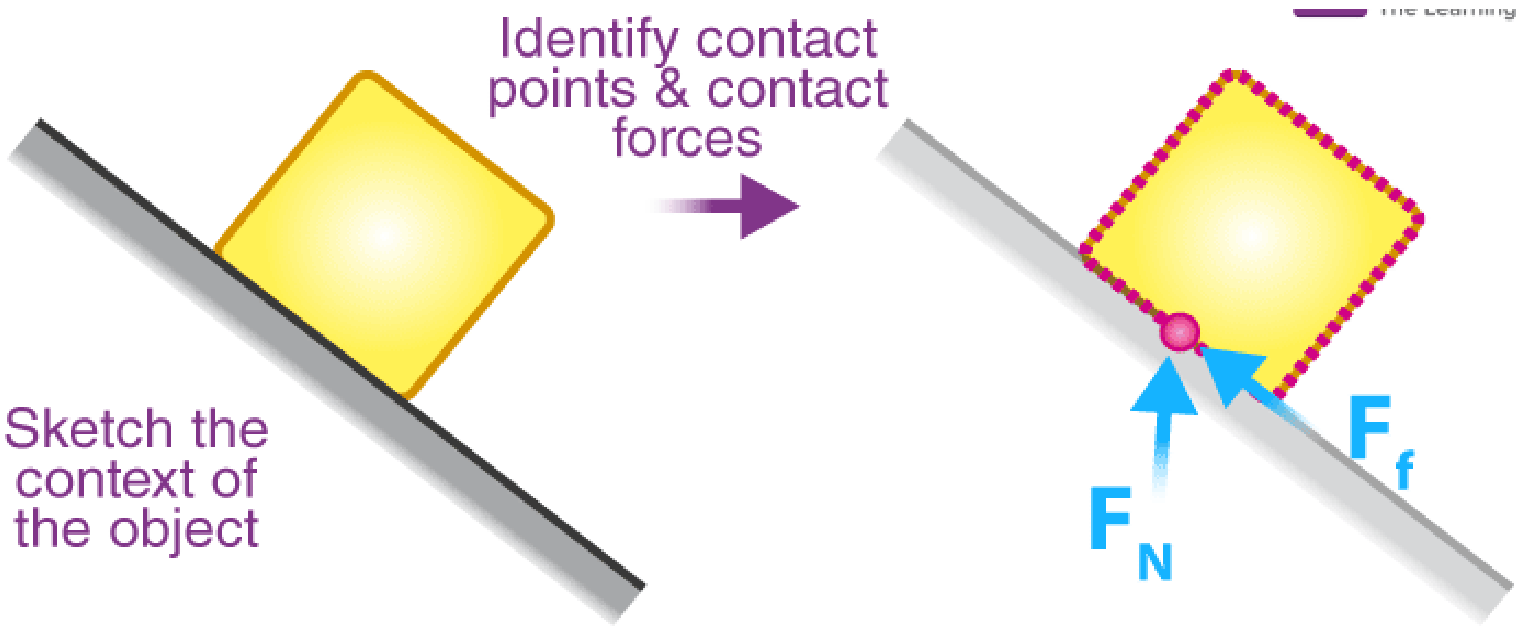
HOW TO MAKE A FREE BODY DIAGRAM?

Step 3. Draw a coordinate system and label positive directions.



HOW TO MAKE A FREE BODY DIAGRAM?

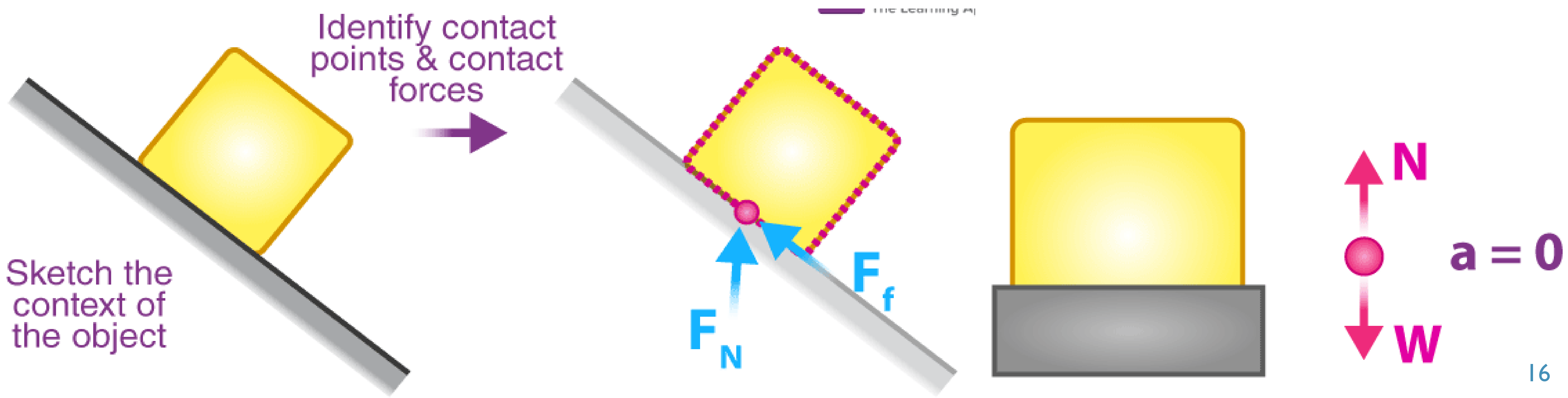
Step 4. Draw the contact forces on the dot with an arrow pointing away from the dot. The arrow lengths should be relatively proportional to each other. Label all forces.



HOW TO MAKE A FREE BODY DIAGRAM?

Step 5. Draw and label our long-range forces. This will usually be weight unless there is an electric charge or magnetism involved.

Step 6. If there is acceleration in the system, then draw and label the acceleration vector.



COMMON MISTAKES MADE WHILE DRAWING FBD

- Avoid drawing forces of the object acting on other objects
- The direction of the different types of forces is denoted wrong.
- The direction of different forces:
  - Weight is always down
  - Friction is always parallel to the contact surface,
  - The normal force is always perpendicular to the contact surface, and tension only pulls.

FBD EXAMPLES

- A bottle is resting on a tabletop. Draw the forces acting on the bottle.

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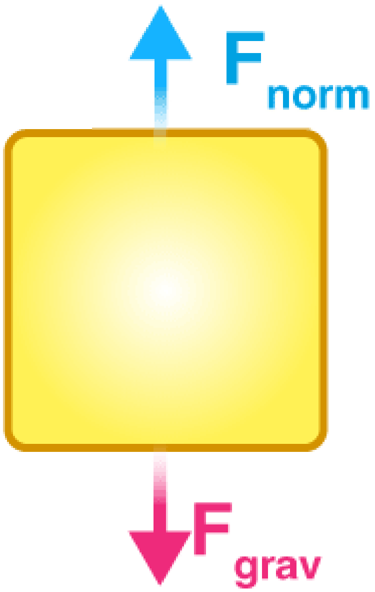

FBD EXAMPLES

- An egg is free-falling from a nest in a tree, neglecting the air resistance, what would the free body diagram look like?

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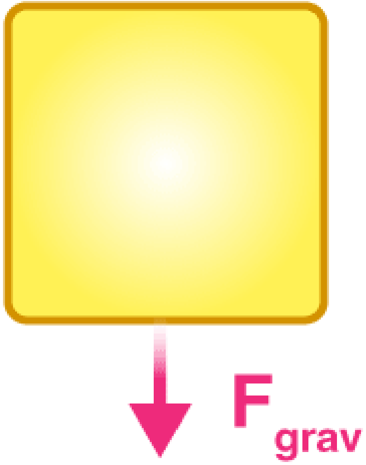

FBD EXAMPLES

- If a rightward force is applied to a book in order to move it across a desk at a constant velocity. Considering only the frictional forces and neglecting air resistance. A free-body diagram for this situation looks like this:

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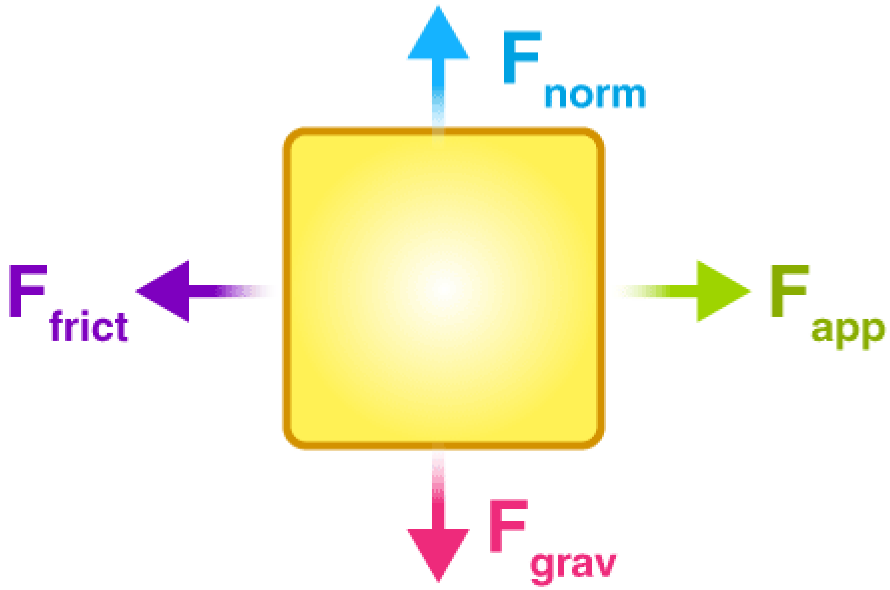

FBD EXAMPLES

- A skydiver is descending at a constant velocity. Considering the air resistance, the free body diagram for this situation would like the following:

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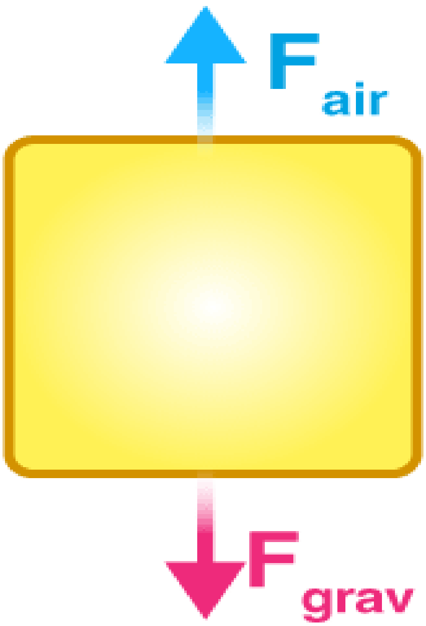
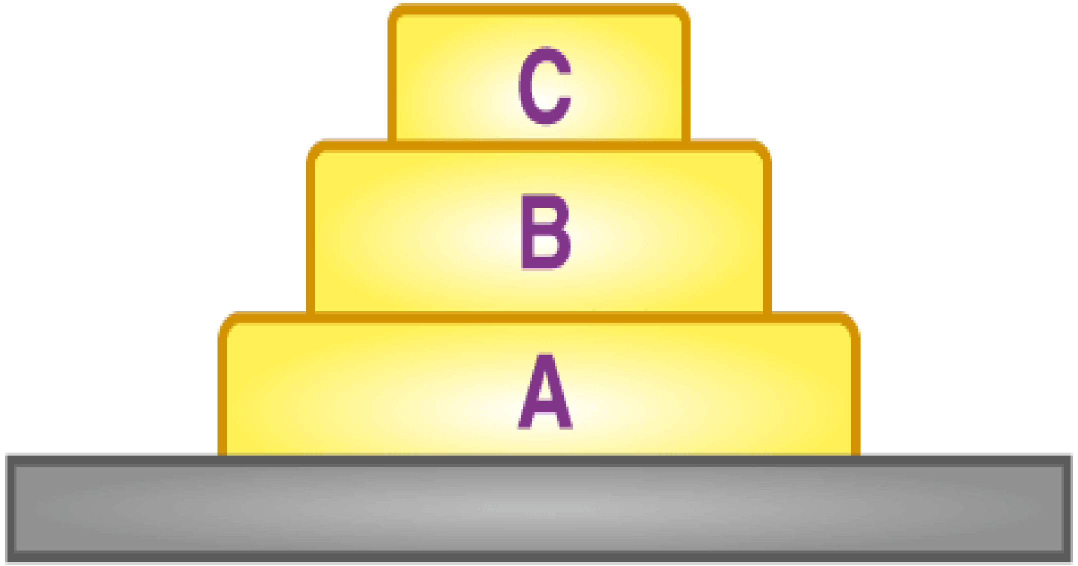


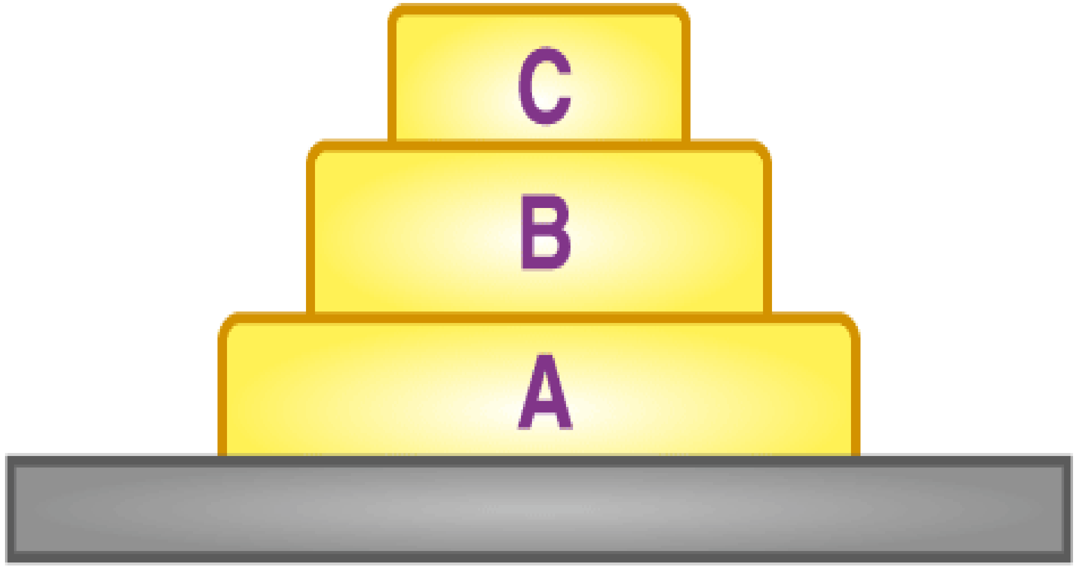
FBD EXAMPLES

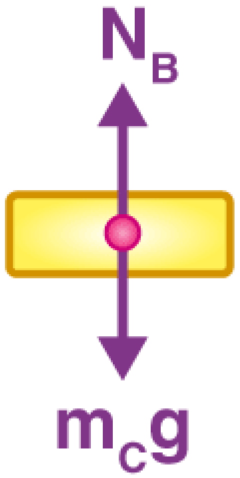
- Draw a free body diagram of three blocks placed one over the other as shown in the figure.



FBD EXAMPLES

- The forces acting on the individual elements of the system are shown below:





C

The forces on block "C" are:

$W_C=m_Cg$= its weight, acting downward

N_B= normal reaction on "C" due to the upper surface of block B, acting upward

FBD EXAMPLES

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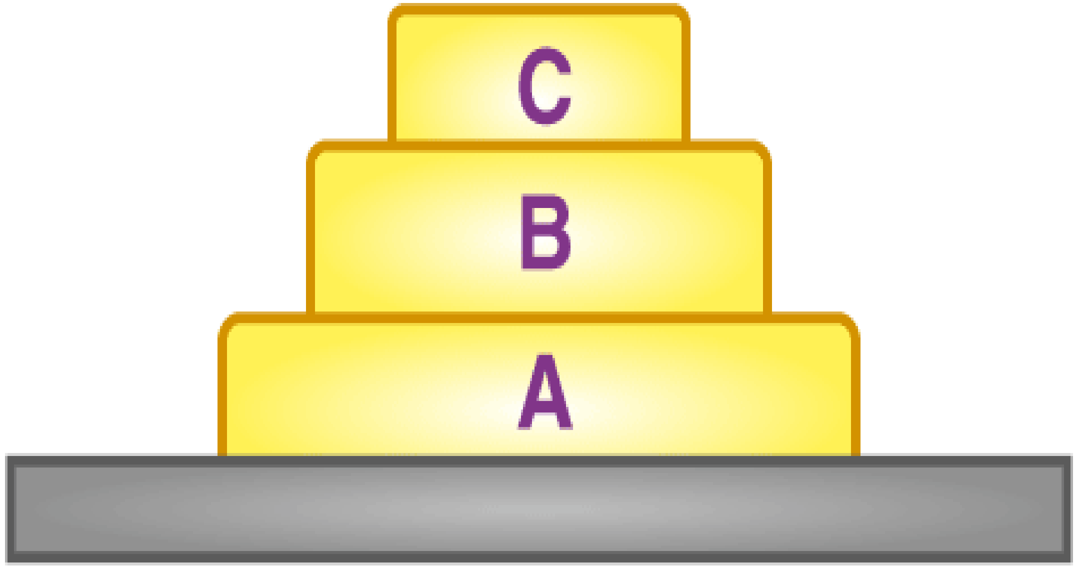
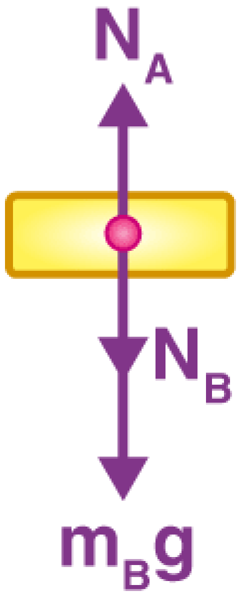




B

The forces on block "B" are:

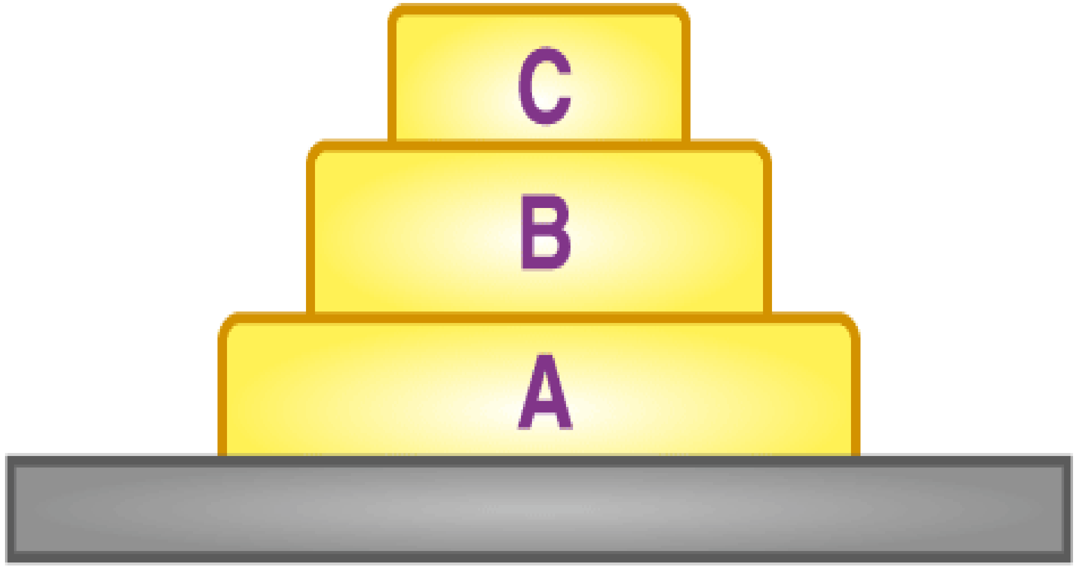
$W_B = m_B g$ = its weight, acting downward

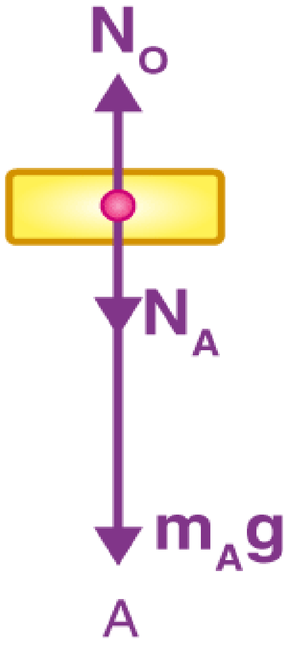
N_B = normal reaction on "B" due to the lower surface of block C, acting downward

N_A = normal reaction on "B" due to the upper surface of block A, acting upward

FBD EXAMPLES

- The forces acting on the individual elements of the system are shown below:





The forces on the block "A" are :

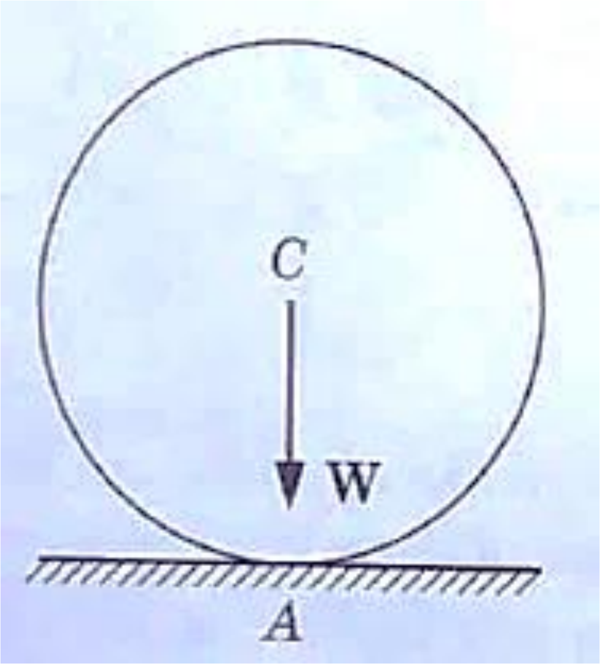
$W_A = m_A g$= its weight, acting downward

N_A= normal reaction on "A" due to the lower surface of block B, acting downward

N_O= normal reaction on "A" due to horizontal surface, acting upward

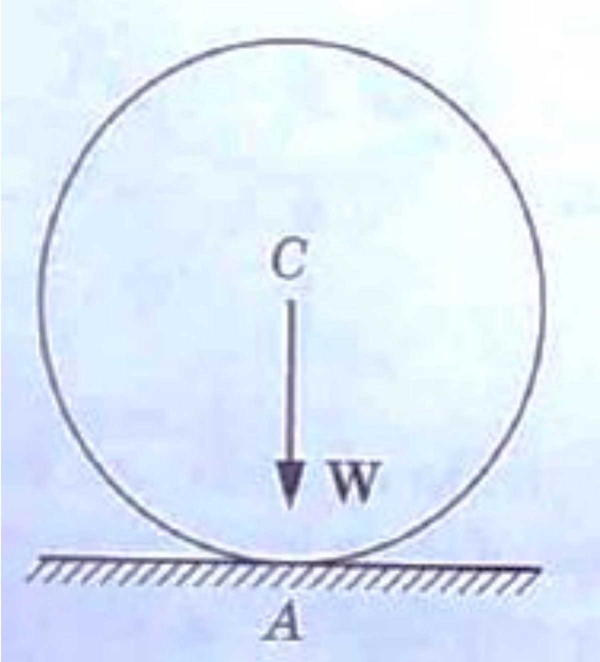
ASSIGNMENT

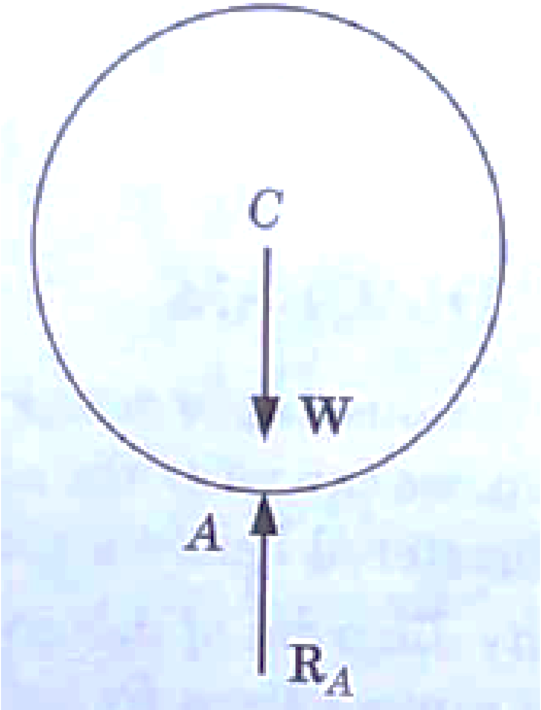
- Question1. Draw the free body diagram of a sphere of weight W is resting on a frictionless plane surface as shown in figure.



SOLUTION 1

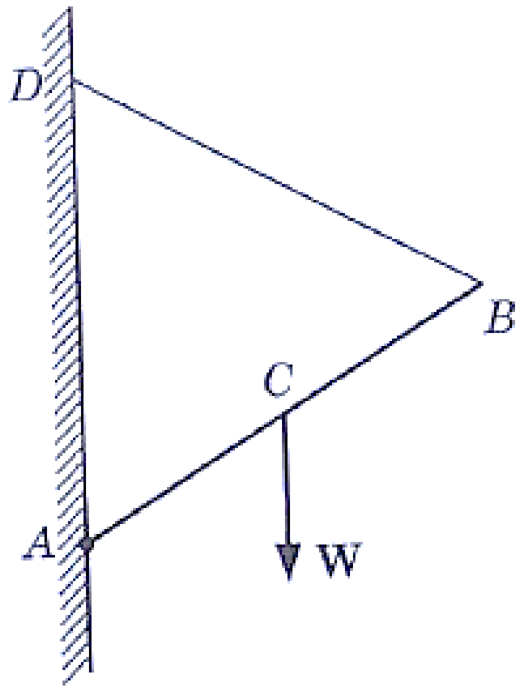
- Solution. The sphere exerts a downward force **W** on the surface acting through the centroid **C** of the sphere. When the sphere is isolated from the surface, a reaction **R_A** which is equal and opposite to the force **W** is exerted on the sphere by the surface at the point of contact **A**. The sphere is in equilibrium under the action of two equal and opposite forces **W** and **R_A** which are collinear.





ASSIGNMENT

- Question 2. A bar **AB** of weight **W** is hinged at A to the wall and is supported in a vertical plane by the string **BD** as shown in figure. Draw the free body diagram of the bar.



SOLUTION 2

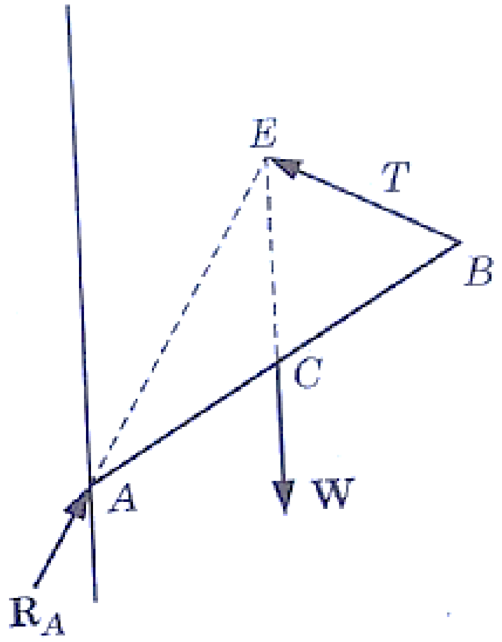
- Solution. The forces acting on the bar when isolated from the support area:

(i) a force **W** equal to its own weight and acting vertically downward

(ii) the pull **T** of the string along **BD** and

(iii) the reaction $\mathbf{R_A}$ at the hinge in an unknown direction.

Since the bar **AB** is in equilibrium under the action of three non parallel forces, they must pass through the point **E**. It thus fixes the unknown direction of the reaction $\mathbf{R_A}$ as shown in figure.



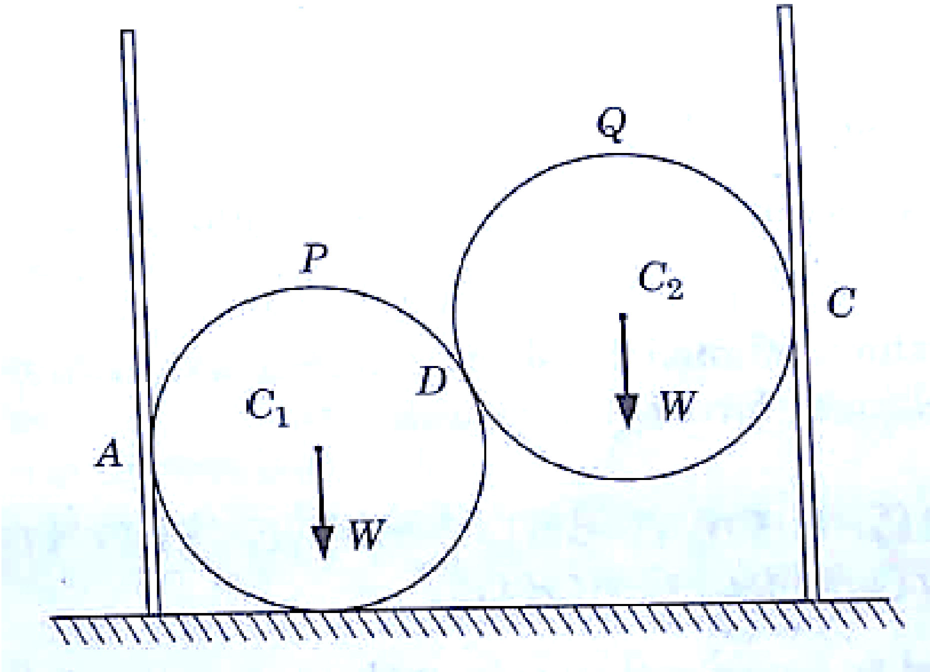
ASSIGNMENT

- Question 3. Two similar spheres **P** and **Q** each of weight **W** rest inside a hollow cylinder which is resting on a horizontal plane. Draw the free body diagram of:

A. both the spheres taken together.

B. The sphere **P**

C. The sphere **Q**.

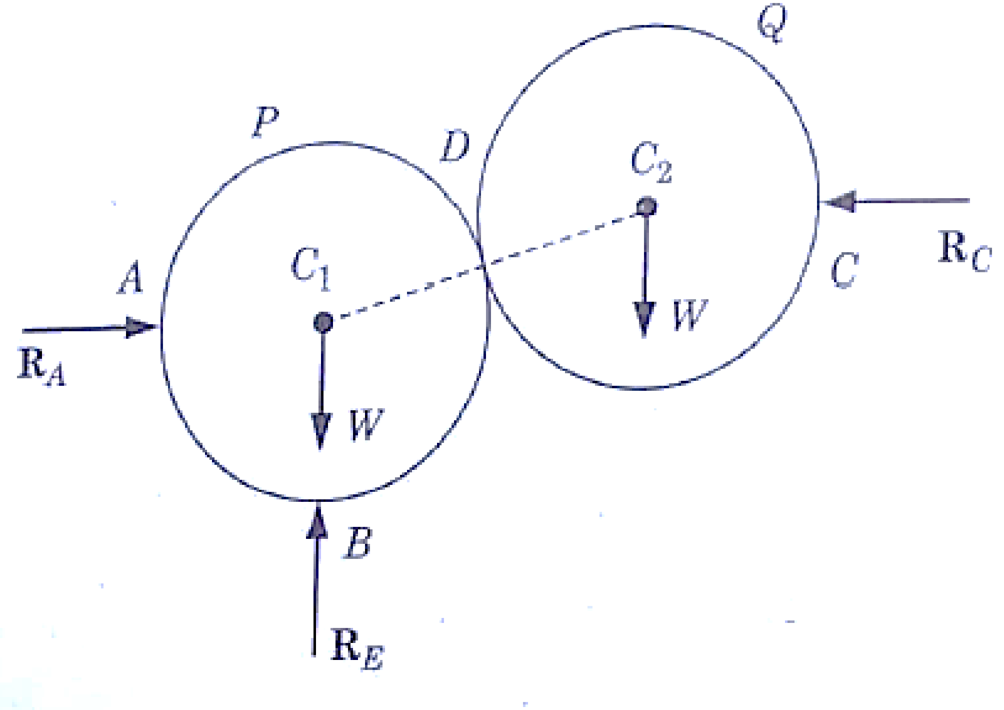


SOLUTION 3

- Let $\mathbf{R_A}$, $\mathbf{R_B}$ and $\mathbf{R_C}$ be the reactions at the points **A, B** and **C** of the cylinder and the plane on the spheres.

A. **Free body diagram of spheres P and Q:**

Note that the reaction at the point of contact D does not appear in FBD, it being an internal force between the two spheres.

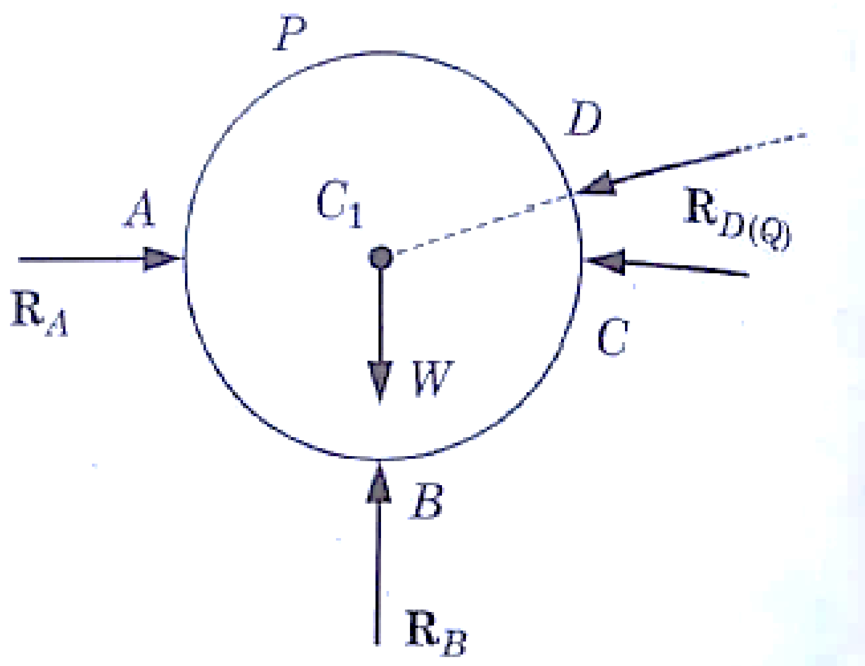


SOLUTION 3

- Let $\mathbf{R_A}$, $\mathbf{R_B}$ and $\mathbf{R_C}$ be the reactions at the points **A, B** and **C** of the cylinder and the plane on the spheres.

B. Free body diagram of sphere P:

$R_{D(Q)}$ is the reaction of the sphere Q on the sphere P at the point of contact D, acting in the direction normal to the surface. That is along C_1C_2.

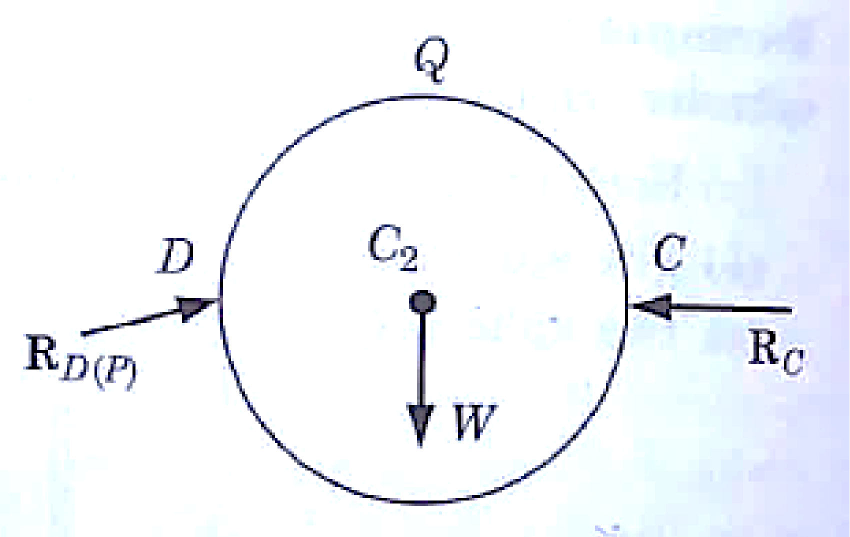


SOLUTION 3

- Let $\mathbf{R_A}$, $\mathbf{R_B}$ and $\mathbf{R_C}$ be the reactions at the points **A, B** and **C** of the cylinder and the plane on the spheres.

C. **Free body diagram of a sphere Q.**

- $R_{D(P)}$ is the reaction of the sphere P on the sphere Q acting at the point of contact D. It should be noted that the reaction of the sphere P on the sphere Q, $R_{D(P)}$ and the reaction of the sphere Q on the sphere P, $R_{D(Q)}$ are equal in magnitude but opposite in direction and are collinear. So,
- $R_{D(P)} = R_{D(Q)} = R_D$ They do not appear in the combined FBD of the two spheres as they form a pair of equal opposite and collinear forces that cancel.



EQUILIBRIUM OF FORCES

- If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium.
- The force, which brings the set of forces in equilibrium is called on equilibrant.
- The equilibrant is equal to the resultant force in magnitude, but opposite in direction.

EQUATIONS OF EQUILIBRIUM FOR A SYSTEM OF CONCURRENT FORCES IN A PLANE

- We can find the resultant **R** of several forces $\mathbf{F_1}, \mathbf{F_2}, \mathbf{F_3}, \mathbf{F_4}$….. using the method of summing the rectangular components of forces as explained earlier.
- If their resultant is zero the particle is said to be in equilibrium.
- i.e., When the resultant of all the forces acting on a particle is zero the particle is said to be in equilibrium.
- For the resultant **R** to be zero it's each of the two rectangular components R_x and R_y must be separately equal to zero. If,

$$R = \sqrt{(R_x)^2 + (R_y)^2} = 0$$

Then, $R_x = 0$ and $R_y = 0$

Therefore, $R_x = F_{x1} + F_{x2} + F_{x3} + \ldots = 0$ and $R_y = F_{y1} + F_{y2} + F_{y3} + \ldots = 0$

or, $\Sigma F_x = 0$ and $\Sigma F_y = 0$

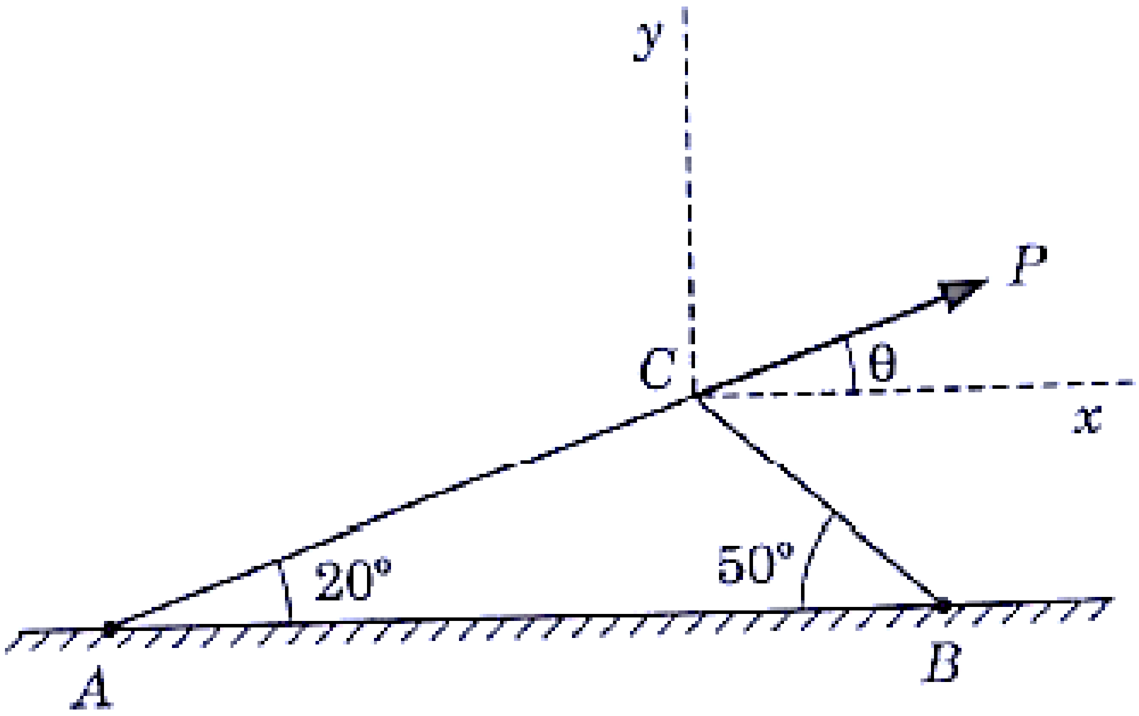
EQUATIONS OF EQUILIBRIUM FOR A SYSTEM OF CONCURRENT FORCES IN A PLANE

- Where, F_{x1} , F_{x2} , F_{x3} , … are the components of the forces F_1, F_2 …. along the x axis.

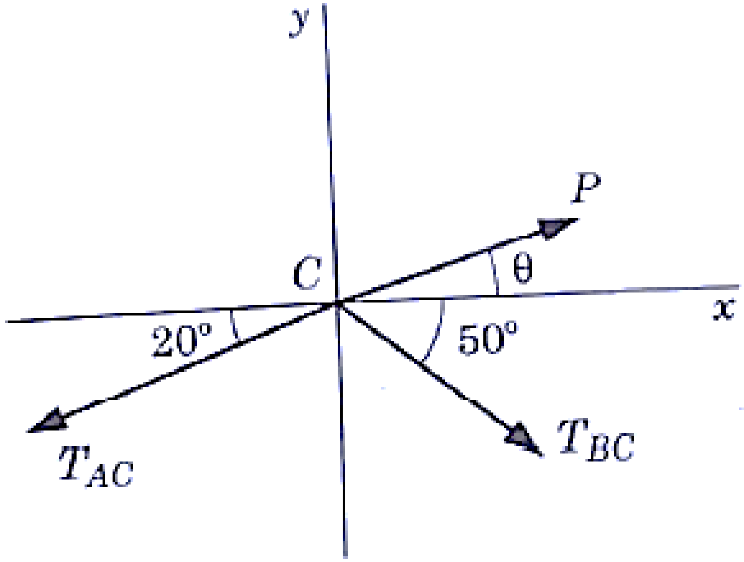
  and F_{y1} , F_{y2} , F_{y3} , … are the components of the forces F_1, F_2 …. along the y axis.
- Equations $\mathbf{\Sigma F_x = 0}$ and $\mathbf{\Sigma F_y = 0}$ are called the equations of equilibrium.
- If a number of concurrent forces lying in a plane are in equilibrium these equations are to be satisfied.
- The x and the y-axis can be arbitrarily chosen to the point of concurrency. For example, in the case of a body resting on an inclined plane, the x and y axis can be chosen along and normal to the inclined plane.
- The two equations of equilibrium can be solved to find a maximum of two unknowns. Determination of a force in magnitude and direction or determination of the magnitudes of two forces whose directions are known amounts to solving for two unknowns.

EXAMPLE

- Two ropes are tied together at C. If the maximum permissible tension in each rope is 3.5 kN. What is the maximum force P that can be applied and in what direction?



SOLUTION



- Consider the equilibrium of the point C,

$\Sigma F_x = 0$: $P\cos\theta + T_{BC}\cos 50^o - T_{AC}\cos 20^o = 0$

$\Sigma F_y = 0$: $P\sin\theta - T_{BC}\sin 50^o - T_{AC}\sin 20^o = 0$

where, T_{AC} and T_{BC} are tension in the strings AC and BC

$P\cos\theta = T_{AC}\cos 20^o - T_{BC}\cos 50^o$ and $P\sin\theta = T_{AC}\sin 20^o + T_{BC}\sin 50^o$

$$\frac{P\sin\theta}{P\cos\theta} = tan\theta = \frac{T_{AC}\sin 20^o + T_{BC}\sin 50^o}{T_{AC}\cos 20^o - T_{BC}\cos 50^o}$$

But, $T_{AC} = T_{BC} = 3.5kN$

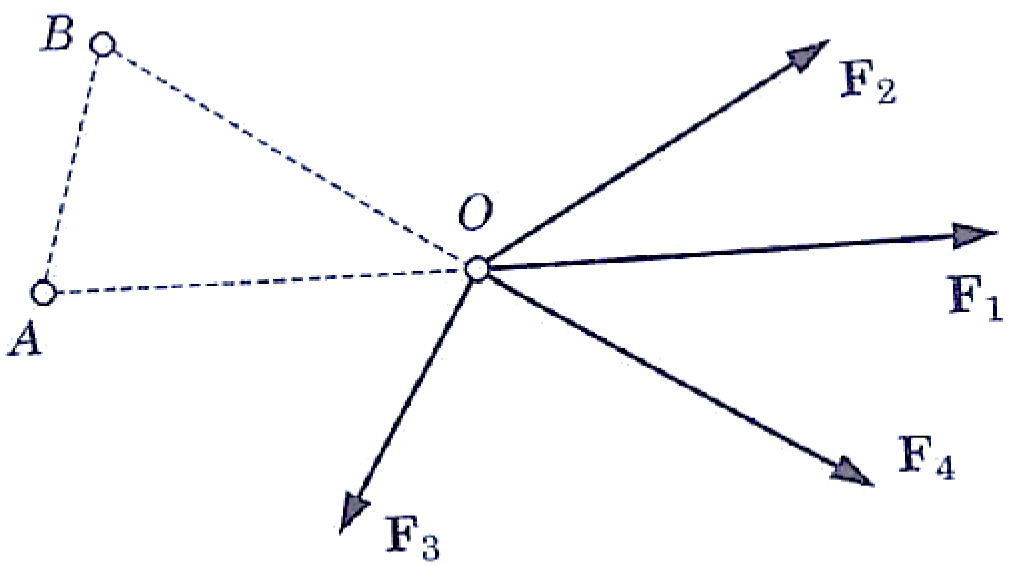
$$\frac{P\sin\theta}{P\cos\theta} = tan\theta = \frac{0.766 + 0.342}{0.94 - 0.643} = 3.703$$

So, $\theta = 75^o$

Thus, $P = \frac{3.5\,(0.94 - 0.643)}{\cos 75} = \frac{1.04}{0.259} = 4.0\ kN$

EQUATIONS OF EQUILIBRIUM

- **System of coplanar concurrent forces.**
- Suppose that the algebraic sum of the moments of a system of coplanar concurrent forces is zero. To possibilities that exist are:-
  a. The resultant of the system of forces is zero and the forces are in equilibrium.
  b. The moment centre lies on the line of action of the resultant.
- To clarify the situation, let us assume that a number of concurrent forces $\mathbf{F_1}$, $\mathbf{F_2}$, $\mathbf{F_3}$ and $\mathbf{F_4}$ are acting at a point O.



- Let the algebraic sum of the moments of these forces about any point A be zero, i.e.,
- $\mathbf{\Sigma M_A = 0}$, then either
  a) Resultant of forces is zero or
  b) Point A lies on the line of action of the resultant. Which implies that the resultant lies along the line OA.

EQUATIONS OF EQUILIBRIUM

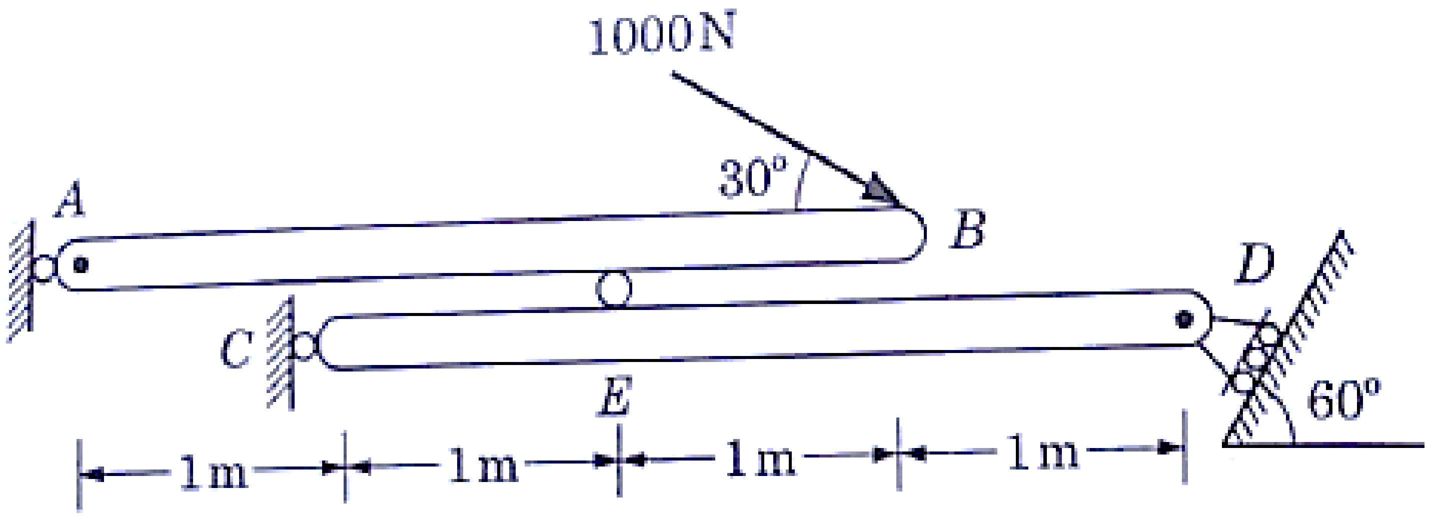
- Let us now choose any other point B and find the sum of the moments of these forces about this point B.
- Suppose it is also zero, i.e., $\Sigma M_B = 0$**, then either**

  (a) Resultant force is zero or (b) Line of action of resultant lies along OB.

- Since resultant cannot have two lines of actions, i.e., along OA as well as OB so resultant must be zero.
- Therefore, $\Sigma M_A = 0$ **and** $\Sigma M_B = 0$ (Moment equation of equilibrium)
- Thus, resultant of system of concurrent forces is zero or system is in equilibrium.
- Provided the points A and B (or the two moment centres) do not lie on the straight line passing through the point of concurrence of the force system.
- Earlier we derived $\Sigma F_x = 0$ **and** $\Sigma F_y = 0$
- Both sets of equations are equivalent and choice depends on problem at hand.
- Sometimes moment equations of equilibrium can offer certain advantage by way of eliminating an unknown reaction or a force provided the moment centre is chosen to lie on the line of action of that force.

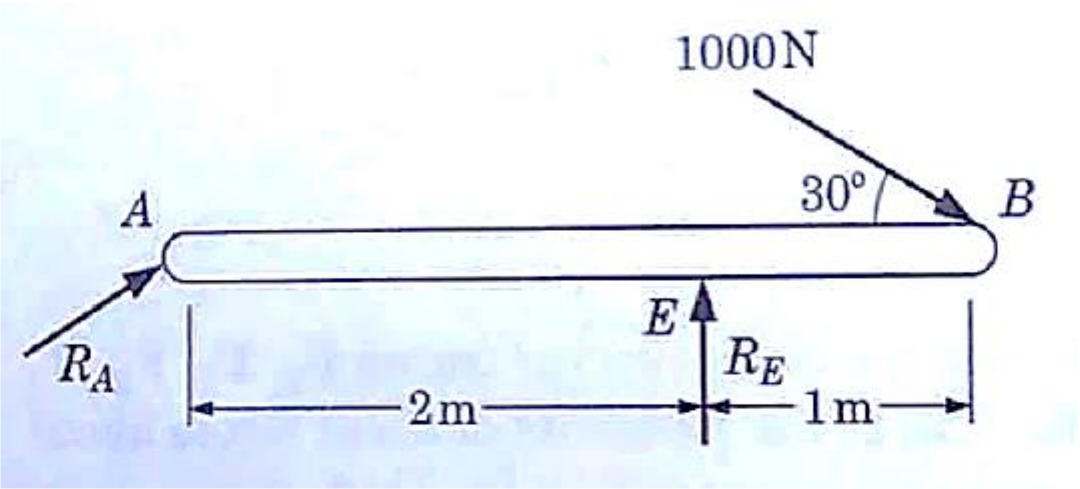
EXAMPLE: EQUATIONS OF EQUILIBRIUM

- Two beams AB and CD are arranged and supported as shown. Find the reaction at D due to force of 1000N acting at B as shown in figure a.



SOLUTION

- Solution. Consider the free body diagram of the beam AB, where R_A is the reaction at A and R_E is the reaction at E.

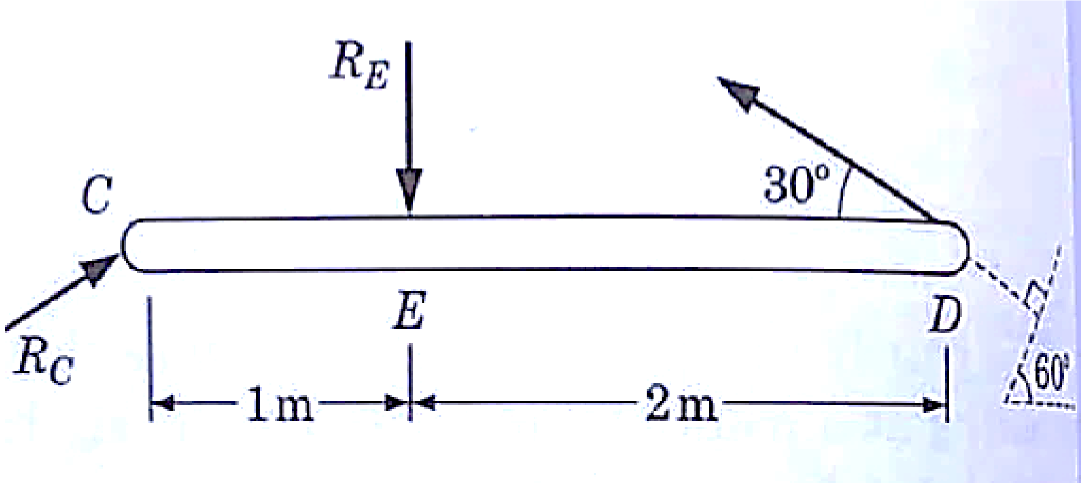


- Taking moment about A,

$\Sigma M_A = 0$: $R_E * 2 - 1000\,(3\sin 30^\circ) = 0$

$R_E = (3000 * 0.5)/2 = 750N$

- Considered the FBD of the beam CD, where R_C is the reaction at C and R_D is the reaction at D.



- Taking moment about C,

$\Sigma M_C = 0$: $R_D * (3\sin 30^\circ) - R_E(1) = 0$

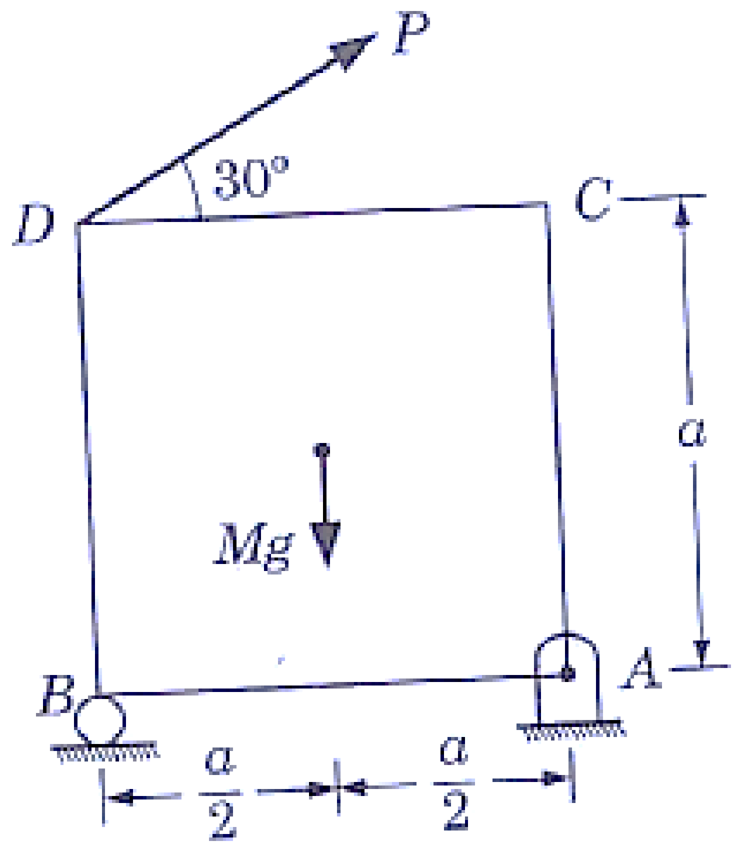
$R_D * (3\sin 30^\circ) - 750(1) = 0$

$R_D = 750 / (3\sin 30^\circ) = 750 / (3*0.5)$

$R_D = 500N$

EXAMPLE: EQUATIONS OF EQUILIBRIUM

- A square block of wood of mass M is hinged at A and rest on a roller at B. It is pulled by means of a string attached at D and inclined at an angle 30 degree with the horizontal. Determine the force P which should be applied to the string to just lift the block off the roller.



SOLUTION

- Solution. The FBD of the block is as shown in figure.
- **Various forces acting are:** (i) Force P, (ii) Weight of the block M_g
  (iii) Reaction R_B and (iv) Reaction at the hinge R_A
- When the block is at the point of being lifted off the roller B, it shall no longer be in contact with the roller. So. R_B =0.
- Reaction R_A at the hinge can be eliminated by writing equation of equilibrium as:

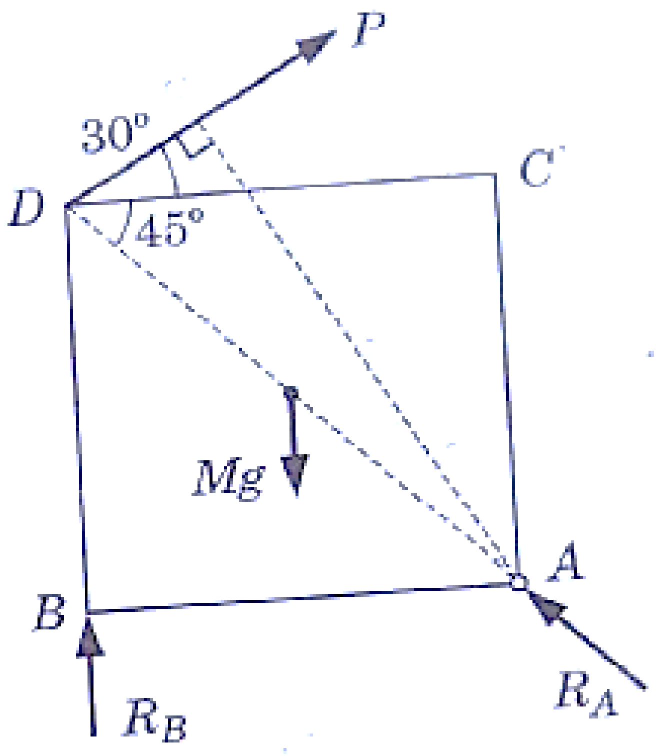
$\Sigma M_A = 0$: $$M_g\left(\frac{a}{2}\right) - \mathrm{P}\,(\mathrm{AD}\sin 75°) = 0$$

But $$AD = \sqrt{AC^2 + CD^2} = \sqrt{a^2 + a^2} = a\sqrt{2}$$

$$M_g\left(\frac{a}{2}\right) - \mathrm{P}\left(a\sqrt{2}\sin 75°\right) = 0$$

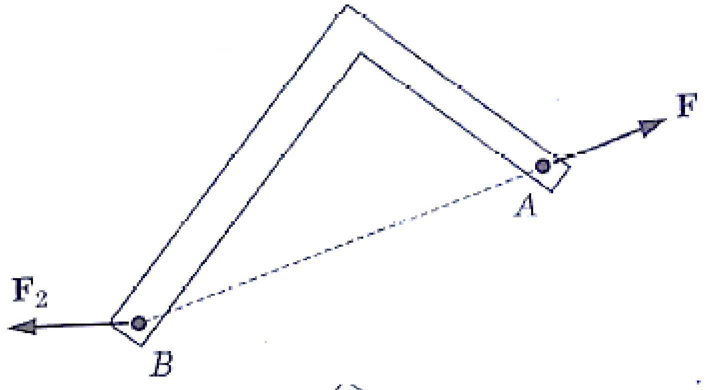
$$M_g\left(\frac{a}{2}\right) - \mathrm{P}\left(a\sqrt{2} * 0.966\right) = 0$$

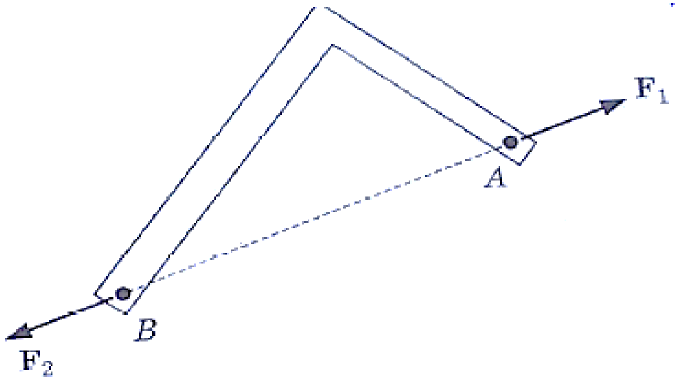
$$\mathrm{P} = 0.374 M_g$$



EQUILIBRIUM OF A BODY SUBJECTED TO TWO FORCES (TWO FORCE BODY)

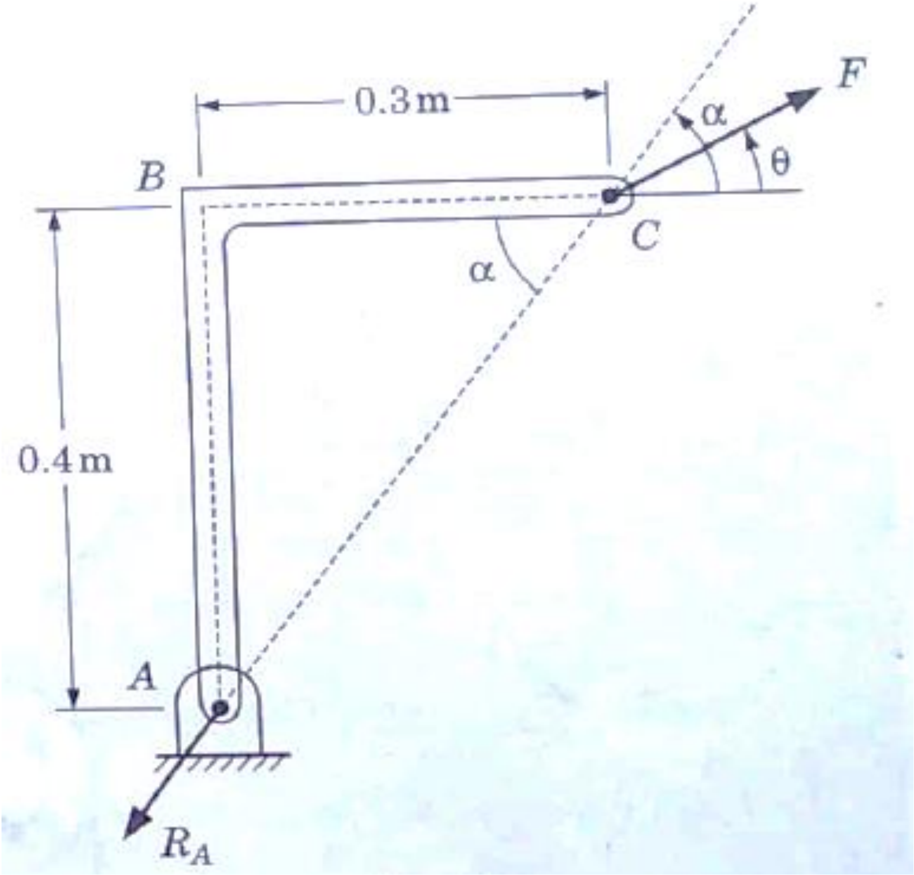
- If a rigid body is subjected to forces acting only at the two points, it is called a two force body. The equilibrium of a rigid body acted upon by only two forces, is of considerable interest and is being separately discussed.
- Consider a rigid body in the shape of a L-shaped plate acted upon by two forces F_1 and F_2 at the ends A and B respectively as shown in figure a.
- Two forces can be in equilibrium only if they are equal in magnitude but opposite in direction and have the same line of action.
- So, the forces F_1 and F_2 as shown in the figure a cannot keep the body in equilibrium when $F_1 = F_2$.
- Where as, forces F_1 and F_2 as shown in figure b, can keep the body in equilibrium when $F_1 = F_2$.
- A prismatic bar AB, with two forces acting at its ends A and B can be in equilibrium if two forces F_1 and F_2 are equal in magnitude but opposite in direction and are collinear with the center line of the bar as shown in figure c.





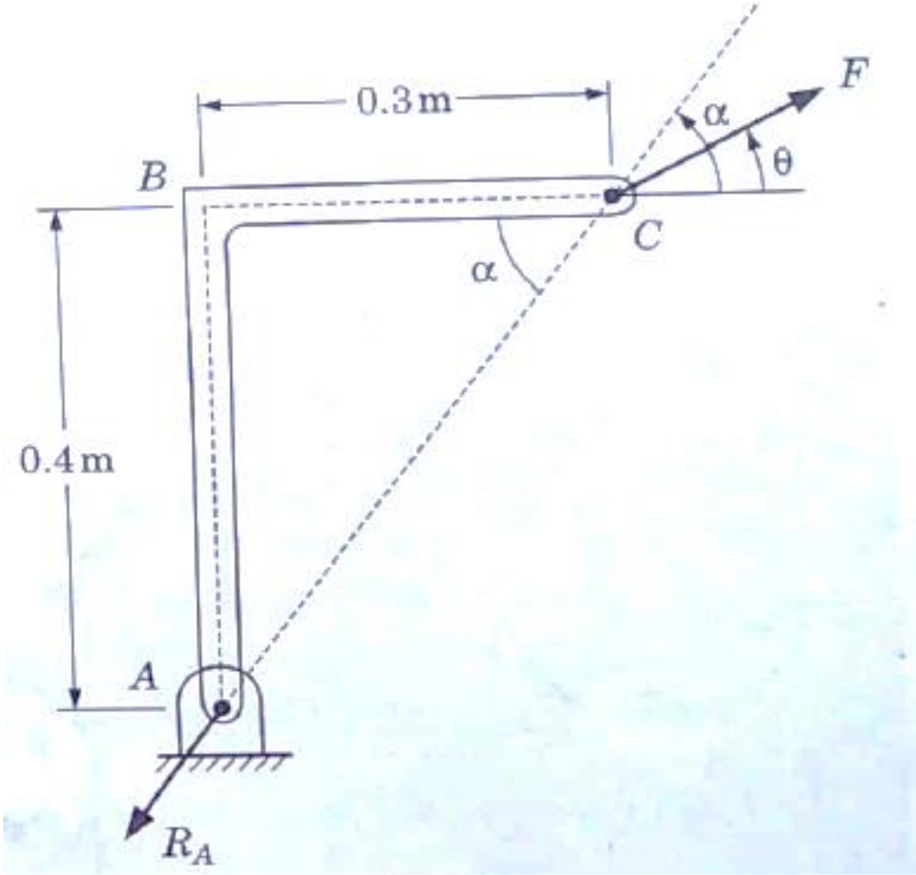
EXAMPLE: EQUILIBRIUM OF A BODY SUBJECTED TO TWO FORCES (TWO FORCE BODY)

- A body ABC of negligible weight is hinged at A with a force F acting at its end C. Determine the angle θ which this force should make with the horizontal to keep the edge AB of the body vertical.



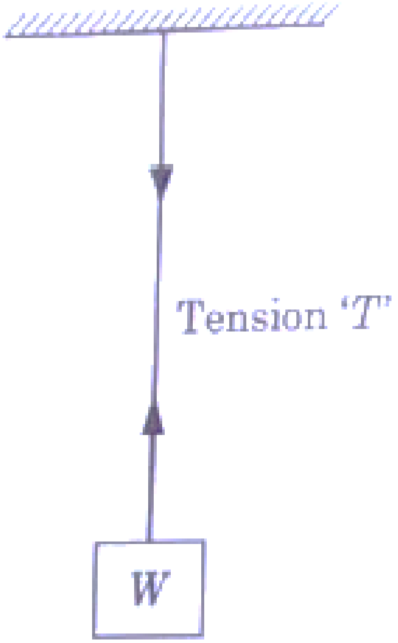
SOLUTION

- The body ABC is a two force body.
- At point C force F is acting at angle θ with the horizontal and at point A, reaction R_A of hinge is acting in an unknown direction.
- These two forces should be equal, opposite and collinear for the body to be in equilibrium.
- To be collinear both forces should act along the line joining points A and C such that the edge AB remains vertical.
- Therefore, $\theta = \alpha$
- $tan\alpha = \frac{0.4}{0.3} = 3.333$
- α = 53.13°



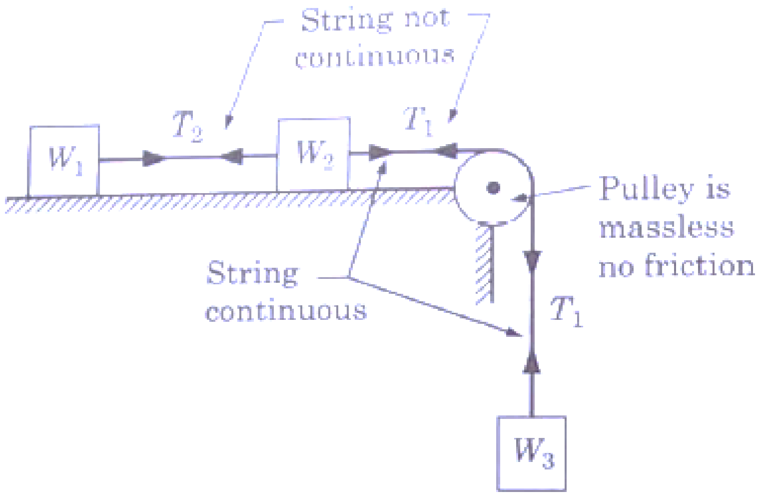
TENSION IN THE STRING, ROPE, BELT, CABLE AND CHAIN

- Consider a weight W shown in figure a, supported at the end of a string attached to a fixed support.
- Tension in the string is an internal force marked as shown.
- In a container string, rope, belt, cable and chain passing over a pulley etc., the tension remains same throughout provided,
  - ❖ String, rope, cable and chain are assumed to be inextensible and massless
  - ❖ Pulleys are assumed to be massless
  - ❖ Frictionless conditions are assumed to exist.

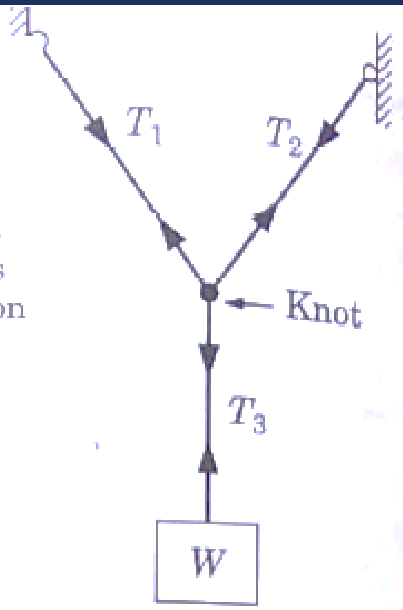


(a)

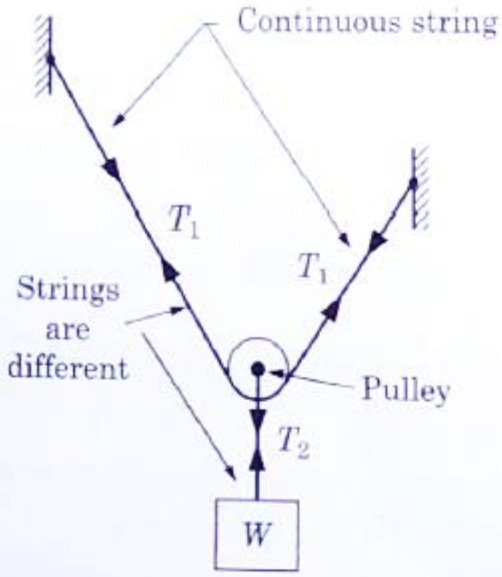
TENSION IN THE STRING, ROPE, BELT, CABLE AND CHAIN



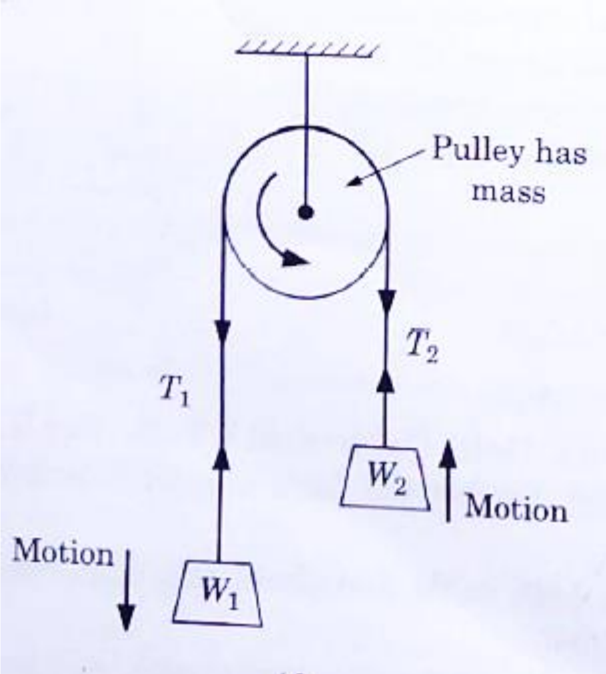
(b)



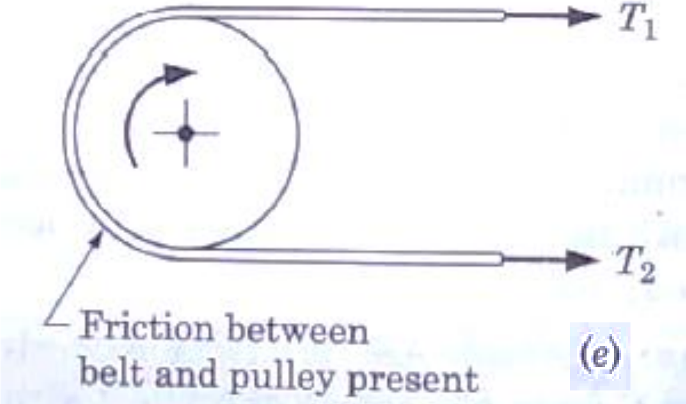
(c)



(d)



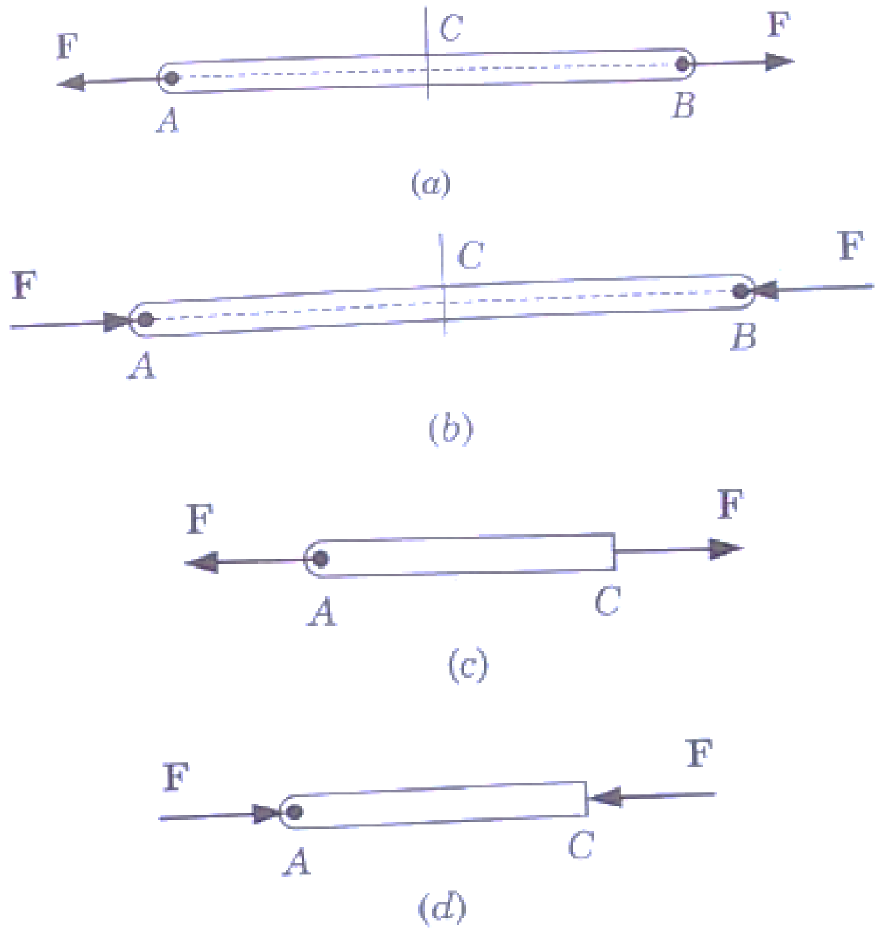
(f)



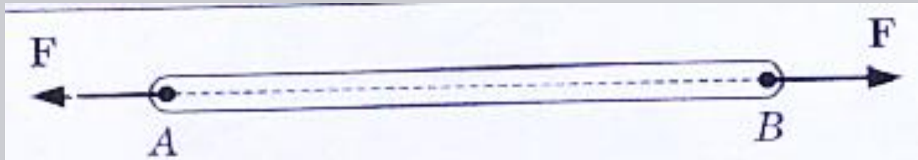
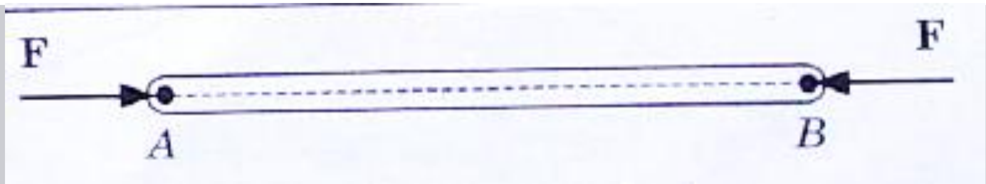
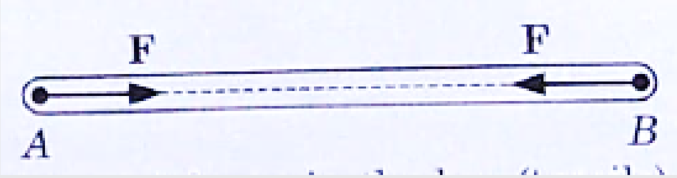
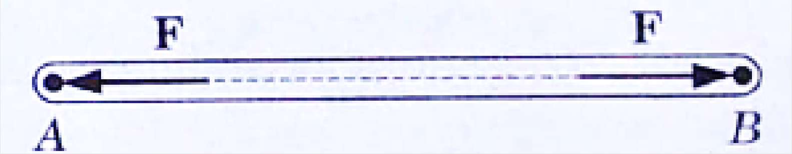
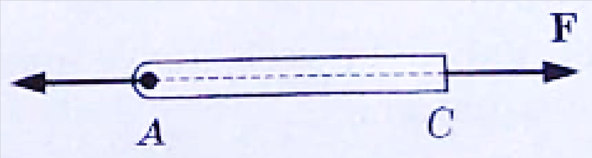
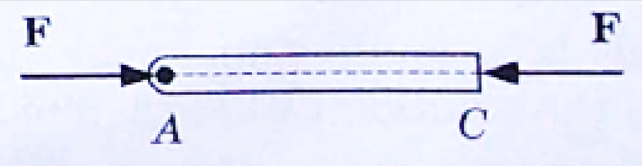
(e)

REPRESENTATION OF AXIAL FORCES IN BAR

- Consider a bar AB with two equal and opposite forces acting at its ends A and B.
- If this bar is in equilibrium, these forces must be collinear with the geometric axis of the bar and this bar is a two force member in equilibrium.
- In figure a, the external forces acting in this fashion are trying to pull or elongate the bar. The bar here is said to be under tensile forces.
- In figure b, the external forces acting in this fashion are trying to push or shorten the bar. The bar here is said to be under compressive forces.
- Now let us cut the bar AB at C. The two portions of the bar is AC and CB are now in equilibrium because of the internal resisting force F coming into play. This internal force is equal and opposite to the external force F and is collinear with it. This is shown in figure c and d.

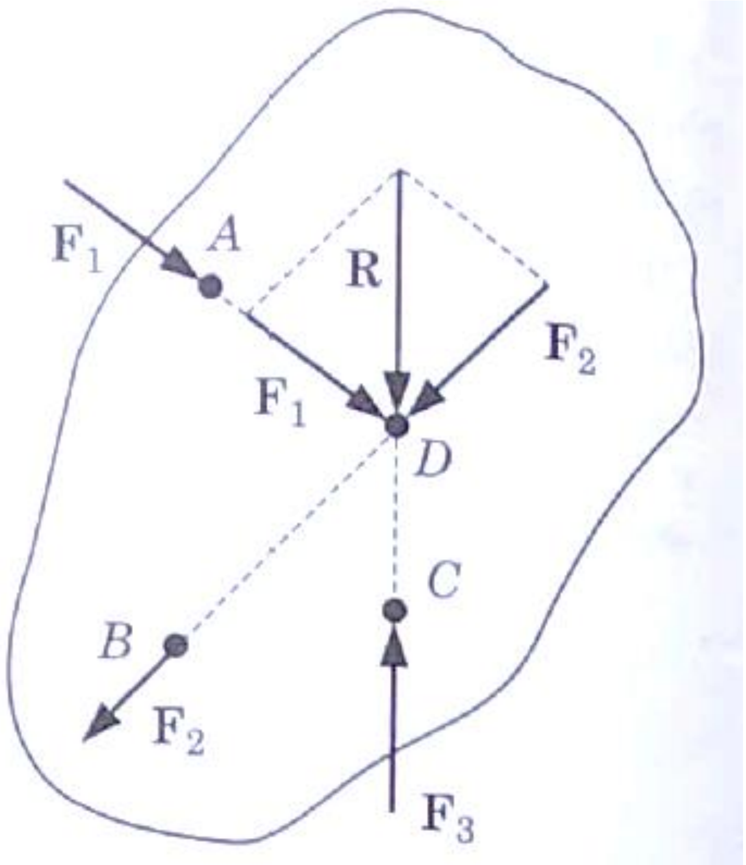


REPRESENTATION OF INTERNAL FORCES IN BAR

Tension	Compression
 External forces on the bar	 External forces on the bar
 Internal forces in the bar (tensile)	 Internal forces in the bar (compressive)
 A portion of the bar in equilibrium	 A portion of the bar in equilibrium

EQUILIBRIUM OF A BODY SUBJECTED TO THREE FORCES

- When a body is acted upon by three coplanar forces it can be in equilibrium if either the lines of action of the three forces intersect at one point (i.e., concurrent) or they are parallel.
- To prove the above statement let us consider a rigid body with three non parallel forces $\mathbf{F_1, F_2}$ **and** $\mathbf{F_3}$ acting at points A, B and C respectively.
- Let lines of action of forces F_1 and F_2 intersect at D.
- Transmit forces F_1 and F_2 to act through point D.
- Replace now forces F_1 and F_2 by their resultant R acting at point D.
- F_3 and resultant R of forces F_1 and F_2 can keep body in equilibrium if they have same lines of action or are collinear.
- So F_3 must also pass through point D.



EQUILIBRIUM OF A BODY SUBJECTED TO THREE FORCES

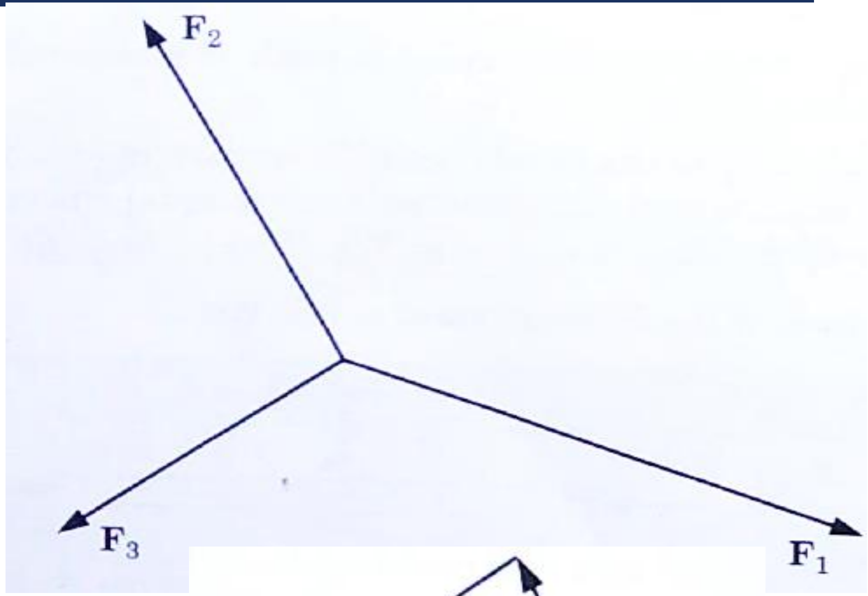
- So we can conclude that the three forces are concurrent. The triangle law for these forces can be stated as below,

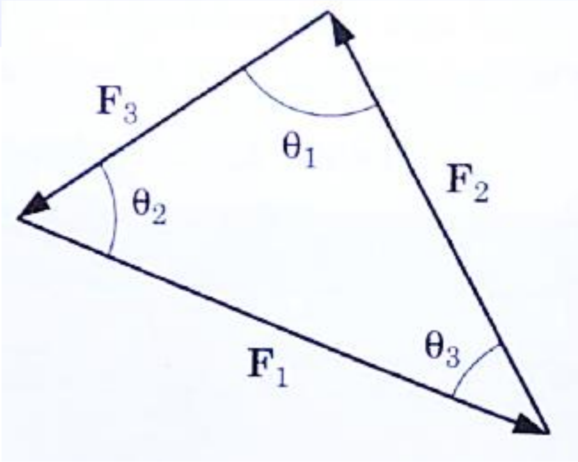
  "Three concurrent forces in equilibrium must form a closed triangle of force when drawn in head to tail fashion as shown in figure".

- Consider the triangle of force shown. Law of sines can be conveniently used to solve the problem. Thus,

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

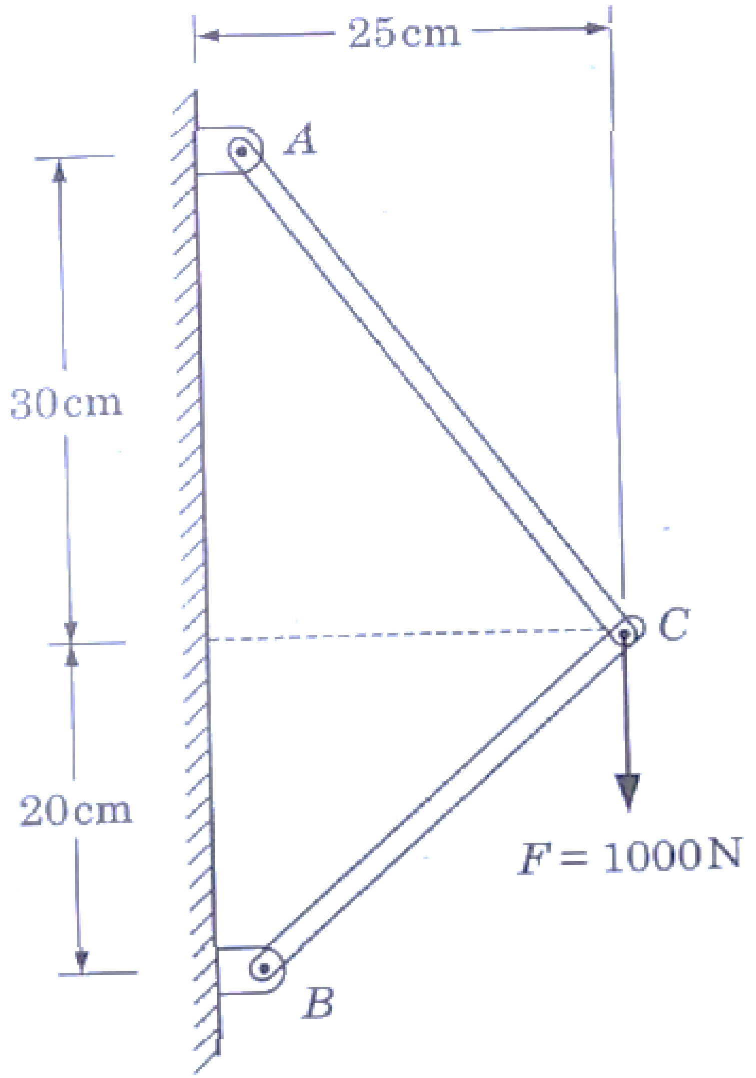
- If a body is in equilibrium under the action of three non parallel forces, the above method of an simplifies the solution rather than using the equilibrium equations.





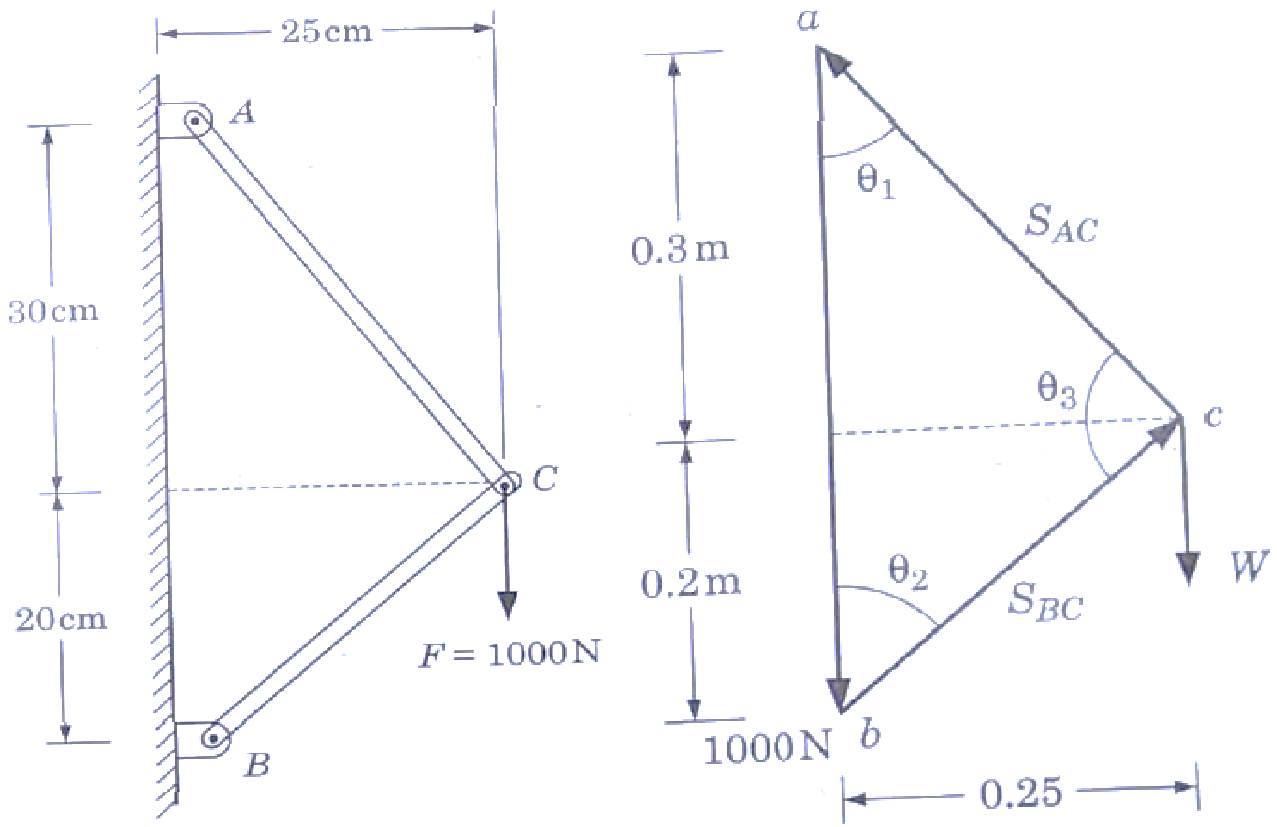
EXAMPLE: EQUILIBRIUM OF A BODY SUBJECTED TO THREE FORCES

- Two bars CB and AC are held together at C. Their other ends are hinged to a vertical wall at A and B as shown. Find the axial forces in the bar.



SOLUTION

- Axial forces included in the bars AC and BC are internal forces. But when we consider FBD of the point C these become the external forces acting at the point C.
- Let these be S_{CA} and S_{BC}.
- It is important to note here that each bar is acted upon by only two forces at its end. One of these forces is the component of 1000N and the other is reaction at hinge so each bar is a two force member.
- Free body of the point C is as shown in figure.
- As the point C is in equilibrium under the action of three forces they must be concurrent and should form a closed triangle of force abc similar to the triangle ABC.



SOLUTION

- $\frac{S_{AC}}{AC} = \frac{S_{BC}}{BC} = \frac{1000}{AB}$
- $\frac{S_{AC}}{0.3905} = \frac{S_{BC}}{0.3206} = \frac{1000}{0.5}$
- $S_{AC} = \frac{0.3905 * 1000}{0.5}$ = 780N
- $S_{BC} = \frac{0.3206 * 1000}{0.5}$ = 640N
- We can also apply sine law to the force triangle abc,
- $\frac{S_{AC}}{sin\theta 2} = \frac{S_{BC}}{sin\theta 1} = \frac{1000}{sin\theta 3}$
- $\frac{S_{AC}}{0.78} = \frac{S_{BC}}{0.64} = \frac{1000}{0.999}$
- S_{AC} = 780N
- S_{BC} = 640 N

where, $AC = \sqrt{(0.3)^2 + (0.25)^2}$

$AC = 0.3905m$

where, $BC = \sqrt{(0.20)^2 + (0.25)^2}$

$BC = 0.3206m$

where, $\sin\theta_1 = \frac{0.25}{0.3905} = 0.64$

$\theta_1 = 39.87°$

$\sin\theta_2 = \frac{0.25}{0.3206} = 0.78$

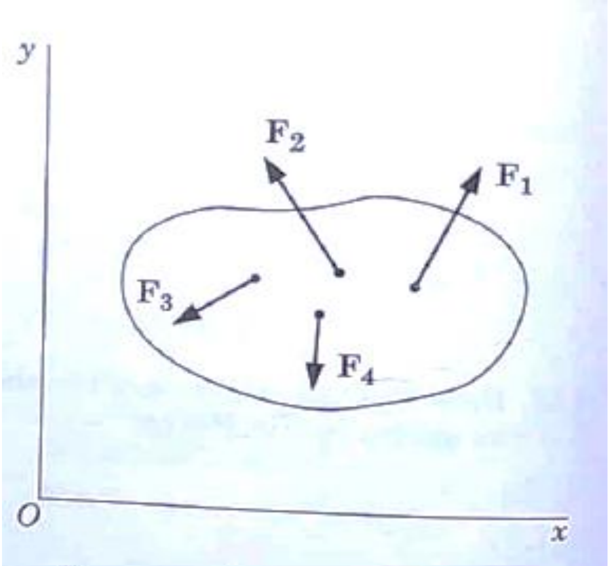
$\theta_2 = 51.26°$

$\theta_3 = 180 - (\theta_1 + \theta_2) = 89.87°$

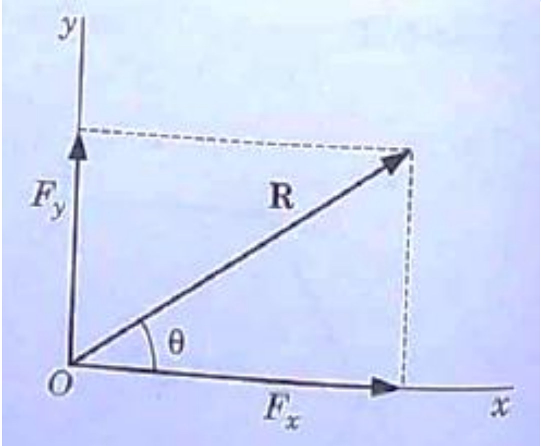
$\sin\theta_3 = 0.9999$

GENERAL CASE OF FORCES ACTING IN A PLANE: EQUATIONS OF EQUILIBRIUM

- If several forces acting in a plane are such that they do not intersect in one (i.e., are not concurrent) and are not parallel then they represent a general system of coplanar forces.
- Consider a body acted by several coplanar forces F_1, F_2, F_3,.....
- Let us choose some convenient co-ordinate axes x-y.
- Let F_{x1}, F_{x2}, F_{x3} represent the components of these forces along the x axis and F_{y1}, F_{y2}, F_{y3} represent their components along the y-axis.
- Now let us study the three possibilities that exist:



Case 1. Let the system of forces reduce to a single resultant force R whose components (along the x and y axes) are F_x and F_y respectively.

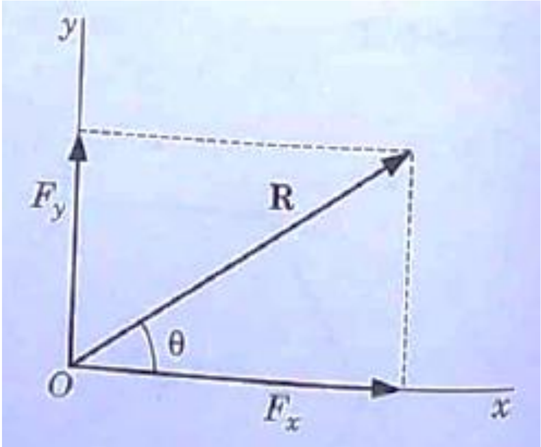


- The sum of the components of all the forces along the x axis = the component along the x-axis of their resultant.
- Therefore, $F_x = F_{x1} + F_{x2} + F_{x3} = \Sigma(F_x)_i$

GENERAL CASE OF FORCES ACTING IN A PLANE: EQUATIONS OF EQUILIBRIUM

- For the same reason, $F_y = F_{y1} + F_{y2} + F_{y3} = \Sigma(F_y)_i$
- Magnitude of the resultant, $R = \sqrt{F_x^{\,2} + F_y^{\,2}}$
- and its direction, $\theta = \tan^{-1}\frac{F_y}{F_x}$
- Line of action of the resultant can be determined using the Varignon's theorem.
- Choose the origin O as the moment centre.
- Let $(M_o)_1$, $(M_o)_2$, $(M_o)_3$ …. be the moments of the given forces w.r.t origin. M_o be the sum of the moments of all the forces about the same point.
- Moment of a force about a point = Sum of moments of its components about the same point.
- Therefore, $M_o = (M_o)_1 + (M_o)_2 + (M_o)_3 + \ldots$
- If the resultant lies at a distance d from the origin then, $M_o = R * d$

$$d = \frac{M_o}{R} = \frac{\sum(M_o)_i}{\sqrt{F_x^{\,2}+F_y^{\,2}}}$$



GENERAL CASE OF FORCES ACTING IN A PLANE: EQUATIONS OF EQUILIBRIUM

Case 2. The system of force reduced to a single couple.

- As a couple is a system of two equal and unlike parallel forces whose resultant is zero, therefore,

$$\Sigma(F_x)_i = \Sigma(F_y)_i = 0$$

- The moment M_o of the resultant is given by $M_o = \sum(M_o)_i$

Case 3. The system is in equilibrium. Then resultant force is zero and resultant couple is zero.

i.e.,

$$\Sigma(F_x)_i = \Sigma(F_y)_i = \sum(M_o)_i = 0$$

- Above equations express the conditions for the equilibrium of a body when acted upon by a general system of forces.
- The conditions of equilibrium can alternatively be expressed by **three moment equation** as,

$$\sum(M_A)_i = 0; \sum(M_B)_i = 0; \sum(M_C)_i = 0$$

GENERAL CASE OF FORCES ACTING IN A PLANE: EQUATIONS OF EQUILIBRIUM

$$\sum (M_A)_i = 0; \sum (M_B)_i = 0; \sum (M_C)_i = 0$$

- Where, $\sum(M_A)_i, \sum(M_C)_i, \sum(M_C)_i$ represent the algebraic sum of the moments of the forces acting on the body about the points A, B and C respectively. But the points A, B and C should not lie on a straight line.
- Thus, the three equations of equilibrium can determine only three unknowns such as,

1. The magnitude of three forces whose directions are known.
2. The magnitude and direction of one force and
3. The magnitude of the second force

- In the case of a system of concurrent forces and in the case of a system of parallel forces, only two of the above equations of equilibrium become available. Therefore in these cases only two unknowns can be determined.

QUESTIONS

- A rigid body is in equilibrium under the action of three forces. It implies that the forces must

  (a) be concurrent (b) be coplanar

  (c) either be concurrent or coplanar (d) pass through the centre of mass

- A rigid body is in equilibrium. Given that the moment of all the forces acting on the body about some axis is zero and also given that forces are concurrent, implies that

  (a) the resultant force is zero (b) forces have a line of action passing through the axis

  (c) resultant forces have a line of action parallel to axis (d) any of (a), (b), (c) can be true

- A body is acted upon by a force system. It can in general be brought to equilibrium by the application of

  (a) a force acting on a suitable point on the body (b) a force acting anywhere along a suitable line

  (c) force acting along a suitable line and moment along direction of force

  (d) a wrench acting anywhere on the body.

QUESTIONS

- Lami's theorem

(a) relates the forces with the sines of angles

(b) state that, for equilibrium under the action of three concurrent forces, there is a unique constant of proportionality between a force and the angle between the other two forces

(c) may be applied to consider a relationship between forces and angles of a polygon representation of forces

(d) may be applied for a body which may or may not be in equilibrium

- If the sum of all the forces acting on a body is zero, it may be concluded that the body

(a) must be in equilibrium (b) cannot be in equilibrium

(c) may be in equilibrium provided the forces are concurrent

(d) may be in equilibrium provided the forces are parallel

QUESTIONS

- A rigid body is in equilibrium under the action of three forces. It implies that the forces must **(b) be coplanar**
- A rigid body is in equilibrium. Given that the moment of all the forces acting on the body about some axis is zero and also given that forces are concurrent, implies that

  (a) the resultant force is zero (b) forces have a line of action passing through axis

  (c) resultant forces have a line of action parallel to axis **(d) any of (a), (b), (c) can be true**
- A body is acted upon by a force system. It can in general be brought to equilibrium by the application of

  (c) force acting along a suitable line and moment along direction of force
- Lami's theorem **(b) state that, for equilibrium under the action of three concurrent forces, there is a unique constant of proportionality between a force and the angle between the other two forces**
- If the sum of all the forces acting on a body is zero, it may be concluded that the body

(c) may be in equilibrium provided the forces are concurrent